Covariant density functional theory for excited states in nuclei

INT, Sept. 29, 2005

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**Content**

- **Covariant Density Functional Theory**
  - parametrization of the Lagrangian

- **Excitations within mean field approximation**
  - rotational excitations (Cranked RHB)
  - vibrational excitations (Rel. QRPA)

- **Methods beyond mean field**
  - projected density functionals (PDFT)
  - relativistic GCM
  - particle vibrational coupling (PVC)
  - decay width of Giant resonances

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Nucleons are coupled by exchange of mesons through an effective Lagrangian (EFT)

\[ (J^\pi, T) = (0^+, 0) \] \[ (J^\pi, T) = (1^-, 0) \] \[ (J^\pi, T) = (1^-, 1) \]

Sigma-meson: attractive scalar field

Omega-meson: short-range repulsive

Rho-meson: isovector field

\[ S(\mathbf{r}) = g_\sigma \sigma(\mathbf{r}) \]

\[ V(\mathbf{r}) = g_\omega \omega(\mathbf{r}) + g_\rho \vec{\tau}\vec{\rho}(\mathbf{r}) + eA(\mathbf{r}) \]
Relativistic Hartree Bogoliubov (RHB)

Ground-state properties of weakly bound nuclei far from stability

Unified description of mean-field and pairing correlations

\[
\begin{pmatrix}
\hat{h}_D - m - \lambda \\
-\hat{\Delta}^* - \hat{h}_D + m + \lambda
\end{pmatrix}
\begin{pmatrix}
\Delta \\
V_k(r)
\end{pmatrix}
= E_k
\begin{pmatrix}
U_k(r) \\
V_k(r)
\end{pmatrix}
\]

Dirac hamiltonian

chemical potential

quasiparticle energy

pairing field

Gogny D1S

quasiparticle wave function

\[
\Delta_{ab}(\vec{r}, \vec{r}') = \frac{1}{2} \sum_{c,d} V_{pp}^{ab} \langle \vec{r}, \vec{r}' \rangle \kappa_{cd}(\vec{r}, \vec{r}')
\]
Effective density dependence:

**non-linear potential:**

$$\frac{1}{2} m_\sigma^2 \sigma^2 \Rightarrow U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$$

**density dependent coupling constants:**

R.Brockmann and H.Toki, PRL 68, 3408 (1992)

S.Typel and H.H.Wolter, NPA 656, 331 (1999)

$$g_\sigma, g_\omega, g_\rho \Rightarrow g_\sigma(\rho), g_\omega(\rho), g_\rho(\rho)$$

$$g \rightarrow g(\rho(r))$$

New

DD-ME1, DD-ME2

Niksic et al, PRC 66, 024306 (2002)

Lalazissis et al Niksic, PRC 71, 024312 (2005)
### Nuclei used in the fit for DD-ME2

