Instabilities in collective neutrino transformations

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Introduction

• $\sim 10^{58}$ neutrinos in $\sim 10$ seconds $\Rightarrow$ dense neutrino medium $\Rightarrow$ collective neutrino flavor transformation, a quantum collective phenomenon mediated by the weak force on scales $\sim 10$-$1000$ km.

• Based on Standard Model physics.

• Can affect many aspects of SN physics (neutrino signals, nucleosynthesis, dynamics? …).
Neutrino Flavor Transport in a Dense Medium

\[ (\partial_t + \mathbf{v} \cdot \nabla) \rho = -i[H, \rho] + C \]

\[ H = \frac{M^2}{2E} + \sqrt{2}G_F \text{diag}[n_e, 0, 0] + H_{\nu \nu} \]

\[ H_{\nu \nu} = \sqrt{2}G_F \int d^3p' (1 - \mathbf{v} \cdot \mathbf{v}') (\rho_{p'} - \bar{\rho}_{p'}) \]
Oscillations in SN

$$H = \frac{M^2}{2E} + \sqrt{2} G_F \, \text{diag}[n_e, 0, 0] + H_{\nu\nu}$$
Numerical Models

Coherent forward scattering outside neutrino sphere

\[ \rho(t; r, \Theta, \Phi; E, \nu, \varphi) \]
Numerical Models

Stationary emission

\[ \rho(r, \Theta, \Phi; E, \theta, \phi) \]
Numerical Models

Axial symmetry around the Z axis

\[ \rho(r, \Theta; E, \vartheta, \varphi) \]
Numerical Models

Spherical symmetry about the center (inconsistent?)

$$\rho(r; E, \vartheta, \varphi)$$
Numerical Models

Azimuthal symmetry around any radial direction

$\rho(r; E, \psi)$

Bulb model
\[ \delta m^2 = -3 \times 10^{-3} \text{ eV}^2 \simeq \delta m^2_{\text{atm}}, \quad \theta_v = 0.1, \quad L_\nu = 10^{51} \text{ erg/s} \]

Duan, Fuller, Carlson & Qian (2006)
Numerical Models

Trajectory independent neutrino flavor evolution

$\rho(r; E)$

Single-angle model

*Equivalent to the expansion of a homogeneous, isotropic gas*
The orientation of trinos are in the collective mode even in the region where the formation in the hot bubble if (1) neutrinos and antineutrinos, stays aligned or antialigned with initial conditions, and will be either aligned or antialigned when $\nu_e$ approaches zero. We define $\nu_e$ as $<E_{\nu_e}> <E_{\nu_\tau}>$, where $\nu_{\text{eff}}$ is the flavor-averaged energy of the total neutrino. There can be a sharp transition in neutrino multi-angle and single-angle calculation results. (Duan+ 2006)
Flavor pendulum

Mono-energetic $\nu - \bar{\nu}$ gas

$\sigma \sim \frac{n_\nu - n_{\bar{\nu}}}{n_\nu + n_{\bar{\nu}}}$

$Mg \sim \frac{B}{n_\nu + n_{\bar{\nu}}}$

Hannestad+ (2006)
Duan+ (2007)
Flavor pendulum

Inverted Mass Hierarchy

$n^\text{tot}_\nu \downarrow, \quad M g \uparrow$

Hannestad+ (2006)
Duan+ (2007)
Dimension matters

Duan & Friedland (2010)
Flavor Instabilities

Electron flavor neutrinos and antineutrinos initially

$$\rho \propto \begin{bmatrix} 1 & \epsilon \\ \epsilon^* & 0 \end{bmatrix} \quad \bar{\rho} \propto \begin{bmatrix} 1 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$$

$$i\partial_z \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix} = \nu_z^{-1} \begin{bmatrix} -\eta\omega - \alpha\mu & \alpha\mu \\ -\bar{\eta}\omega & \eta\omega + \mu \end{bmatrix} \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix}$$

$$\eta \quad \text{Hierarchy}$$

$$\omega \quad \text{Osc. Freq.}$$

$$\alpha = n_{\bar{\nu}} / n_{\nu}$$

$$\mu \propto n_{\nu}$$

Flavor Instabilities

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\[ i \partial_z \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix} = \nu_z^{-1} \begin{bmatrix} -\eta \omega - \alpha \mu & \alpha \mu \\ -\mu & \eta \omega + \mu \end{bmatrix} \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix} \]

