Lec 5: multi-hadron properties

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Lecture content

- Multi-particle systems at finite temporal extent
- Medium effects and multi-hadron matrix elements
- Open Issues for the future
Multi-particle systems at finite temporal extent
Multi-pion correlation function

- Consider N pion correlation function

\[ C_n(t) \propto \left( \sum_x \pi^-(x, t) \right)^n \left( \pi^+(0, 0) \right)^n \]

- For a lattice of temporal extent \( T \) (inverse temperature)

\[
C_n(t) = \text{Tr} \left[ e^{-HT} \left( \sum_x \pi^-(x, t) \right)^n \left( \pi^+(0) \right)^n \right] \\
= \sum_m \left\langle m \left| e^{-HT} \left( \sum_x \pi^-(x, t) \right)^n \left( \pi^+(0) \right)^n \right| m \right\rangle \\
= \sum_{m, \ell} \left\langle m \left| e^{-HT} \left( \sum_x \pi^-(x, t) \right)^n \right| \ell \right\rangle \left\langle \ell \left| \left( \pi^+(0) \right)^n \right| m \right\rangle \\
= \sum_{m, \ell} \left\langle m \left| e^{-H(T-t)} \left( \sum_x \pi^-(x, 0) \right)^n e^{-Ht} \right| \ell \right\rangle \left\langle \ell \left| \left( \pi^+(0) \right)^n \right| m \right\rangle \\
= \sum_{m, \ell} e^{-E_m(T-t)} e^{-E_{\ell} t} Z_{\ell, m}
\]

- Many states contribute (ignore excitations)

\[ \{ |m\rangle = |0\rangle, |\ell\rangle = |n\pi\rangle \}, \{ |m\rangle = |\pi\rangle, |\ell\rangle = |(n-1)\pi\rangle \}, \ldots, \{ |m\rangle = |n\pi\rangle, |\ell\rangle = |0\rangle \} \]
Four pion correlation

t=0
Four pion correlation
Four pion correlation

\[ t = 0 \]
Four pion correlation

t=0
Four pion correlation

\[ Z_{4\pi} \left( e^{-4E_{4\pi} t} + e^{-4E_{4\pi} (T-t)} \right) \]
Four pion correlation

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Four pion correlation

\[ Z_{4\pi} \left( e^{-4E_{4\pi}t} + e^{-4E_{4\pi}(T-t)} \right) \]

\[ Z_{3/1\pi} \left( e^{-E_{3\pi}t} e^{-E_{1\pi}(T-t)} + e^{-E_{3\pi}(T-t)} e^{-E_{1\pi}t} \right) \]

\[ t=0 \]
Four pion correlation

\[ Z_{4\pi} \left( e^{-4E_{4\pi} t} + e^{-4E_{4\pi} (T-t)} \right) \]

\[ Z_{3/1\pi} \left( e^{-E_{3\pi} t} e^{-E_{1\pi} (T-t)} + e^{-E_{3\pi} (T-t)} e^{-E_{1\pi} t} \right) \]
Four pion correlation

\[ Z_{4\pi} \left( e^{-4E_{4\pi} t} + e^{-4E_{4\pi}(T-t)} \right) \]

\[ Z_{3/1\pi} \left( e^{-E_{3\pi} t} e^{-E_{1\pi}(T-t)} + e^{-E_{3\pi}(T-t)} e^{-E_{1\pi} t} \right) \]

\[ Z_{2/2\pi} e^{-E_{2\pi} t} e^{-E_{2\pi}(T-t)} = Z_{2/2\pi} e^{-E_{2\pi} T} \]

\( t=0 \)
Many meson 2-point correlator

- Consider $\pi^+$ correlator ($m_u=m_d$)

$$C^{(1)}(t) = \left\langle 0 \left| \sum_x \bar{d}\gamma_5 u(x, t)\bar{u}\gamma_5 d(0, 0) \right| 0 \right\rangle$$

$$t \rightarrow \infty \quad A_1 e^{-E_1 t}$$
Many meson 2-point correlator

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\[ \overset{t \text{ large}}{\longrightarrow} A_1 e^{-E_1 T} \cosh(E_1(t - T/2)) \]
Many meson 2-point correlator

