Dynamical pairing effects on soft quadrupole vibrations in deformed unstable nuclei close to the neutron drip line

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Collective vibrations ~ superposition of particle-hole excitations

stable nuclei

drip-line nuclei

coupling to continuum states
  • Excitations into the continuum
  • Pair correlations in the continuum

Mechanism for generation of collective modes in the new situation?
Dominance of single-particle excitation

Octupole excitations on SD state in $^{50}$S

K. Yoshida et al., PTP113 (2005), 1251

Deformed RPA without pairing

Single-particle excitation from weakly bound to resonant state is dominant.

$X_{ph} \approx 0.97$

Collective modes disappear??

Effect of pairing
QRPA calculations for investigation properties of excitation modes in neutron-rich nuclei

**Spherical nuclei**
- e.g. Matsuo et al., PRC71 (2005), 064326
- pairing: Hartree-Fock-Bogoliubov method
- continuum: Out-going boundary condition

**Deformed nuclei**
- Hagino et al., NPA731 (2004), 264c
- pairing: BCS approximation, neglect dynamical pairing
- continuum: Box boundary condition
- deformation: Expansion in spherical basis
Low-lying modes unique to neutron-rich nuclei

Quadrupole vibration of neutron skin

In deformed superfluid system close to the drip line

Effect of Deformation?  Pairing?  Continuum?

Soft $K=0^+$ mode?
Deformation Pairing Continuum

Directly solve HFB eq. in coordinate-space mesh-representation

H.O. basis

Spatially extended structure

First results of such a calculation
Investigation of deformed neutron-rich nuclei

**Ground state**

Coordinate-space HFB equation

\[
\begin{pmatrix}
h(r) - \lambda & \tilde{h}(r) \\
\tilde{h}(r) & -h(r) + \lambda
\end{pmatrix}
\begin{pmatrix}
\varphi_1(E, r) \\
\varphi_2(E, r)
\end{pmatrix}
= E
\begin{pmatrix}
\varphi_1(E, r) \\
\varphi_2(E, r)
\end{pmatrix}
\]

Mean-field  Deformed Woods-Saxon potential

\[
h = -\frac{\hbar^2}{2m} \nabla^2 + V_{WS}\rho + V_{SO}\nabla f \cdot (\sigma \times p)
\]

\[
f(\rho, z) = \frac{1}{(1 + \exp[(r - R(\theta))/a])}, \quad r^2 = \rho^2 + z^2
\]

Pair-field  Density-dependent delta interaction

\[
\tilde{h} = \frac{V_0}{2} \left(1 - \frac{\rho(r)}{\rho_0}\right)
\]

\[
V_0 = -450 \text{ MeV fm}^3, \quad \rho_0 = 0.16 \text{ fm}^{-3}
\]
Excited state

QRPA equation in the AB matrix formulation

\[
\sum_{\gamma\delta} \begin{pmatrix} A_{\alpha\beta\gamma\delta} & B_{\alpha\beta\gamma\delta} \\ B_{\alpha\beta\gamma\delta} & A_{\alpha\beta\gamma\delta} \end{pmatrix} \begin{pmatrix} X_{\gamma\delta}^\lambda \\ Y_{\gamma\delta}^\lambda \end{pmatrix} = \hbar \omega^\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X_{\alpha\beta}^\lambda \\ Y_{\alpha\beta}^\lambda \end{pmatrix}
\]

Residual interaction

p-h channel \[ v_{\text{ph}}(\mathbf{r}, \mathbf{r}') = [t_0 (1 + x_o P_\sigma) + \frac{t_3}{6} (1 + x_3 P_\sigma) \rho(\mathbf{r})] \delta(\mathbf{r} - \mathbf{r}') \]

p-p channel \[ v_{\text{pp}}(\mathbf{r}, \mathbf{r}') = V_0 \left(1 - \frac{\rho(\mathbf{r})}{\rho_0}\right) \delta(\mathbf{r} - \mathbf{r}') \]

Spurious modes should be zero excitation energy.

\[ \mathbf{v} \rightarrow f \cdot \mathbf{v} \]

Normalize the interaction strength
Deformed unstable nuclei

As an example of deformed neutron drip-line nuclei

Mg region

J.Terasaki et al., NPA621(1997)706
Isoscalar quadrupole transition strengths (intrinsic)

Low-frequency quadrupole vibrations in deformed Mg isotopes close to the drip line

Neutron number increasing

Enhancement of neutron excitation
Soft K=0\(^+\) mode in deformed \(^{40}\)Mg

\[
\left| \frac{M_n}{M_p} \right| \sqrt{\frac{N}{Z}} = \frac{6.78}{2.33} = 2.9
\]

Isoscalar \(^{40}\)Mg \(K^\pi=0^+\)

Unperturbed

Strength (fm\(^4\))

\[|X|^2 - |Y|^2\]

\[\langle \lambda | \hat{Q}_{20} | 0 \rangle = \sum_{\alpha \beta} M_{20}^{\alpha \beta} \]

\(h\omega=2.90\) MeV

1 W.u.=8.13 fm\(^4\)
Physically meaningful or box artificial?

Box calculation
Discretized spectrum even above the threshold

?
Structure of soft $K=0^+$ mode in $^{40}\text{Mg}$

Pairing

$|X_{\alpha\beta}| \geq 0.1$
Properties of positive energy states

\[ \Omega^\pi = 1/2^- \]

\[ \Omega^\pi = 3/2^- \]
Eigenphase sum \[ \Delta(E) = \sum_a \delta_a(E), \quad (U^\dagger SU)_{aa'} = e^{2i\delta_a(E)} \delta_{a,a'} \]

Hagino et al., NPA735(2004)55

Main 2qp excitations generating the K=0+ state are associated with one-particle resonant or bound states.

