(i) Isospin breaking effects in $\bar{B} \rightarrow \pi\pi, \rho\rho, \rho\pi$

(ii) $\alpha$ from two-body $\rho\pi$

Jure Zupan

Carnegie Mellon University
Isospin breaking effects

- sources of isospin breaking
  - \( d \) and \( u \) charges different
  - \( m_u \neq m_d \)
- typical effect of isospin breaking
  \[ \sim \frac{m_u - m_d}{\Lambda_{QCD}} \sim \alpha_0 \sim 1\% \]

Questions:
- Are the isospin breaking effects that we can calculate of this order?
- Does any of the methods fare better?
Manifestations of isospin breaking

- extends the basis of operators to EWP $Q_7, ..., 10$
- mass eigenstates do not coincide with isospin eigenstates: $\pi - \eta - \eta'$ and $\rho - \omega$ mixing
- reduced matrix elements between states in the same isospin multiplet may differ e.g.
  \[ \langle \pi^+ \pi^- | Q_1 | B^0 \rangle \neq \frac{1}{\sqrt{2}} \langle \pi^+ \pi_3 | Q_1 | B^0 \rangle \]
- may induce $\Delta I = 5/2$ operators not present in $H_W$
$B(t) \rightarrow \pi \pi$
\[
\sin 2\alpha \text{ from } \\
\Gamma(B^0(t) \to \pi^+\pi^-) \propto [1 + C_{\pi\pi} \cos \Delta mt - S_{\pi\pi} \sin \Delta mt]
\]

\[
\sin(2\alpha_{\text{eff}}) = \frac{S_{\pi\pi}}{\sqrt{1 - C_{\pi\pi}^2}}
\]

\[
2\alpha = 2\alpha_{\text{eff}} - 2\theta
\]
Electroweak penguins

- separate triangle relations still hold
- neglecting $Q_{7,8}$

\[
H_{\Delta I = 3/2}^{\text{eff, EWP}} = -\frac{3}{2} \frac{C_9 + C_{10}}{C_1 + C_2} \frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} H_{\Delta I = 3/2}^{\text{eff, c-c}}
\]

\[\Rightarrow e^{i\gamma} A_{+0} = e^{-i(\gamma + 2\delta)} \bar{A}_{+0}, \quad \text{but still } |A_{+0}| = |\bar{A}_{+0}|
\]

\[\Rightarrow \alpha = \alpha_{\text{eff}} - \theta - \delta \quad \text{with } \delta = (1.5 \pm 0.3 \pm 0.3)^\circ \]

\[\text{conservatively? } \sim 2(|c_7| + |c_8|)/(|c_9|) < 0.2 \]

- the same shift in $\alpha$ from $\rho \rho$ and $\rho \pi$
- Q: how large error due to $Q_{7,8}$
\[ \pi^0 - \eta - \eta' \text{ mixing} \]

- \( \pi^0 \) w.f. has \( \eta, \eta' \) admixtures
  \[
  |\pi^0\rangle = |\pi_3\rangle + \epsilon|\eta\rangle + \epsilon'|\eta'\rangle
  \]
  where \( \epsilon = 0.017 \pm 0.003, \ \epsilon' = 0.004 \pm 0.001 \)  
  Kroll (2004)

- GL triangle relations no longer hold
  \[ A_{+} - \sqrt{2}A_{00} - \sqrt{2}A_{+0} \neq 0 \]

- estimate using QCD factorization
  - obtained \( \Delta \alpha \sim 5^\circ \) (including EWP)  
  Gardner (1999, 2005)

- use SU(3) decomposition for \( A_{0\eta'} \), \( A_{+\eta'} \) + neglect annihilation-like contribs. (\( @ \ 90\% \text{ CL} \))  

\[ |\Delta \alpha| \leq \sqrt{2 \frac{\tau_+}{\tau_0}} \left( \epsilon \sqrt{\frac{B_{0\eta}}{B_{+0}}} + \epsilon' \sqrt{\frac{B_{0\eta'}}{B_{+0}}} \right) < 1.05 \epsilon + 1.28 \epsilon' = 1.6^\circ \]

- Q: how well do we know \( \epsilon, \epsilon' \)?
Open questions

- Isospin breaking due to $\pi - \eta' \ (')$ mixing and EWP is of expected order
- How large other isospin breaking, still assumed
  - SU(2) for $A_{+-}, A_{+3}, A_{33}$ reduced matrix elements
  - $A_{5/2}$ set to zero
- There is an experimental check, due to $\pi - \eta - \eta'$ mixing
  - $|A_{+0}| \neq |\bar{A}_{+0}|$
$B(t) \rightarrow \rho\rho$
Isospin breaking

- shift due to EWP exactly the same as in $\pi\pi$
- as in $\pi\pi$ not much can be said about breaking in the reduced matrix elements and $A_{5/2} \neq 0$
- couplings $g_I \equiv g(\rho_I \rightarrow \pi^+\pi^-)$ and $g_c \equiv g(\rho^+ \rightarrow \pi^+\pi_3)$ may not be equal
  - $g_c = g(\rho^+ \rightarrow \pi^+\pi^0) + O(\epsilon^2)$
  - PDG: $g_c/g_I - 1 = (0.5 \pm 1.0\%)$
- since $\Gamma_\rho \neq 0 \Rightarrow I = 1$ contributions possible
- $\rho - \omega$ mixing