- Now an $n \pi^+$ correlator ($m_u = m_d$)

\[
C^{(n)}(t) = \left\langle 0 \left| \sum_x \bar{d}_x \gamma_5 u(x, t) \bar{u}_0 \gamma_5 d_0 \right| 0 \right\rangle
\]

\[
t \to \infty \quad A_n e^{-E_n t}
\]
Many meson 2-point correlator

- Now an \( n \pi^+ \) correlator (\( m_u=m_d \))

\[
C^{(n)}(t) = \left\langle 0 \left| \left[ \sum_{\mathbf{x}} \bar{d}\gamma_5 u(\mathbf{x}, t)\bar{u}\gamma_5 d(0, 0) \right]^n \right| 0 \rightangle
\]

\[
t \to \infty \quad A_n e^{-E_n t}
\]
Many meson 2-point correlator

- Now an $n \pi^+$ correlator ($m_u = m_d$)

$$C^{(n)}(t) = \left\langle 0 \left| \left( \sum_x \bar{d} \gamma_5 u(x, t) \bar{u} \gamma_5 d(0, 0) \right)^n \right| 0 \right\rangle$$

$$t \overset{t \text{ large}}{\longrightarrow} \sum_{m=0}^\lfloor \frac{n}{2} \rfloor A_{n,m} e^{-(E_m + E_{n-m})T/2} \cosh \left( (E_m - E_{n-m})(t - T/2) \right)$$
Analysis on finite T correlators

- Can rewrite the t dependence as

\[ C_{n\pi}(t) = \sum_{m=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{m} A_m^n Z_m^m e^{-(E_{n-m}+E_m)T/2} \cosh((E_{n-m} - E_m)(t - T/2)) + \ldots \]

- Extracting the eigen-energies from these correlators is difficult
  - Many parameters appear in each correlator
  - Correlations between different C\textsubscript{j} as the energy E\textsubscript{k} occurs in all C\textsubscript{j} (j\geq k)
  - Various ways to deal with this: eg cascading fits
At no point does the ground state dominate the correlator!!!
Medium effects and matrix elements

- So far we have only investigated spectroscopy of multi-hadron systems
- What about the structure and other properties of such systems?
  - Moments, form factors, polarisabilities, weak interactions...
  - Probed by matrix elements in multi-hadron eigenstates
- What about in medium properties – how does a proton get modified in a nucleus (intrinsically not a well defined separation)?
  - Really an interpretation of the above
- Very new direction of investigation
Colour screening of static charges

- Static quark potential
Colour screening of static charges

- Static quark potential

V(r)

[Leinweber][Bali et al.][Petreczky/Petrov]
Colour screening of static charges

- Static quark potential
- Screening: evidence for quark-gluon plasma

[Leinweber][Bali et al.][Petreczky/Petrov]
Color screening

- Static quark potential

\[ C_W(R,t_w,t) = \left\langle 0 \left| \sum_{y, |\mathbf{r}|=R} \mathcal{W}(\mathbf{y} + \mathbf{r}, t; \mathbf{y}, t_w) \right| 0 \right\rangle \rightarrow Z \exp[-V(R)(t - t_w)] \]
Color screening

- Static quark potential

\[ C_W(R, t_w, t) = \left\langle 0 \left| \sum_{y, |r|=R} \mathcal{W}(y + r, t; y, t_w) \right| 0 \right\rangle \]

\[ \longrightarrow Z \exp[-V(R)(t - t_w)] \]
Color screening

• Static quark potential

\[ C_W(R, t_w, t) = \left\langle 0 \left| \sum_{y, |r|=R} \mathcal{W}(y + r, t; y, t_w) \right| 0 \right\rangle \]

\[ \longrightarrow Z \exp[-V(R)(t - t_w)] \]

• Modified by condensate? Hadronic screening?
Color screening

- Static quark potential

\[ C_W(R, t_w, t) = \left\langle 0 \left| \sum_{y, |r|=R} W(y + r, t; y, t_w) \right| 0 \right\rangle \]

\[ \rightarrow Z \exp[-V(R)(t - t_w)] \]

