Neutrino-nucleus Interactions and Nuclear Structure.
"I have done a terrible thing. I have invented a particle that cannot be detected."

W. Pauli
"Neutrinos, they are very small
They have no charge and have no mass
And do not interact at all
The earth is just a silly ball
To them, through which they simply pass
Like dust maids down a drafty hall."

- John Updike, 1960
Neutrino-Nucleus Interactions.

A. Charged Current (CC) interactions

\[ zA (\nu_e, e^-)_{z+1} A^* \]  \text{LSND, KARMEN}

\[ zA (\nu_\mu, \mu^-)_{z+1} A^* \]  \text{LSND ($C$-target)}

These are charge-exchange (CE) transitions (therefore of isovector type).

Inclusive and exclusive (to g.s.) cross-sections are known for \( A=12 \). These are flux averaged cross-sections.

In addition, the \( \mu^- \) capture rates were considered in the theoretical work.

\[ zA (\mu^-, \nu_\mu)_{z+1} A^* \]  \text{inclusive and exclusive.}

B. RPA Calculations and the role of GRs.

C. Shell-Model Calculations.
FIG. 1. Flux shape of neutrinos from pion and muon decay at rest.
Neutrino Energy (MeV)

DiF flux

(Decay in Flight)
The importance of neutrino-nucleus reactions.

1. The neutrinos are "messengers" bringing information about the insides of stars. The detection of such neutrinos through the $\nu$-nucleus interactions.

2. The study of $\nu$-oscillations and $m_\nu$ is via $\nu$-nucleus reactions. LSND, KARMEN experiments.

3. The study of atmospheric neutrinos is possible because of $\nu$-nucleus interactions.


D. Comparison between the RPA and SM.

E. Quenching factors.
   (or effective transition operators).
1. Exclusive cross-sections (to g.s.)
   Easy in the extended shell-model
   Difficult in RPA
   (correlations important).

2. Inclusive cross-sections
   Easy in the RPA
   Difficult in the shell-model
   (high-lying states, many two).
occupied states and by \( I, J, \ldots \) \((A, B, \ldots)\) the proton (neutron) unoccupied states. The proton (neutron) creation and annihilation operators are respectively \( p_i^+ \) and \( p_i \) \((n_i^+ \) and \( n_i)\). In reactions of the \((\nu_e, \mu^-)\) or \((\nu_e, e^-)\) type the final states \(|\lambda\rangle\) belong to the \(\Delta T_2 = -1\) daughter nucleus (e.g., \(^{12}\text{N}\)) and they can be described by the charge-exchange RPA \([12,13]\) model:

\[
|\lambda\rangle = \left( \sum_{i,a} X_{i,a}^{(\lambda)} p_i^+ n_a + \sum_{i,A} Y_{i,A}^{(\lambda)} p_i^+ n_A \right) |\bar{0}\rangle ,
\]  

(1)

where \(|\bar{0}\rangle\) is the correlated RPA ground state. The \(X^{(\lambda)}\) and \(Y^{(\lambda)}\) are solutions of the charge-exchange RPA equations \([12,13]\). For a one-body charge-exchange operator of the general form:

\[
O = \sum_{\alpha,\beta} O_{\alpha,\beta} p_\alpha^+ n_\beta ,
\]  

(2)

the transition amplitude \(\langle \lambda | F | \bar{0} \rangle\) can be expressed simply as:

\[
\langle \lambda | O | \bar{0} \rangle = \sum_{i,s} X_{i,s}^{(\lambda)^*} O_{i,s} - \sum_{i,A} Y_{i,A}^{(\lambda)^*} O_{i,A} .
\]  

(3)

In the case of a parent nucleus with zero angular momentum in the ground state the cross-section is \([2,3]\):

\[
\sigma = \frac{G^2}{2\pi} \cos^2(\theta_C) \sum_{\lambda} p_i E_i \mathcal{F}(Z, E_i) \int_{-1}^1 d(\cos(\theta)) \mathcal{M}_{\lambda 0} ,
\]  