- Normal modes \( \rightarrow \) Collective oscillations \( (\epsilon, \bar{\epsilon} \sim e^{-i\Omega z}) \)

- \( \kappa = \text{Im}(\Omega) > 0 \rightarrow \) Flavor instabilities

\[ \eta \quad \text{Hierarchy} \]
\[ \omega \quad \text{Osc. Freq.} \]
\[ \alpha = \frac{n_{\bar{\nu}}}{n_{\nu}} \]
\[ \mu \propto n_{\nu} \]

Directional Symmetry

time independent, x translation symmetry, left-right symmetry

\[ \epsilon_{\pm} = \frac{\epsilon_L \pm \epsilon_R}{2} \]

\[ \begin{bmatrix} \epsilon_+ \\ \bar{\epsilon}_+ \\ \epsilon_- \\ \bar{\epsilon}_- \end{bmatrix} = \begin{bmatrix} \Lambda_+ \\ \Lambda_- \end{bmatrix} \begin{bmatrix} \epsilon_+ \\ \bar{\epsilon}_+ \\ \epsilon_- \\ \bar{\epsilon}_- \end{bmatrix} \]

Break left-right symmetry

Raffelt+ (2013), Duan (2015)
Directional Symmetry

\[
H_{\nu \nu} = \sqrt{2}G_F \int d^3p' (1 - \hat{v} \cdot \hat{v}') (\rho_{p'} - \bar{\rho}_{p'})
\]

\[
(1 - \hat{v} \cdot \hat{v}') = 4\pi \left[ Y_{0,0}(\hat{v})Y_{0,0}^*(\hat{v}') - \frac{1}{3} \sum_{m=0,\pm1} Y_{1,m}(\hat{v})Y_{1,m}^*(\hat{v}') \right]
\]

- Monopole \((l=0)\) and dipole \((l=1)\) modes are unstable in opposite neutrino mass hierarchies.
- Unstable dipole \((l=1)\) modes break the directional symmetry.

Duan (2013)
Inverted Hierarchy

\[ |\bar{q}_{00}|, |\bar{q}_{10}|, |\bar{q}_{1c}|, |\bar{q}_{1s}| \]

\[ \tau \]

Duan (2013)
Normal Hierarchy

\[ |\tilde{q}_{00}| \]
\[ |\tilde{q}_{10}| \]
\[ |\tilde{q}_{1c}| \]
\[ |\tilde{q}_{1s}| \]

Duan (2013)
Directional Symmetry

\[ \rho(r; E, \vartheta) \]

\[ \rho(r; E, \vartheta, \varphi) \]
FIG. 5: Case A. MZA and MAA flavor evolution for ν’s (left panel) and ¯ν’s (right panel) in NH (upper panels) and IH (lower panels). Energy spectra initially for νe (black dashed curves) and νx (light dashed curves) and after collective oscillations for νe (black continuous curves) and νx (light continuous curves).

Indeed we have seen with the stability analysis that the MAA instability is more suppressed with respect to the bimodal one. Indeed ¯Pϕ remains identically null. In conclusion in this case, the flavor evolution is governed by the only bimodal and MZA instabilities.

The evolution of the gω is shown in Fig. 8 with the same format of Fig. 4. We realize that in the SZA case the MAA effects would produce flavor decoherence with a swap function s(ω) tending to zero. This is consistent with what found in [34] for the case of small flavor asymmetry. Conversely in the MZA case, since MAA effects are suppressed one recovers the splitting configurations already found in the axial symmetric case [22, 26].

The oscillated spectra for e and x flavor are shown in Fig. 9. These are very similar to what shown in [22, 26] to which we address the reader for a detailed discussion. Here we only remark that MZA effects produce a smearing of the splitting features with respect to what found in the SZA case.