- Modified by condensate? Hadronic screening?
In medium effects

- n pion correlator
  \[ C_n(t_\pi, t) = \left\langle 0 \left| \left( \sum_x \chi_{\pi^+}(x, t)\chi_{\pi^+}^\dagger(0, t_\pi) \right)^n \right| 0 \right\rangle \]
  \[ \longrightarrow Z' \exp[-E_{n\pi}(t - t_\pi)] \]

- Wilson loop correlator
  \[ C_W(R, t_w, t) = \left\langle 0 \left| \sum_{y, |r| = R} \mathcal{W}(y + r, t; y, t_w) \right| 0 \right\rangle \]
  \[ \longrightarrow Z \exp[-V(R)(t - t_w)] \]

- Pions and Wilson loop
  \[ C_{n,W}(R, t_\pi, t_w, t) = \left\langle 0 \left| \left( \sum_x \chi_{\pi^+}(x, t)\chi_{\pi^+}^\dagger(0, t_\pi) \right)^n \sum_{y, |r| = R} \mathcal{W}(y + r, t; y, t_w) \right| 0 \right\rangle \]

- Ratio gives shift in potential due to interaction of potential with pion system
  \[ G_{n,W}(R, t_\pi, t_w, t) = \frac{C_{n,W}(R, t_\pi, t_w, t)}{C_n(t_\pi, t)C_W(R, t_w, t)} \]
  \[ \longrightarrow \# \exp[-\delta V(R, n)(t - t_W)] \]
Wilson line
source $t=t_w$

$n$-$\pi$
source $t=t_{\pi}$

Euclidean time
Effective $\delta V$ plots

$\delta V(R = 2b, n = 1) [\text{MeV}], N_{\text{HYP}}=1$

$\delta V(R = 6b, n = 1) [\text{MeV}], N_{\text{HYP}}=2$

$\delta V(R = 6b, n = 4) [\text{MeV}], N_{\text{HYP}}=4$

- DWF on MILC: $a=0.09$ fm, $28^3x96$, $m_\pi=318$ MeV

[WD & M Savage, PRL 102:032004, 2009]
$\delta V(R, n)$

- 100 keV effect at small $n$, small $R$
- Approximately linear in isospin density

[WD & M Savage, PRL 102:032004, 2009]
\[ \delta F(R, \ n=1 \ & \ 5) \]

[WD & M Savage, PRL 102:032004, 2009]

- Small effect: \( \delta F(n=1)/F = 2/1000 \) at large \( R \)
- Constant at large \( R \)
- Dielectric medium inside flux tube
Hadron structure in QCD

- Deep inelastic scattering experiments probe parton distribution functions $q_H(x)$
- Probability of finding a parton $(q,g)$ in hadron $h$ carrying longitudinal momentum fraction $x$
- Operator product expansion: Mellin moments of PDFs defined by forward matrix elements of local operators
  \[
  \langle x^n \rangle_H = \int_{-1}^{1} dx \, x^n q_H(x)
  \]
  \[
  \langle H | \bar{\psi} \gamma^{\mu_0} D^{\mu_1} \ldots D^{\mu_n} | H \rangle = p^{\mu_0} \ldots p^{\mu_n} \langle x^n \rangle_H
  \]
- $n=1$ corresponds to LC momentum fraction carried by quarks inside $H$
- Phenomenologically find DIS on nuclei
Hadron structure in QCD

- Proton structure intensively studied in QCD using 3-pt functions (see James Zanotti’s lectures next week)

\[
C_2(t, p) = \sum_x e^{ip \cdot x} \langle 0 | \chi_H(0) \chi_H^\dagger(x, t) | 0 \rangle \\
C_3(t, p) = \sum_{y, x} e^{ip \cdot x} \langle 0 | \chi_H(0) \mathcal{O}(y, \tau) \chi_H^\dagger(x, t) | 0 \rangle \\
R = \frac{C_3(t, p)}{C_2(t, p)} \xrightarrow{t \to \infty} \langle H | \mathcal{O} | H \rangle
\]