(4)

where \(G\) and \(\theta_C\) are the Fermi constant and the Cabibbo angle, \(p_i\) and \(E_i\) are the momenta and energies of outgoing leptons (muon or electron), \(\theta\) is the angle between the momenta of the lepton and the incoming neutrino. The factor \(\mathcal{F}\) accounts for the effects of the final state interaction (FSI) of the outgoing lepton with the daughter nucleus of charge \(Z\) \([2,3]\). For the case of \((\nu_e, e^-)\) reactions the mass of the outgoing lepton is small and the effect is not so important. The effect of the FSI for the negatively charged muon is more significant, increasing the cross-section approximately by \(15 - 20\%\). In Eq.(4), the sum goes over the available nuclear excitations, denoted by \(\lambda\). The nuclear structure effects are incorporated into \(\mathcal{M}_{\lambda 0}\), the bilinear combination of the nuclear matrix elements between the ground state \(|\bar{0}\rangle\) and the excited states \(|\lambda\rangle\) of the daughter nucleus. These are given by \([3]\):
\[ M_{\lambda 0} = M_F \left| \langle \lambda | F | \bar{0} \rangle \right|^2 + M_{GT} \left| \langle \lambda | GT | \bar{0} \rangle \right|^2 + M'_{GT} \Lambda \]  

(5)

The coefficients \( M_i \) are obtained by the Foldy-Wouthuysen transformation of the weak Hamiltonian where the terms up to third order in the momentum transfer \( q/M \) are kept (\( M \) is the nucleon mass) [3]. The first matrix element squared of Eq. (5) is:

\[ \left| \langle \lambda | F | \bar{0} \rangle \right|^2 = 4\pi \sum_l J_l \left| \langle \lambda, J || t_{-j L}(q r) Y_J || \bar{0} \rangle \right|^2 \]  

(6)

Here, \( \vec{q} \) is the momentum transfer, \( q = |\vec{q}| \), \( \vec{r} \) and \( t_- \) refer to the nucleon spin Pauli matrices and isospin-lowering operator, respectively, \( ||...|| \) stands for the standard definition of the reduced matrix elements, \( j_L \) are the spherical Bessel functions and \( Y_J \) are the spherical harmonics. The remaining combinations of the matrix elements are:

\[ \left| \langle \lambda | GT | \bar{0} \rangle \right|^2 = 4\pi \sum_l J_l \left| \langle \lambda, J || t_{-j L}(q r) [Y_l \times \vec{\sigma}]_J || \bar{0} \rangle \right|^2 , \]  

(7)

\[ \Lambda = 4\pi \left( \frac{5}{6} \right)^{1/2} \sum_{l,l',J} (-1)^{(l-l')/2+J} ((2l + 1)(2l' + 1))^{1/2} \times \left\{ \begin{array}{c} l \\ l' \\ 2 \end{array} \right\} \left\{ \begin{array}{c} 1 \\ 1 \\ 2 \end{array} \right\} \langle \lambda, J || t_{-j L}(q r) [Y_l \times \vec{\sigma}]_J || \bar{0} \rangle \]  

\[ \times \langle \lambda, J || t_{-j L}(q r) [Y_{l'} \times \vec{\sigma}]_J || \bar{0} \rangle^* . \]  

(8)

Here, \( \left[ \; \times \; \right]_J \) means the coupling to the total angular momentum \( J \). In our calculation we take into account states with \( J \leq 3 \) with positive and negative parity.

We perform the integration over angle \( \theta \) in Eq.(4) with the step size \( \Delta \theta = 2\pi/30 \). The single-particle matrix elements of the operators \( O \) are calculated using the Hartree-Fock wave functions in steps of \( \Delta r = 0.1 \) fm in the radial coordinate.