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>B.E (MeV)</th>
<th>r_c (fm)</th>
<th>r_α − r_p (fm)</th>
<th>dE</th>
<th>d_r_c</th>
<th>d_r_αp</th>
</tr>
</thead>
<tbody>
<tr>
<td>^{16}\text{O}</td>
<td>127.801 (127.619)</td>
<td>2.727 (2.730)</td>
<td>-0.03</td>
<td>0.1</td>
<td>-0.1</td>
<td></td>
</tr>
<tr>
<td>^{40}\text{Ca}</td>
<td>342.741 (342.052)</td>
<td>3.464 (3.485)</td>
<td>-0.05</td>
<td>0.2</td>
<td>-0.6</td>
<td></td>
</tr>
<tr>
<td>^{48}\text{Ca}</td>
<td>414.770 (415.991)</td>
<td>3.481 (3.484)</td>
<td>0.18</td>
<td>-0.3</td>
<td>-0.1</td>
<td></td>
</tr>
<tr>
<td>^{72}\text{Ni}</td>
<td>612.655 (613.173)</td>
<td>3.914</td>
<td></td>
<td>0.28</td>
<td>-0.1</td>
<td></td>
</tr>
<tr>
<td>^{90}\text{Zr}</td>
<td>783.155 (783.893)</td>
<td>4.275 (4.272)</td>
<td>0.07</td>
<td>-0.1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>^{116}\text{Sn}</td>
<td>986.928 (988.681)</td>
<td>4.615 (4.626)</td>
<td>0.12 (0.12)</td>
<td>-0.2</td>
<td>-0.2</td>
<td>3.8</td>
</tr>
<tr>
<td>^{124}\text{Sn}</td>
<td>1048.859 (1049.962)</td>
<td>4.671 (4.674)</td>
<td>0.21 (0.19)</td>
<td>-0.1</td>
<td>-0.1</td>
<td>10.7</td>
</tr>
<tr>
<td>^{132}\text{Sn}</td>
<td>1103.469 (1102.860)</td>
<td>4.718</td>
<td></td>
<td>0.26</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>^{204}\text{Pb}</td>
<td>1608.506 (1607.520)</td>
<td>5.500 (5.486)</td>
<td>0.17</td>
<td>0.1</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>^{208}\text{Pb}</td>
<td>1639.826 (1636.446)</td>
<td>5.518 (5.505)</td>
<td>0.19 (0.20)</td>
<td>0.2</td>
<td>0.2</td>
<td>-4.7</td>
</tr>
<tr>
<td>^{214}\text{Pb}</td>
<td>1661.182 (1663.298)</td>
<td>5.568 (5.562)</td>
<td>0.24</td>
<td>-0.1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>^{210}\text{Po}</td>
<td>1649.695 (1645.228)</td>
<td>5.552</td>
<td></td>
<td>0.17</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

**Nuclear matter:**

- E/A = -16 MeV (5%), \( \rho_0 = 1.53 \text{ fm}^{-1} \) (10%)
- K = 250 MeV (10%), \( a_4 = 33 \text{ MeV} \) (10%)
How many parameters?

7 parameters

- Symmetric nuclear matter: $E/A, \rho_0$
  - $g_\sigma/m_\sigma$
  - $g_\omega/m_\omega$

- Finite nuclei (N=Z): $E/A$, radii
  - Spinorbit o.k.
  - $m_\sigma$

- (N\neq Z): Coulomb, symmetry energy: $a_4$
  - $g_\rho/m_\rho$

- Density dependence: $T=0$
  - $K_\infty$
  - $g_2, g_3$

- $T=1$
  - $r_n - r_p$
  - $a_\rho$

Nuclear matter equation of state

![Graph showing the equation of state for neutron matter with curves for DD-ME2, DD-ME1, and NL3 models.](image)

Symmetry energy

\[ E(\rho, \alpha) = E(\rho, 0) + S_2(\rho)\alpha^2 \]
\[ + S_4(\rho)\alpha^4 + \cdots \]
\[ \alpha \equiv \frac{N-Z}{N+Z} \]
\[ S_2(\rho) = a_4 + \frac{p_0}{\rho_{\text{sat}}} (\rho - \rho_{\text{sat}}) + \frac{\Delta K_0}{18\rho_{\text{sat}}^2} (\rho - \rho_{\text{sat}})^2 + \cdots \]

empirical values:

\[ 30 \text{ MeV} < a_4 < 34 \text{ MeV} \]
\[ 2 \text{ MeV/fm}^3 < p_0 < 4 \text{ MeV/fm}^3 \]
\[ -200 \text{ MeV} < \Delta K_0 < -50 \text{ MeV} \]

Furnstahl, NPA 705 (2002) 85
**rms-deviations:**

- **Masses:** $\Delta m = 900$ keV
- **Radii:** $\Delta r = 0.015$ fm

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**Graph:**

- **$B_{\text{exp}} - B_{\text{th}}$ (MeV)**
- **A**

**Legend:**

- **RHB/DD-ME2**

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29.09.2005  
Towards a Universal Density Functional for the Nucleus, INT, Sept. 2005
Excited States: Time dependence:

\[ \delta \int dt \left\{ \Phi(t) \left| i \partial_t \right| \Phi(t) \right\} - E[\hat{\rho}(t)] = 0 \]

\[ i \partial_t \hat{\rho} = [\hat{h}(\hat{\rho}) + \hat{f}, \hat{\rho}] \]