- Limited to low moments by reduced lattice symmetry
- Most studies for nucleon, but also pion, rho, ...
-Disconnected term often neglected (absent for isovector quantities)
- What about multi-baryon structure (EMC effect)?
Many meson 3-point correlator

[WD & H-W Lin, in progress]

- Pionic analogue of EMC effect
- \( n \pi^+ \) 3-point correlator

\[
C_m^{(n)}(\tau, t, p) = \left\langle 0 \left| \prod_{i=1}^{m} e^{i p_i \cdot x} \chi(x, t) \sum_y e^{i q \cdot y} \mathcal{J}(y, \tau) [\chi^\dagger(x_0)]^m \right| 0 \right\rangle
\]
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\]
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\[
C^{(n)}_m(\tau, t, p) = \left\langle 0 \left| \prod_{i=1}^{m} \sum_{x} e^{i p_i \cdot x} \chi(x, t) \right| \sum_{y} e^{i q \cdot y} \mathcal{J}(y, \tau) \left[ \chi^\dagger(x_0) \right]^m \right| 0 \right\rangle
\]

\[
\rightarrow Z_m \langle \mathcal{O}^{(n)}_m \rangle e^{-E_m t}
\]

where \( \langle \mathcal{O}^{(n)}_m \rangle = \langle m\pi|\mathcal{J}^{(n)}|m\pi\rangle \)
Many meson 3-point correlator

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\[
C_m^{(n)}(\tau, t, p) = \langle 0 \left| \prod_{i=1}^{m} \sum_{x} e^{i p_i \cdot x} \chi(x, t) \right| \sum_{y} e^{i q \cdot y} J(y, \tau) [\chi^\dagger(x_0)]^m | 0 \rangle \\
\rightarrow Z_m \langle \mathcal{O}_m^{(n)} \rangle e^{-E_m t}
\]

where \( \langle \mathcal{O}_m^{(n)} \rangle = \langle m\pi | J^{(n)} | m\pi \rangle \)
Many meson 3-point correlator

[WD & H-W Lin, in progress]

- Pionic analogue of EMC effect
- $n \pi^+$ 3-point correlator

$$C_m^{(n)}(\tau, t, p) = \langle 0 \mid \prod_{i=1}^{m} \sum_{x} e^{ip_i \cdot x} \chi(x, t) \sum_{y} e^{iq \cdot y} J(y, \tau) [\chi^\dagger(x_0)]^m \mid 0 \rangle$$

$$\longrightarrow \sum_{\ell=0}^{m} \binom{m}{\ell} Z_m^{(\ell)} \langle O_m^{(n)} \rangle e^{-E_m - \ell t} e^{-E_\ell (T-t)}$$

where $\langle O_m^{(n)} \rangle = \langle m\pi | J^{(n)} | m\pi \rangle$
Backwards propagator contamination

- Thermal contamination gets very bad near the midpoint of the temporal extent.
- Fraction of non-thermal contributions to 2pt correlator (T=64 here).

- Trying to measure three point function at $t>T/4$ is problematic – nothing to do with physically relevant state.
Many meson 3-point correlator

- Pionic analogue of EMC effect
- \( n \pi^+ \) 3-point correlator

\[
C_m^{(n)}(\tau, t, p) = \left< 0 \left| \prod_{i=1}^{m} e^{i p_i \cdot x} \chi(x, t) \right| \sum_y e^{i q \cdot y} J(y, \tau) [\chi^\dagger(x_0)]^m \right> \]

\[ \xrightarrow{\text{excitations and thermal effects}} Z_m \left< O_m^{(n)} \right> e^{-E_m t} \]

- Constructions performed by treating the struck meson as a separate species

\[
\Pi = \sum_x \gamma_5 S(x, t; 0) \gamma_5 S^\dagger(x, t; 0), \quad \tilde{\Pi} =_{x,y} \gamma_5 S(x, t; y, \tau)\Gamma O S(y, \tau; 0) \gamma_5 S^\dagger(x, t; 0)
\]

- System now looks like \((m-1)\) pions + 1 “kaon”