The \(^{12}\text{C}\) nucleus is not described well by a closed \( 3p_{3/2} \) subshell and configuration mixing is important in the ground state. Besides the RPA correlations, one of the important correlations is introduced by the pairing force. Here, we estimate the effect of pairing on the exclusive and inclusive neutrino cross-sections. In the expressions (4-8) for the cross-section, two types of single-particle matrix elements enter: a) those which do not contain the spin
To obtain the cross-sections that allow for comparison with the experimental data one has to fold the energy-dependent cross-section of Eq.(4) with a corresponding neutrino flux \( f(E) \)

\[
\langle \sigma \rangle_f = \int dE \sigma(E) \tilde{f}(E),
\]

(11)

where \( \tilde{f}(E) \) is a properly normalized neutrino flux from an available neutrino source,

\[
\tilde{f}(E) = \frac{f(E)}{\int_{E_0}^{\infty} dE' f(E')},
\]

(12)

and \( f(E') \) is the initial (unnormalized) flux from the source. Here, the value of \( E_0 \) depends on the neutrino source used in each experiment. It is taken to be zero for the case of the electron neutrino [15], while for the case of the muon neutrino source \( E_0 = E_{thr} \) [6], where
related weak processes

\[ 1^+ T=1 \quad T_2=+1 \quad 1^+ T=1 \quad T_2=0 \quad 1^+ T=1 \quad T_2=-1 \]

\[ \begin{array}{c}
\mu^- \\
\beta^-
\end{array} \quad \begin{array}{c}
\gamma
\end{array} \quad \begin{array}{c}
\beta^+
\end{array} \]

\[ 0^+ T=0 \quad T_2=0 \quad 12_6 C_6 \quad 12_7 N_5 \]

\[ A=12 \text{ TRIAD} \]

\[ \text{A TEST OF THE WAVEFUNCTIONS} \]
Isovector RPA ($\Delta T = 1$)
(Charge-Exchange RPA)

\[ \begin{align*}
T+1 & \\
T & \\
T & \\
T+1 &
\end{align*} \]

\[ \begin{align*}
p \to n \\
\Delta T_e = 1 \\
(p \to n) + (p \to n) \\
\Delta T_e = 0 \\
(n \to p) \\
\Delta T_e = -1
\end{align*} \]
E. Kelbe et al. PRC 52, 3442 (1995)  

A. C. Hayes and I. S. Towner  

RPA+PAIRING  
N. Auerbach, N. Van Giai and Vorov PRC 56, R2328 (1997)  
C. Volpe, N. Auerbach, G. Colo  
T. Suzuki and N. V. Giac  


N. Auerbach and B. A. Brown  

None recent work (Kudak, Belgium, Spain)
The latest result from LSND for the $^\text{12}\text{C}(\nu_e,\mu^-)^\text{12}\text{N}$ (inclusive) is:

$\sigma_{\text{exp}} = (10.6 \pm 3 \pm 1.8) \times 10^{-40} \text{cm}^2$

Typical RPA results

$\sigma_{\text{RPA}} = 20 \times 10^{-40} \text{cm}^2$

large shell-model space

$\sigma_{\text{SM}} = 15 \times 10^{-40} \text{cm}^2$
Inclusive ($\nu\mu, \mu^-$) Diff cross-section.

<table>
<thead>
<tr>
<th>$SM$ (HF WF)</th>
<th>RPA</th>
<th>RPA ($3\hbar\omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.2</td>
<td>19.2</td>
<td>15.8</td>
</tr>
</tbody>
</table>

When the RPA is restricted to $3\hbar\omega$ the result is similar to the Shell-Model result (which includes up to $3\hbar\omega$ excitations).
The "universal" quenching problem.

About 30\% to 40\% of Gamow-Teller strength is missing from the main peak (\( \sigma_t \)).

\[ \sigma_t \rightarrow \sqrt{0.6} \sigma_t. \]

What is the origin of this quenching?
Two theories considered (in the 80s).

1. D_33 region

2. \( D_{33} \) region

There is also the question whether the above quenching occurs also in the \( \delta r \), \( \delta r^2 \) etc. spin-dependent operators.
N. Auerbach and B.A. Brown

"... our strategy will be to assume that the role of ground-state correlations beyond O(\hbar) can be taken into account by effective operators. In this paper, we show that the simplest possible choice of an overall renormalization can account for most of the experimental data.......

* This renormalization may have several sources including the long-range RPA-type and short-range correlations in the ground state.