Rotational Motion:

\[ \rho(t) = e^{-i\vec{\Omega} \cdot \vec{j}} \rho_\Omega e^{i\vec{\Omega} \cdot \vec{j}} \]

\[ [h - \vec{\Omega} \cdot \vec{j}, \rho_\Omega] = 0 \]

Vibrational Motion:

\[ \hat{\rho}(t) = \hat{\rho}^{(0)} + \delta \hat{\rho}(t) \]

\[ \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar \omega \begin{pmatrix} X \\ Y \end{pmatrix} \]

\[ \delta \rho_{ph} \]

\[ \delta \rho_{hp} \]

Cranked relativistic Hartree+Bogoliubov theory

CRHB equations for the fermions in the rotating frame

\[
\begin{pmatrix}
    \hat{h}_D - \lambda_\tau - \Omega \hat{J}_x \\
    -\hat{\Delta}^*
\end{pmatrix}
\begin{pmatrix}
    \hat{\Delta} \\
    \hat{h}_D + \lambda_\tau + \Omega \hat{J}_x
\end{pmatrix}
\begin{pmatrix}
    U_k \\
    V_k
\end{pmatrix}
= E_k
\begin{pmatrix}
    U_k \\
    V_k
\end{pmatrix}
\]

Coriolis term

\[
\hat{h}_D = \alpha (-i \vec{\nabla} - \vec{V}(\vec{r})) + V_0(\vec{r}) + \beta (m - S(\vec{r}))
\]

Magnetic potential

\[
\vec{V}(\vec{r}) = g_\omega \vec{\omega}(\vec{r}) + g_\rho \tau_3 \vec{\rho}(\vec{r}) + e \frac{1 - \tau_3}{2} \vec{A}(\vec{r})
\]

space-like components of vector mesons behaves in Dirac equation like a magnetic field

Nuclear magnetism
Towards a Universal Density Functional for the Nucleus, INT, Sept.2005


solid diamonds - ol
solid lines - calcula

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Excitation energy of the superdeformed minimum:

$^{194}\text{Hg}$

$E_x = 6.02$, $V = 5.6$

$E_x = 6.0$, $V = 5.6$

$E_x = 6.9$, $V = 5.0$

$E_x = 4.6$

G. A. Lalazissis, P. Ring, PLB 427 (1998) 225
Cranked RHB

A.V. Afanasjev, P. Ring, J. König

N=110  N=112  N=114  N=116

experiment: unlinked symbols
CRHB calculations: lines

black - $J^{(1)}$  
blue - $J^{(2)}$

$^{198}$Po  $^{192}$Pb  $^{194}$Pb  $^{196}$Pb  $^{198}$Pb  $^{190}$Hg  $^{192}$Hg  $^{194}$Hg

$Z=84$  $Z=82$  $Z=80$

Time-dependent RMF: breathing mode, $^{208}$Pb:

\[ \langle \Phi(t) | r^2 | \Phi(t) \rangle \]

$K_\infty = 211$

$K_\infty = 271$

$K_\infty = 355$
Pb: lowlying discrete spectrum

Calculated and experimental excitation energies, and $B(EL)$ values for the low-lying vibrational states in $^{208}$Pb

<table>
<thead>
<tr>
<th>$L^\pi$</th>
<th>$E_{th}$</th>
<th>$E_{exp}$</th>
<th>$B(EL)_{th}$</th>
<th>$B(EL)_{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3$^-$</td>
<td>2.76</td>
<td>2.61</td>
<td>$499 \times 10^3$</td>
<td>$(540 \pm 30) \times 10^3$</td>
</tr>
<tr>
<td>5$^-$</td>
<td>3.26</td>
<td>3.71</td>
<td>$201 \times 10^6$</td>
<td>$330 \times 10^6$</td>
</tr>
<tr>
<td>2$^+$</td>
<td>4.99</td>
<td>4.07</td>
<td>2816</td>
<td>2965</td>
</tr>
<tr>
<td>4$^+$</td>
<td>4.95</td>
<td>4.32</td>
<td>$998 \times 10^4$</td>
<td>$1287 \times 10^4$</td>
</tr>
</tbody>
</table>

The calculated values correspond to NL3 parameterization, the data are from Ref. [29]. Energies are in MeV, $B(EL)$ values in $e^2 \text{ fm}^{2L}$.