- Can be written as products of traces of two matrices

[WD & B Smigielski, arXiv:1103.4362]
Double ratio

- Define ratio to extract matrix elements (eg for momentum fraction)

\[ R^{(n)}(t, \tau) = \frac{C^{(n)}_3(t; \tau)}{C^{(n)}_2(t)} \xrightarrow{t \gg \tau} \frac{1}{E_{n\pi}} \langle n \pi^+ | O^{44} | n \pi^+ \rangle \]

- Double ratio – allows direct investigation of ratio of moments

\[ \frac{R^{(n)}(t, \tau)}{R^{(1)}(t, \tau)} \rightarrow \frac{m_\pi}{E_{n\pi}} \frac{\langle n \pi^+ | O^{44} | n \pi^+ \rangle}{\langle \pi^+ | O^{44} | \pi^+ \rangle} \rightarrow \frac{E_{n\pi}}{m_\pi} \frac{\langle x \rangle_{n\pi^+}}{\langle x \rangle_{\pi^+}} \]

- No need to renormalise operator!

- Calculate ratios for various quark masses [DWF valence on MILC sea]
Double ratio

DWF on MILC
\( m_\pi = 350 \text{ MeV} \)
\( a=0.12 \text{ fm}, 20^3 \times 64 \)
Medium modification

- Extracted ratio of moments is not unity – medium modification of pion structure

- Extension to baryons certainly possible but messier as usual!
Matrix elements in multi-hadron systems

- Many pion PDF moments are one example of matrix elements of multi-hadron systems

- Other theoretical investigations
  - WD & M Savage “Electroweak matrix elements in the two nucleon sector from lattice QCD” hep-lat/0403005
  - H Meyer, “Photodisintegration of a Bound State on the Torus“, 1202.6675
• Consider QCD in the presence of a constant background magnetic field

• Implement by adding term to the action (careful with boundaries)

• Shifts spin-1/2 particle masses

\[ M_{\uparrow\downarrow} = M_0 \pm \mu|B| + 4\pi \beta|B|^2 + \ldots \]

• Changing strength of background field allows \( \mu, \beta \) to be extracted

• Two nucleon states

• Levels split and mix

• Landau levels:

• Similar for electro-weak fields and twist-two fields
EFT two-body currents

- Two-body contributions

\[ \langle d | \mathcal{O} | d \rangle = \] \hspace{1cm} \begin{tikzpicture}
    \node at (0,0) [circle,fill,inner sep=0.5em,draw,ultra thick] {	extcolor{blue}{K_0}};
    \node at (2,0) [circle,fill,inner sep=0.5em,draw,ultra thick] {};
    \draw [->,thick] (0,0) -- (1,1); \draw [->,thick] (0,0) -- (1,-1);
\end{tikzpicture} \hspace{1cm} \begin{tikzpicture}
    \node at (0,0) [circle,fill,inner sep=0.5em,draw,ultra thick] {	extcolor{blue}{L_2}};
    \node at (2,0) [circle,fill,inner sep=0.5em,draw,ultra thick] {};
    \draw [->,thick] (0,0) -- (1,1); \draw [->,thick] (0,0) -- (1,-1);
\end{tikzpicture} + \ldots

- Magnetic moment: two body modification \( L_2 \)

\[ \mu_d = \frac{2}{1 - \gamma r_3} (\gamma L_2 + \kappa_0) \]

- Twist-two current: leading EMC effect \( \alpha_n \) (more complicated as necessary to include pions)

\[ \langle x^n \rangle_d = 2 \langle x^n \rangle_N + \alpha_n \langle d | (N^\dagger N)^2 | d \rangle + \ldots \]
EFT two-body currents

- Two-body contributions

\[
\langle d | \mathcal{O} | d \rangle = \langle x^n \rangle + \ldots
\]

- Magnetic moment: two body modification \( L_2 \)

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- Twist-two current: leading EMC effect \( \alpha_n \) (more complicated as necessary to include pions)

\[
\langle x^n \rangle_d = 2 \langle x^n \rangle_N + \alpha_n \langle d | (N^\dagger N)^2 | d \rangle + \ldots
\]
• Background field modifies eigenvalue equation for m=±1 states

\[ p \cot \delta(p) - \frac{1}{\pi L} S \left( \frac{L^2}{4\pi^2} [p^2 \pm e|B|\kappa_0] \right) \mp \frac{e|B|}{2} (L_2 - r_3\kappa_0) = 0 \]