"We wish to restate the factors that contribute to the reduction factor: (a) the RPA-type correlations reduce the strength due to the repulsive nature of the isovector particle-hole interaction; (b) other correlations due to the short-ranged interaction that are also responsible for a reduction in the absolute spectroscopic factors; (c) removal of strength in the spin-channel due to the coupling to the \Delta_{33} resonance...."

* for Fermi-type and Gamow-Teller type
The "otw g.s." model means that the ground states have described with a otw configuration (in $^{16}$O just the closed shell and in $^{12}$C the full p-shell), and the excited states are formed from otw, 1tw, and 2tw configurations. The "strategy" is to assume that the role of g.s. correlations beyond otw is taken into account by effective operators.

(This is new because we assume that both the Fermi type and Gamow-Teller type operators are affected).
<table>
<thead>
<tr>
<th>Process</th>
<th>Exp.</th>
<th>othw</th>
<th>othw x 0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \mu^-$ capture</td>
<td>162</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>$^{16}O \rightarrow ^{16}N$ (incl.) $(10^3 s)$ $102.5 \pm 1.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda \mu^-$ capture</td>
<td>66 $\pm$ 5</td>
<td>108</td>
<td>68</td>
</tr>
<tr>
<td>$^{14}N \rightarrow ^{14}C$ (incl.) $(10^3 s)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z(DAR) $^{12}C(\nu_e, e^-)N \times 10^{2 m^2}$</td>
<td>14.1 $\pm$ 1.2</td>
<td>23.4</td>
<td>15.1</td>
</tr>
<tr>
<td>total (incl.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z(DAR) $^{12}C(\nu_e, e^-)N \times 10^{2 m^2}$</td>
<td>0.66 $\pm$ 0.14</td>
<td>1.4</td>
<td>0.9</td>
</tr>
<tr>
<td>to $1^+$ (exclus.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z(DIF) $^{12}C(\nu_e, \mu^-)N \times 10^{2 m^2}$</td>
<td>10.6 $\pm$ 3.3 $\pm$ 18</td>
<td>30</td>
<td>19.2</td>
</tr>
<tr>
<td>(incl.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda \mu^-$ capture $(10^3 s)$ $C \rightarrow B$</td>
<td>6.0 $\pm$ 0.4</td>
<td>9.4</td>
<td>6.0</td>
</tr>
<tr>
<td>to $1^+$ (exclus.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(GT) $^{12}C \rightarrow ^{12}N(9 S)$ (incl.)</td>
<td>37.9 $\pm$ 0.5</td>
<td>52.4</td>
<td>33.5</td>
</tr>
<tr>
<td>(exclus.)</td>
<td>0.99 $\pm$ 0.01</td>
<td>1.45</td>
<td>0.93</td>
</tr>
<tr>
<td>B(GT) $^{12}C \rightarrow ^{12}B(9 S)$ (exclus.)</td>
<td>0.87 $\pm$ 0.01</td>
<td>1.45</td>
<td>0.93</td>
</tr>
</tbody>
</table>
TABLE I. Results for cross sections, rates, and δ(GT) values. The units for (δ)(DAR) are 10^{-31} cm^2, the units for (δ)(DIF) are 10^{-40} cm^2, and the units for Λ are 10^12/s. The experimental values are compared to the calculated values based on the 0hω ground state wave function.