Z.Y. Ma, A. Wandelt et al., NPA 694 (2001) 249
The ISGMR represents the essential source of experimental information on the nuclear incompressibility:

\[ K_0 = p_f^2 \left. \frac{d^2 E}{dp_f^2} \right|_{p_f=0} \]

The nuclear incompressibility constrains the nuclear matter compressibility:

\[ 250 \text{ MeV} \leq K_0 \leq 270 \text{ MeV} \]
the IVGDR represents one of the sources of experimental informations on the nuclear matter symmetry energy

constraining the nuclear matter symmetry energy

the position of IVGDR is reproduced if

\[ 34 \text{ MeV} \leq a_4 \leq 36 \text{ MeV} \]
DD-ME2

IV-GDR in Sn-isotopes

E_{th} = 15.59 MeV
E_{exp} = 15.68 MeV

E_{th} = 15.53 MeV
E_{exp} = 15.59 MeV

E_{th} = 15.40 MeV
E_{exp} = 15.36 MeV

E_{th} = 15.28 MeV
E_{exp} = 15.19 MeV
Experimental indications of the soft dipole mode
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Photoneutron Cross Sections for Unstable Neutron-Rich Oxygen Isotopes


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GeSELL Forschungsgesellschaft mbH, D-64291 Darmstadt, Germany

Institut für Kernchemie, Johannes Gutenberg-Universität, D-55099 Mainz, Germany

Institut für Kernphysik, Technische Universität, D-84259 Darmstadt, Germany

Institut für Physik, Universität Jena, Institut für Physik, Jena, 07743 Jena, Germany

Sektion Physik, Ludwig-Maximilians-Universität, D-85748 Garching, Germany

(Received 19 December 2000)

The dipole response of stable and unstable neutron-rich oxygen nuclei of masses $A=17$ to $A=22$ has been investigated experimentally utilizing electromagnetic excitation in heavy-ion collisions at beam energies about 600 MeV/nucleon. A kinematically complete measurement of the photoneutron decay channel in inelastic scattering of the secondary beam properties from a Pb target was performed. Differential electromagnetic excitation cross sections, derived up to 30 MeV excitation energy. In contrast to stable nuclei, the deduced dipole strength distribution appears to be strongly fragmented and systematically exhibits a considerable fraction of low-lying strength.

DOI: 10.1103/PhysRevLett.86.2909

The study of the response of a clear or electromagnetic isotope to the properties of the nuclear excitation energies above the particular response of stable nuclei is dominantly determined by various multipole transitions. The giant resonance strength is usually stable to excitation energies above the giant resonance energy. In particular, the response to stable nuclei is the weakly bound gamma-ray transitions. For neutron-rich nuclei, more strongly bound and resonant effects, in particular the strength towards lower excitation energies of the giant resonance region. The general behavior of the giant resonance of the exotic nuclei can be understood via the isospin dependent electromagnetic interaction [2].

Systematical experimental information on the response of exotic nuclei, however, is very scarce. In several cases, e.g., for some light halo nuclei, low-lying resonances have been observed in electromagnetic excitation measurements. For example, in the oxygen isotopes $^{16}$O [8-11] and the one-neutron deficient $^{15}$O, the observed electromagnetic excitation energies were interpreted as evidence for the existence of a covalent neutron into the core [12].

FIGURE 2. Photoneutron cross sections for $^{16}$O (upper panel) and for the unstable isotopes $^{17}$O (lower panel) as extracted from the measured electromagnetic excitation cross section (symbols). The inset displays the cross section for the neutron threshold on an expanded energy scale. The thresholds for decay channels involving protons (which were not observed in the present experiment) are indicated by arrows.
Evolution of IV dipole strength in Oxygen isotopes

RHB + RQRPA calculations with the NL3 relativistic mean-field plus D1S Gogny pairing interaction.

What is the structure of low-lying strength below 15 MeV?

Effect of pairing correlations on the dipole strength distribution

Isovector dipole strength in $^{132}$Sn.