IV. CONCLUSIONS

It has been recently pointed out that removing the axial symmetry in the SN self-induced neutrino flavor evolution, MAA effects can trigger new flavor conversions. In this paper we have investigated the dependence of the MAA effects on the initial supernova neutrino fluxes. We performed a stability analysis of the linearized equations of motion to classify the cases unstable under the MAA effects. Then, we looked for a local solution along a given line of sight, working under the assumption that the transverse variations of global solution are small. If one considers neutrino fluxes typical of the accretion phase, with a pronounced νe excess, de facto behaving like spectra with a single crossing, the MAA effects produce an instability in NH. In our simplified scheme for the flavor evolution, this seems to produce spectral swaps and splits very similar to what produced by the bimodal instability in IH, as expected from the recent analysis of [32].

Another interesting spectral class is the one with neutrino fluxes typical of the central post-bounce phase, when the MAA effects are expected to be more pronounced. In this case, the spectral features are significantly altered, with the emergence of new spectral lines and significant modifications of the existing ones. This suggests that the MAA effects play a crucial role in the flavor evolution of neutrinos during the SN core collapse.

Chakraborty, Mirizzi (2013)
Spatial Symmetry

time independent, x translation symmetry, left-right symmetry

\[
\epsilon_m^{\pm} = \frac{1}{L} \int_0^L \left[ \frac{\epsilon_L(x) \pm \epsilon_R(x)}{2} \right] e^{-i(2\pi m)(x/L)} \, dx
\]

\[
\begin{bmatrix}
\epsilon_m^+ \\
\epsilon_m^- \\
\bar{\epsilon}_m^+ \\
\bar{\epsilon}_m^-
\end{bmatrix}
= \Lambda_m \cdot
\begin{bmatrix}
\epsilon_m^+ \\
\epsilon_m^- \\
\bar{\epsilon}_m^+ \\
\bar{\epsilon}_m^-
\end{bmatrix}
\]

Duan & Shalgar (2015)
Spatial Symmetry

$\alpha = n_\nu / n_\nu$

$\mu \propto G_F n_\nu$

Duan & Shalgar (2015)
Spatial Symmetry

\[ \rho(r; E, \vartheta, \varphi) \]

\[ \rho(r, \Theta, \Phi; E, \vartheta, \varphi) \]
Figure 1. Footprint of the MAA instability region in the parameter space of effective neutrino density $\mu = \frac{p}{2G_F n_\nu} \left( \frac{R}{r} \right)^2$, where $R$ is the neutrino-sphere radius, and matter density $\rho \propto L/r^4$, for the schematic SN model described in the text. Because $\mu / r^4$, the horizontal axis is equivalent to the distance from the SN as indicated on the lower horizontal axis. We also show a representative schematic SN density profile where the sharp density drop marks the shock wave. We also show the instability footprint explicitly for co-moving wave numbers $k = 10^2$ and $k = 10^3$ in units of the vacuum oscillation frequency. Notice that for the same value of $k$ there are two separate instability strips. The collection of all small-scale instabilities fill the gray-shaded region below the traditional $k = 0$ (blue shaded) instability region, whereas they leave the space above untouched. The rest of our paper is devoted to substantiating this main point and to explain our exact assumptions. We stress that our simplifications may be too restrictive to provide a reliable proxy for a realistic SN. In particular, we assume stationary neutrino emission and that the solution is stationary as well, i.e., we assume that the evolution can be expressed as a function of distance from the surface alone. We also ignore the “halo flux” caused by residual scattering which can be a strong effect. Our study would not be applicable at all in regions of strong scattering, i.e., below the neutrino sphere. We assume that the original neutrino flux is homogeneous and isotropic in the transverse directions, i.e., global spherical symmetry of emission at the neutrino sphere. It has not yet been studied if this particular assumption has any strong impact on the stability question, i.e., if violations of such an ideal initial state substantially change the instability footprint, or if such disturbances would simply provide seeds for instabilities to grow. It is impossible to understand and study all effects at once, so here we only attempt to get a grasp of the differential impact of including spatial inhomogeneities in the form of self-induced small-scale flavor instabilities. All the other questions must be left for future studies.
Temporal Symmetry

\[ \epsilon_\omega = \int \epsilon(t) e^{i\omega t} \, dt \]
Summary

• Research of collective neutrino oscillations in SNe is at a major turning point.

• Linear flavor-stability analysis suggests nontrivial results in multi-D models.

• The existence of “neutrino halo” (Cherry?) and fast modes (Raffelt) can further enrich the problem.

• Simulations with multi-spatial dimensions are necessary.