Using GW Data to Constrain Neutron Star EOS

Mohammad Al-Mamun and Andrew W. Steiner
July 18, 2019

Department of Physics and Astronomy
University of Tennessee, Knoxville
Introduction
Neutron Stars (NS) are the remnant of supernova explosions.

- NS’s are the densest object (except black holes).
- Typical NS has mass $1.4 \, M_{\odot}$ with radius 10-15 km.
- Density of a NS core is likely to be 3 to 4 times the nuclear saturation density.

The ultimate goal here is to find the EOS which can describe dense matter composition inside a NS. But,

- At present, Nuclear experiments can constrain the EOS up to the saturation density.
- Nuclear theory also cannot yet provide a proper description for the dense matter of NS core.

Fortunately, Tolman-Oppenheimer-Volkoff (TOV) equations provide an one-to-one correspondence between mass-radius curve and the EOS. So, we’ll be using this relationship in our model to constrain the desired quantities.
**Tolman-Oppenheimer-Volkoff (TOV) equations**, a set of coupled ODE, constrains the structure of a spherically symmetric body of isotropic material which is in static gravitational equilibrium.

For a spherically symmetric metric,

\[ ds^2 = -e^{2\phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \]  

the **TOV** equations has the form of

\[ \frac{dP}{dr} = -\frac{G\epsilon m}{r^2} \left(1 + \frac{P}{\epsilon}\right) \left(1 + \frac{4\pi Pr^3}{m}\right) \left(1 - \frac{2Gm}{r}\right)^{-1} \]  

\[ \frac{dm}{dr} = 4\pi r^2 \epsilon \]  

**Equation of State (EOS)** relates the energy density (\(\epsilon\)) with pressure P. A polytropic EOS has the form of

\[ \epsilon = \kappa P^{1+1/n} \]  

where

- \(\kappa\) = fitting const.  
- \(n\) = polytropic index.

The **EOS** also depends on the number density of protons, electrons and neutrons as well as some exotic high energy particles. Adding these interactions to the **EOS** can produce a realistic model of the star.
Methods
Quiescent low mass X-ray binaries (qLMXB) are the primary sources of spectral data for NS M and R data. List of qLMXBs used for parameter properties and atmosphere informations are given below\(^1\):

- X5
- \(\omega\) cen
- NGC6397
- NGC6304
- M13
- M30
- M28

We also have couple of inferences of mass-radius from PRE X-ray bursts\(^2\),

- SAX J1810.8–429
- 4U 1702–429
- 4U 1724–307

\(^1\) Steiner et. al. (2018); https://doi.org/10.1093/mnras/sty215.

\(^2\) J. Nattila et. al.; https://doi.org/10.1051/0004-6361/201527416.
Observables used in the prior distribution of GW170817\(^3\) data source properties paper are as follows:

chirp mass, \( M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \) \hspace{1cm} (5)

mass ratio, \( q = \frac{m_2}{m_1} \) \hspace{1cm} (6)

and linear combination of tidal deformabilities(\( \tilde{\Lambda} \)),

\[ \tilde{\Lambda} = f(m_1, m_2) \Lambda_1 + g(m_1, m_2) \Lambda_2 \] \hspace{1cm} (7)

here, \( m_1, m_2, \Lambda_1 \) and \( \Lambda_2 \) are the masses and tidal deformabilities of the NS binaries respectively. And for our models,

- we’ve selected low spin priors from the GW170817 inspiral.
- we’ve done bayesian analysis with these joint priors to produce the posterior distribution of the observables.

Results
Figure 1: $q$ vs $\tilde{\Lambda}$ from LIGO

Figure 2: $q$ vs $\tilde{\Lambda}$

Figure 2 represent the 2d-hitogram for $q$ and $\tilde{\Lambda}$. The peak densities occur at an approximate value of 0.85 for $q$, which is in good agreement with LIGO results shown in figure 1.
2D-histogram plot for $\Lambda_2$ vs $\Lambda_1$ is given in figure 3 which is compared with the LIGO’s version in figure 4. We can see that $\Lambda_2$ is shifted to a higher value with our combined prior.

Figure 3: $\Lambda_2$ vs $\Lambda_1$

Figure 4: $\Lambda_2$ vs $\Lambda_1$ from LIGO
Mass-Radius Curve for NS

In figure:5, we’ve generated Mass-Radius distribution with the dashed black lines showing the 90% confidence intervals. We can see that the radial range for a 1.4 solar mass NS is in between 11.5-12.8 km.

And in figure:6, we’ve shown the posterior distribution of pressure with varying energy densities. Here we can see that the maximum pressure for NS occurs at \( \epsilon = 550 \text{Mev/fm}^3 \).

**Figure 5: Mass-Radius Curve**

**Figure 6: log pressure vs energy density**
Concluding Remarks

- This work is one of the first joint X-ray and LIGO analysis to constrain **NS EOS**.
- Constrains on parameter space from our posterior distribution is in good agreement with LIGO results.
- Radial range for 1.4M\(_\odot\) **NS** is approximately 11.5-12.8 km.
- We would also like to use **GW** data as standalone prior and compare the results with our recent work.
Thank you