• Asymptotic expansion of lowest scattering level

\[ E_{0}^{m=\pm 1} = \mp \frac{e|B|\kappa_0}{M} + \frac{4\pi A_3}{M L^3} \left[ 1 - c_1 \frac{A_3}{L} + c_2 \left( \frac{A_3}{L} \right)^2 + \ldots \right] \]

where \( \frac{1}{A_3} = \frac{1}{a_3} \pm \frac{e|B|L_2}{2} \)

• Mixes \(^1S_0\) and \(^3S_1\) m=0 states (coupled channels – but perturbative)

\[ \left[ p \cot \delta_1(p) - \frac{S_+ + S_-}{\pi L} \right] \left[ p \cot \delta_3(p) - \frac{S_+ + S_-}{\pi L} \right] = \left[ \frac{e|B|L_1}{2} + \frac{S_+ - S_-}{2\pi L} \right]^2 \]

where

\[ S_\pm = S \left( \frac{L^2}{4\pi^2} [p^2 \pm e|B|\kappa_1] + \ldots \right) \]

[WD & M] Savage Nucl Phys A 743, 170]
Energy levels in B field

EFT prediction for behaviour of $m=\pm 1$ energy levels

$|e B| = 1000 \text{ MeV}^2$
Energy levels in B field

EFT prediction for behaviour of $m=\pm 1$ energy levels

$|e B| = 1000 \text{ MeV}^2$

Matching to lattice measurement determines $L_2$
$np \rightarrow d\gamma: {^3S_1} - {^1S_0} m=0$

$|e B| = 4000 \text{ MeV}^2$

NB: box is asymmetric: $4 \times 4 \times 40 \text{ fm}^3$
\[ np \rightarrow d\gamma: {}^3S_1 - {}^1S_0 \ m=0 \]

\[ |e \ B| = 4000 \text{ MeV}^2 \]

Matching to lattice measurement determines \( L_1 \)

NB: box is asymmetric: 4x4x40 fm\(^3\)
\[ np \rightarrow d\gamma: ^3S_1-^1S_0 \quad m=0 \]

\[ |e B| = 500 \text{ MeV}^2 \]

Matching to lattice measurement determines \( L_1 \)

NB: box is asymmetric: 4x4x40 fm\(^3\)
\( \nu d \rightarrow np: EW \text{ BF} \)

\[ |g_W| = 6 \text{ MeV} \]

Matching to lattice measurement determines \( L_{1,A} \)

NB: box is asymmetric: 4x4x40 fm\(^3\)
Open issues
Noise

• Noise in QCD correlators is generically a problem – somehow related to the sign problem discussed in Gert Aarts' lectures

• There are hints that we can suppress noise for certain choices of correlation functions
  • How effectively can this be systematised?
  • Can this be done for large A systems that we afford to perform contractions for?

• Are we measuring things the most sensible way?

• David K will say a lot more about noise on Friday
Density of states

- One specific issue that is a bit frightening at the moment is the density of scattering states in multi-hadron systems.

- States far below thresholds are presumably OK, but how do we learn about d–d scattering?

- Back to Maiani-Testa No-go Theorem
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States far below thresholds are presumably OK, but how do we learn about d–d scattering?

Back to Maiani-Testa No-go Theorem.
Theoretical problems

- For large $A$ systems, how do we control the volume, lattice spacing, unphysical quark mass artefacts?
  - Maybe just empirically?
  - Can we have a better theoretical understanding?
- What other kinds of observables can we calculate?
Summary

- Nuclear physics and multi-hadron systems are a frontier for QCD calculations

- Major advances in the last few years ($A_{\text{max}}=2 \rightarrow A_{\text{max}}=28$)

- Definitely a difficult problem – noise, contractions, theoretical understanding,...

- Lots of possibilities for new calculations – new observables, new approaches

- Lots of room for improvements (theoretical, algorithmic and computational)

- How far can we go?
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