This K=0+ mode can be considered as a resonance.
Box size dependence

\[ \rho_{\text{max}} \times z_{\text{max}} = 10.0 \text{ fm} \times 12.8 \text{ fm} \]

\[ \rho_{\text{max}} \times z_{\text{max}} = 13.2 \text{ fm} \times 16.0 \text{ fm} \]
Coupling with spurious mode

Trial wave functions

\[ K^\pi = 0^+ \] Pair rotation

Restoration of broken symmetry

|\langle 0 | N | \lambda \rangle |^2 |

\[ \hbar \omega \text{ (MeV)} \]

- Particle number violation
- HFB approximation
Two-neutron pair transition strengths

\[ \hat{T}_{\text{add}} = \int d\mathbf{r} \mathbf{r}^2 Y_{20} \varphi^+(\mathbf{r}, \uparrow) \varphi^+(\mathbf{r}, \downarrow), \quad \hat{T}_{\text{rem}} = \int d\mathbf{r} \mathbf{r}^2 Y_{20} \varphi(\mathbf{r}, \downarrow) \varphi(\mathbf{r}, \uparrow) \]

\[ |< \lambda | \hat{T} | 0 >|^2 \]

Pair creation

Pair annihilation
Neutron-pair creation/annihilation

\[ \sum_{\alpha \beta} M_{\text{pair-rem}}^{\alpha \beta} (\text{fm}^2) \]

\[ \sum_{\alpha \beta} M_{\text{pair-add}}^{\alpha \beta} (\text{fm}^2) \]

\[ \langle \lambda | \hat{T}_{\text{rem}} | 0 \rangle = \sum_{\alpha \beta} M_{\text{pair-rem}}^{\alpha \beta} \]

\[ \langle \lambda | \hat{T}_{\text{add}} | 0 \rangle = \sum_{\alpha \beta} M_{\text{pair-add}}^{\alpha \beta} \]
Mechanism for generation of soft $K=0^+$ mode

Collective both in p-h and in p-p channel

How to generate the coherent mode?

Why are the transition strengths large? \( \sim 10\text{-}20 \text{ W.u. (intrinsic)} \)

Two key points

**Pair correlation**

Effect of dynamical pairing

**Weakly bound system**

Spatial structure of quasiparticle wave functions
Dynamical pairing

Superposition of p-h, p-p and h-h vibrations

→ Generation of the coherence

\[ \begin{align*}
\text{40 Mg} & \quad K^\pi=0^+ \\
\text{\(K^\pi=0^+\)} & \\
\text{\(K^\pi=2^+\)} & \\
\text{\(K^\pi=2^+\)} & \\
\end{align*} \]
Spatial structure of 2qp excitations (p-h channel)

$$< \alpha \beta | \hat{Q}_{20} | 0 > \equiv \int d \rho dz Q_{20}^{\alpha \beta} (\rho, z) \quad \hat{Q}_{20} = \sum_{\sigma} \int d \rho r^2 Y_{20} \psi^+ (\mathbf{r}, \sigma) \psi (\mathbf{r}, \sigma)$$

(a) $[310]1/2 \rightarrow [310]1/2$

(b) $[312]3/2 \rightarrow [312]3/2$

(c) $[310]1/2 \rightarrow [301]1/2$

(d) $[301]1/2 \rightarrow [301]1/2$

(e) $[303]7/2 \rightarrow [303]7/2$

(f) $[321]3/2 \rightarrow [321]3/2$

$$Q_{20}^{\alpha \beta} (\rho, z)$$
Spatial structure of 2qp excitations (p-p, h-h channel)

\[
\langle \alpha\beta | \hat{T} | 0 \rangle \equiv \int d\rho dz Q_{\text{pair-add}}^{\alpha\beta}(\rho, z) \quad \hat{T} = \int drr^2 Y_{20} \psi^+(\mathbf{r}, \uparrow)\psi^+(\mathbf{r}, \downarrow)
\]
Soft K=2\(^+\) mode in deformed \(^{40}\text{Mg}\)

\[
\frac{M_n}{M_p} \left/ \frac{N}{Z} \right. = \frac{9.43}{3.02} = 3.13
\]

\[
40\text{Mg} \\
K^{\pi}=2^+
\]

\[
\lambda(\hat{Q}_{22}|0> = \sum_{\alpha\beta} M_{22}^{\alpha\beta}
\]

\[
\hbar \omega = 2.78 \text{ MeV}
\]

1 W.u. = 8.13 fm\(^4\)
Neutron-pair creation/annihilation

Quadrupole pairing

\[
\hat{T}_{\text{add}} = \int d\mathbf{r} \mathbf{r}^2 Y_{22} \psi^+(\mathbf{r}, \uparrow) \psi^+(\mathbf{r}, \downarrow)
\]

\[
\hat{T}_{\text{rem}} = \int d\mathbf{r} \mathbf{r}^2 Y_{22} \psi(\mathbf{r}, \downarrow) \psi(\mathbf{r}, \uparrow)
\]

\[
|< \lambda | \hat{T} | 0 >|^2
\]
We have investigated properties of excitation modes in deformed Mg isotopes close to the neutron drip line.

**Deformed QRPA calculation based on coordinate-space HFB including the continuum**

We have obtained soft $K=0^+$ and $2^+$ modes in $^{36-40}$Mg.

- **Spatial extension of two-quasiparticle wave functions**
  - Large transition strengths

- **Coupling between quadrupole vib. and pairing vib.**

- **Similar spatial structure of quasiparticle w.f. near the Fermi level**
  - Generating coherent mode

$K=0^+$ mode is particularly sensitive to the dynamical pairing.

- **Good indicator of pair correlation in deformed drip-line nuclei**