$^{132}$Sn at 7.6 MeV
- 28.2% $2d_{3/2} \rightarrow 2f_{5/2}$
- 21.9% $2d_{5/2} \rightarrow 2f_{7/2}$
- 19.7% $2d_{3/2} \rightarrow 3p_{1/2}$
- 10.5% $1h_{11/2} \rightarrow 1i_{13/2}$
- 3.5% $2d_{5/2} \rightarrow 3p_{3/2}$
- 1.9% $1g_{7/2} \rightarrow 2f_{5/2}$
- 1.5% $1g_{7/2} \rightarrow 1h_{9/2}$
- 0.6% $1g_{7/2} \rightarrow 2f_{7/2}$
- 0.6% $2d_{3/2} \rightarrow 3p_{3/2}$

In heavier nuclei low-lying dipole states appear that are characterized by a more distributed structure of the RQRPA amplitude.
E1-strength distribution in deformed $^{20}\text{Ne}$
IV-E1-Resonances in deformed $^{26}\text{Ne}$

$B(E1)$ in Ne $^{26}$ ($J^{\pi}=1^-$)

Arbitrary units

E [MeV]

NL3
Transition density of the upper E1-peak

E1 $^{20}\text{Ne}$ Transition Density, Peak at 21.4 MeV
Pygmy-Resonance in deformed $^{26}\text{Ne}$

Gibelin et al, prelim.

Low Lying $B(E1)$ in $\text{Ne}^{26}$ ($J^\pi=1^+$)

NL3
Isoscalar dipole compression - toroidal modes

Isoscalar GMR in spherical nuclei $\rightarrow$ nuclear matter compression modulus $K_{nm}$.

Giant isoscalar dipole oscillations $\rightarrow$ additional information on the nuclear incompressibility.

\[ \hat{Q}_{1\mu}^{T=0} = \sum_{i=1}^{A} \gamma_0 \left( r^3 - \eta r \right) Y_{1\mu}(\theta_i, \varphi_i) \]

ISGDR strength distributions
Effective interactions with different $K_{nm}$.

The low-energy strength does not depend on $K_{nm}$!

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**Toroidal motion:**

ISGDR transition densities for $^{208}\text{Pb}$ (NL3 interaction)

- multipole expansion of a four-current distribution:
  - charge moments
  - magnetic moments
  - electric transverse moments $\rightarrow$ toroidal moments

- toroidal dipole moment: poloidal currents on a torus

**Isoscalar toroidal dipole operator:**

\[
\hat{I}_{1\mu}^{T=0} \sim \int [r^2 \left( \vec{Y}_{10\mu}^* + \frac{\sqrt{2}}{5} \vec{Y}_{12\mu}^* \right) - < r^2 >_0 \vec{Y}_{10\mu}^* \cdot \vec{J}(\vec{r}) d^3r]
\]
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Toroidal dipole strength distributions.


Velocity distributions in $^{116}$Sn

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Spin-Isospin Resonances: IAR - GTR

\[ |\text{GTR}\rangle = S \cdot T \cdot |Z, N\rangle \]

Spin flip \( \sigma \)

\[ |Z, N\rangle \rightarrow |\text{IAR}\rangle = T \cdot |Z, N\rangle \]

Isospin flip \( \tau \)

\[ E_{\text{GTR}} - E_{\text{IAR}} \sim \Delta (l \cdot s) \sim \frac{dV}{dr} \sim \text{neutron skin} = r_n - r_p \]
Spin-Isospin Resonances: IAR and GTR

Charge-exchange excitations

$\pi$ and $\rho$-meson exchange generate the spin-isospin dependent interaction terms

\[ \mathcal{L}_{\pi N} = -\frac{f_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \partial_\mu \pi \bar{\pi} \psi \]

The Landau-Migdal zero-range force in the spin-isospin channel

\[ V(1, 2) = g'_0 \left( \frac{f_\pi}{m_\pi} \right)^2 \bar{\pi}_1 \cdot \bar{\pi}_2 \Sigma_1 \cdot \Sigma_2 \delta(r_1 - r_2) \]

GAMOW-TELLER RESONANCE: \( S=1 \quad T=1 \quad J^\pi = 1^+ \) \( (g'_0=0.55) \)