<table>
<thead>
<tr>
<th>Process</th>
<th>Final states</th>
<th>Expt.</th>
<th>(0+2)hω δs Gs</th>
<th>0hω δs Gs</th>
<th>0hω δs Gs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(δ)(DAR) 16O(ν, e−)16F</td>
<td>+ Parity</td>
<td>0.6</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>16.3</td>
<td>11.5</td>
<td>10.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>16.9</td>
<td></td>
<td>10.8</td>
<td></td>
</tr>
<tr>
<td>(δ)(DIF) 16O(ν, μ−)16F</td>
<td>+ Parity</td>
<td>13.1</td>
<td>8.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>15.8</td>
<td>11.3</td>
<td>10.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>31.7</td>
<td></td>
<td>20.3</td>
<td>10.4</td>
</tr>
<tr>
<td>A μ− capture 16O to 15N</td>
<td>+ Parity</td>
<td>23</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>139</td>
<td>89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>162</td>
<td></td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>(δ)(DAR) 15N(ν, e−)15O</td>
<td>+ Parity</td>
<td>33.0</td>
<td>21.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>11.8</td>
<td>7.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>44.8</td>
<td></td>
<td>28.9</td>
<td></td>
</tr>
<tr>
<td>(δ)(DIF) 15N(ν, μ−)15O</td>
<td>+ Parity</td>
<td>11.9</td>
<td>7.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>14.1</td>
<td>9.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>25.0</td>
<td></td>
<td>16.6</td>
<td></td>
</tr>
<tr>
<td>A μ− capture 15N to 14C</td>
<td>+ Parity</td>
<td>35</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>73</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>108</td>
<td></td>
<td>68</td>
<td></td>
</tr>
<tr>
<td>(δ)(DAR) 12C(ν, e−)12N</td>
<td>1^+ Gs</td>
<td>9.1±0.4±0.9 (Ref. [9,10])</td>
<td>14.6</td>
<td>9.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Parity (other)</td>
<td>0.7</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>8.4</td>
<td>5.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>14.1±1.2 (Refs. [9,10])</td>
<td>23.7</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>(δ)(DIF) 12C(ν, μ−)12N</td>
<td>1^+ Gs</td>
<td>0.65±0.14 (Ref. [10])</td>
<td>1.4</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Parity (other)</td>
<td>9.9</td>
<td>6.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>15.3</td>
<td>9.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity (π=0)</td>
<td>3.4</td>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>24.8</td>
<td></td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>A μ− capture 12C to 12B</td>
<td>1^+ Gs</td>
<td>6.0±0.4 (Ref. [8])</td>
<td>9.4</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Parity (other)</td>
<td>5.7</td>
<td>3.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>− Parity</td>
<td>37.3</td>
<td>23.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>37.9±0.5 (Ref. [6])</td>
<td>52.4</td>
<td>33.3</td>
<td></td>
</tr>
<tr>
<td>δ(GT) for 12C to 12N</td>
<td>1^+ Gs</td>
<td>0.99±0.01 (Ref. [29])</td>
<td>1.45</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>δ(GT) for 12C to 12B</td>
<td>1^+ Gs</td>
<td>0.87±0.01 (Ref. [29])</td>
<td>1.45</td>
<td>0.93</td>
<td></td>
</tr>
</tbody>
</table>

The flux of DAR neutrinos contains electron neutrinos with a maximum energy of 50 MeV. The DIF νμ flux has a maximum neutrino energy of about 300 MeV, but the threshold for the 12C(νμ, μ−)12N reaction is about 120 MeV. One should also note that the flux of νμ neutrinos drops steeply with energy above 150 MeV [10].

The experimental results for the DIF and DAR neutrino-12C flux-averaged cross sections are given in Table I. Note that in the case of the νμ neutrinos the exclusive cross section to the 12N ground state is a very small fraction of the inclusive cross section, while for the ννe neutrinos the exclusive cross section is more than half of the inclusive one. The reason is simply that the ννe cross section are due to the DAR neutrinos that contain only a low flux of neutrinos that can excite high-lying states and therefore the exclusive cross section is large compared to the inclusive one.

New 10.6±0.3±1.8 ** New 0.5±0.08±0.1 *** New 8.9±0.3±0.9
Older work W. Haxton and C. Johnson PRL 65, 1325 (1990) (0.6)
"Quenching of Weak Interactions in Nucleon Matter"

S. Cowell and V.R. Pandharipande

nucl-th/0211013 (Nov. 2002)
PRC 67, 035504, 2003

"The squares of charge current matrix elements are found to be quenched by 20-25% by short-range correlations in nucleon matter."

* Note that this reduction applies to both spin-independent and spin-dependent isovector matrix elements. The exception is the super-allowed (analog state) transition that is not quenched in this theory in agreement with experiment."