ISOBARIC ANALOG STATE: \( S=0 \quad T=1 \quad J^\pi = 0^+ \)
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GTR GT-Resonances


Experiment
Isobaric Analog Resonance: IAR

The isotopic dependence of the energy spacings between the GTR and IAS provides direct information on the evolution of the neutron skin along the Sn isotopic chain.
* Important points:
  - the tail of the GT-strength distribution at low energies
  - the position of specific single particle levels (i.e. effective mass)
  - effective pairing force in the T=1 and T=0 channel.
  - in simple QRPA the lifetimes are too big

* Possible methods to improve the results:
  - coupling to surface vibrations (difficult and beyond mean field)
  - use of a tensor coupling in the $\omega$-channel (one phenomen. param.)
  - $T=0$ pairing force with Gaussian character (one phen. parameter)
enhanced value of the effective mass

increased density of states around the Fermi surface

The nucleon effective mass $m^*$:

$m^*$ represents a measure of the density of states around the Fermi surface.

**nonrelativistic mean-field models**

- effective mass: $m^*/m = 0.8 \pm 0.1$

**relativistic mean-field models**

- Dirac mass: $m_D = m + S(r)$
- effective mass: $m^* = m - V(r)$

**conventional RMF models**

- spin-orbit splittings + nuclear matter binding
- $0.55m \leq m_D \leq 0.60m$
- $0.64m \leq m^* \leq 0.67m$
- small density of states -> overestimated $\beta$-decay lifetimes
The spin-orbit potential originates from the addition of two large fields: the field of the vector mesons (short range repulsion), and the scalar field of the sigma meson (intermediate attraction).

\[
V_{\text{s.o.}} \approx \frac{1}{r} \frac{\partial}{\partial r} V_{ls}(r)
\]

\[
V_{ls} = \frac{m}{m_{\text{eff}}} (V - S)
\]

Weakening of the effective single-neutron spin-orbit potential in neutron-rich isotopes:

Reduced energy spacings between spin-orbit partners

\[
\Delta E_{ls} = E_{n,l,j=l-1/2} - E_{n,l,j=l+1/2}
\]
tensor omega-nucleon coupling enhances the spin-orbit interaction

\[ V_{SO} = \left[ \frac{1}{4M^2} \frac{1}{r} \frac{d}{dr} (V - S) + \frac{f_V}{2MM} \frac{1}{r} \frac{d\omega}{dr} \right] \hat{l} \cdot \hat{s} \]

scalar and vector self-energies can be reduced

Cadmium isotopes: $\pi_{1g9/2}$ level is partially empty

$T=0$ pairing has large influence on the $\nu_{1g7/2} \rightarrow \pi_{1g9/2}$ transition which dominates the $\beta$-decay process

An increase of the T=0 pairing partially compensates for the fact that the density of states is still rather low. 

Niksic et al, PRC 71, 014308 ('05)

\[ \nu h_{9/2} \rightarrow \pi h_{11/2} \]

G. Martinez-Pinedo and K. Langanke, PRL 83, 4502 (1999)
Correlations beyond mean field

• Conservation of symmetries by projection before variation
• Motion with large amplitude by Generator Coordinates
• Coupling to collective vibrations
  - shifts of single particle energies
  - decay width of giant resonances
Projected Density Functionals

\[
|\Psi^N\rangle = \hat{P}^N|\Phi\rangle = \delta(\hat{N} - N)|\Phi\rangle = \int \frac{d\varphi}{2\pi} e^{i\varphi(\hat{N} - N)}|\Phi\rangle
\]

Projected density functional:

\[
E^N[\hat{\rho}, \hat{\kappa}] = \frac{\langle \Phi | \hat{H}\hat{P}^N | \Phi\rangle}{\langle \Phi | \hat{P}^N | \Phi\rangle}
\]

Projected HFB-equations (variation after projection):

\[
\begin{pmatrix}
\hat{h}^N & \hat{\Delta}^N \\
-\hat{\Delta}^{N*} & -\hat{h}^{N*}
\end{pmatrix}
\begin{pmatrix}
U_k(\mathbf{r}) \\
V_k(\mathbf{r})
\end{pmatrix}
= \begin{pmatrix}
U_k(\mathbf{r}) \\
V_k(\mathbf{r})
\end{pmatrix} E_k
\]

J. Sheikh and P. Ring NPA 665 (2000) 71

analytic expressions

\[\hat{h}^N = \frac{\delta E^N}{\delta \hat{\rho}}\]

\[\hat{\Delta}^N = \frac{\delta E^N}{\delta \hat{\kappa}}\]
Halo-formation in Ne-isotopes

Pairing energies

Binding energies

RMS-radii


Graph showing:
- Pairing energy [MeV] vs. mass number A
- Binding energy / A [MeV] vs. mass number A
- RMS-radii [fm] vs. mass number A

Legend:
- Solid line: projected
- Dashed line: unprojected
Generator Coordinate Method (GCM)

\[ \langle \delta\Phi | \hat{H} - q\hat{Q} | \Phi \rangle = 0 \]

Constraint Hartree Fock produces wave functions depending on a generator coordinate \( q \)

\[ |q\rangle = |\Phi(q)\rangle \]

GCM wave function is a superposition of Slater determinants

\[ |\Psi\rangle = \int dq \ f(q) \ |q\rangle \]

Hill-Wheeler equation:

\[ \int dq \left[ \langle q | H | q' \rangle - E\langle q | q' \rangle \right] f(q') = 0 \]

with projection:

\[ |\Psi\rangle = \int dq \ f(q) \hat{P}^N \hat{P}^I \ |q\rangle \]
GCM without projection:

$^\text{194}\text{Hg}$

$N_{\text{sh}} = 10$

$N_{\text{sh}} = 12$

$N_{\text{sh}} = 14$

$N_{\text{sh}} = 16$
GCM-wave functions of the lowest states
Ang. momentum projected energy surfaces:
Vibrational Couplings: energy dependent self-energy:

\[ \Sigma = S + V + \Sigma(\omega) \]

- mean field
- pole part
- RPA-modes

single particle strength:

\[ z_\nu = \left[ 1 - \frac{d\Sigma_{\nu\nu}}{d\omega} \bigg|_{\omega = \epsilon_\nu} \right]^{-1} \]
Distribution of single-particle strength in $^{209}\text{Bi}$

- $^{209}\text{Bi} \, 1h9/2$
- $^{209}\text{Bi} \, 1i13/2$
- $^{209}\text{Bi} \, 2f5/2$
- $^{209}\text{Bi} \, 2h11/2$

E, MeV

Spectroscopic factor
Level scheme for $^{207}$Pb

- **RMF**: $1h9/2$
- **RMF + PVC**: $2f7/2$
- **EXP**: $1i13/2$
- **EXP**: $3p3/2$
- **EXP**: $2f5/2$
- **EXP**: $3p1/2$

E, MeV

Contributions of complex configurations

The full response contains energy dependent parts coming from vibrational couplings.

\[ \text{Self energy} \]

\[ g - \text{phonon amplitudes (QRPA)} \]

\[ \text{ph interaction amplitude} \]

\[ R^e = - \quad + \quad \text{QRPA} \]

\[ + R^e + \quad \text{QRPA} - R^e \]
Decay-width of the Giant Resonances

\[ S(E) = -\frac{1}{\pi} \text{Im} \Pi(E + i\Delta) \]

**E1 photoabsorption cross section**

\[ \sigma_{E1}(E) = \frac{16\pi^3 e^2}{9\hbar c} E S_{E1}(E) \]
Conclusions

On the way to a universal covariant density functional adjusted to ground state properties of finite nuclei.

≈7 parameters necessary for high precision

Time-dependent mean field theory provides a parameter-free theory for excited states
- rotational spectra (cranked RHB-theory)
- vibrational excitations (rel. QRPA)

Method beyond mean field:
- Projected funcionals (PDFT)
- Generator Coordinate Method (GCM)
- Particle-Vibrational Coupling (PVC)
Open Problems:

Fock terms and tensor forces:
- why is the first order pion-exchange quenched?

Vacuum polarization:
- renormalization in finite systems

Simpler parametrizations:
- point coupling
- simple pairing

Do we have to change the functional, if we go beyond mean field?
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