---

Falk, Ligeti, Nir, Quinn (2003)

J. Zupan  Isospin breaking effects...  INT2005 – p. 10
Effect of $\rho - \omega$ mixing

- large effect expected because
  \[ |A(B^+ \rightarrow \rho^+ \omega)/A(B^+ \rightarrow \rho^+ \rho^0)| = 0.69 \pm 0.14 \]

- an estimate: use $SU(3)$ relation & $P/T = 0.2$
  & same strong phase for $P, T$

- similar effect seen in $D^0 \rightarrow K_S \pi^+ \pi^-$

- integrated effect of $\omega$ resonance is $< 2\%$
\[ B(t) \rightarrow \rho \pi \]
\[ B \rightarrow \pi^+\pi^-\pi^0 \text{ Dalitz plot} \]

- model the Dalitz plot (similarly for \( A(\bar{B}^0 \rightarrow 3\pi) \))

\[
A(B^0 \rightarrow \pi^+\pi^-\pi^0) = A(B^0 \rightarrow \rho^+\pi^-) D_{\rho\rho}(s_+) \cos \theta_+ + A(B^0 \rightarrow \rho^-\pi^+) D_{\rho\rho}(s_-) \cos \theta_- + A(B^0 \rightarrow \rho^0\pi^0) D_{\rho\rho}(s_0) \cos \theta_0
\]

- other resonances need to be included in the fit
- \( \rho - \omega \) mixing treated in the same way as in \( \rho \rho \)
- possible to determine

\[
A_+, \bar{A}_+, A_-, \bar{A}_-, A_0, \bar{A}_0
\]

up to overall phase \( \Rightarrow 11 \) independent measurable
Snyder-Quinn

Snyder, Quinn (1993), Lipkin et al. (1991), Gronau (1991)

- rescale $A_i(\bar{A}_i) \rightarrow e^{i\beta} A_i(e^{-i\beta} \bar{A}_i)$

- tree and penguin defined according to CKM

\[ \mathcal{A}_{\pm,0} = e^{-i\alpha} T_{\pm,0} + P_{\pm,0}, \quad \bar{\mathcal{A}}_{\pm,0} = e^{+i\alpha} T_{\pm,0} + P_{\pm,0} \]

- an isospin relation only between penguins

\[ P_0 + \frac{1}{2}(P_+ + P_-) = 0 \]

(EWP and isospin breaking neglected)

- 10 unknowns: e.g. $\alpha, |t_\pm|, |t_0|, \arg t_\pm, |p_\pm|, \arg p_\pm$

enough info to determine them
Effect of isospin breaking

- Isospin breaking affects only the relation between penguins!
- Largest contribution from EWP because they are related to tree

\[ P_- + P_+ + 2P_0 = P_{EW} \]

Where \( P_{EW} \) can be obtained from

\[ \mathcal{A}_+ + \mathcal{A}_- + 2\mathcal{A}_0 = Te^{-i\alpha} + P_{EW} \]

\[ \frac{P_{EW}}{T} = -\frac{3}{2} \left( \frac{c_9 + c_{10}}{c_1 + c_2} \right) \frac{|V_{tb}V_{td}|}{|V_{ub}V_{ud}|} = +0.013 \frac{\sin(\beta + \alpha)}{\sin \beta} \]

Other isospin breaking effects are \( P/T \sim 0.2 \) suppressed, e.g. \( \pi^0 - \eta - \eta' \) mixing \[ |\Delta \alpha_{\pi^-\eta-\eta'}| \leq 0.1^\circ \]

Gronau, JZ (2005)
$\alpha$ from two-body $\rho^{\pm} \pi^{\mp}$
$B \rightarrow \rho^\pm \pi^\mp$

- use just $\rho^\pm \pi^\mp$, no info from interference regions
- tree and penguin defined according to CKM

$$A(B^0 \rightarrow \rho^\pm \pi^\mp) = e^{i\gamma t_\pm} + p_\pm$$

$$= V_{ub}^* V_{ud} + V_{cb}^* V_{cd}$$

- 8 unknowns:
  $$|t_\pm|, |p_\pm / t_\pm|, \arg(p_\pm / t_\pm), \alpha,$$
  $$\delta_t = \arg(t_- / t_+)$$

- 6 measurables: $C, \Delta C, S, \Delta S, \Gamma^{\rho\pi}, A_{CP}$
- penguin contributions small
  - use SU(3) to constrain them
  - calculate them

Gronau, JZ (2004)
4 new observables: \[ \Gamma(B^0 \rightarrow K^{*+}\pi^-), \Gamma(B^+ \rightarrow K^{*0}\pi^+), \Gamma(B^0 \rightarrow \rho^-K^+), \Gamma(B^+ \rightarrow \rho^+K^0) \]
\[ \Leftarrow \text{not yet measured} \]
Info on penguins

- SU(3) bounds on penguins (90% CL)
  \[ 0.14 \leq |p_+/t_+| \leq 0.25 \quad 0.14 \leq |p_-/t_-| \leq 0.34 \]

- QCD factorization
  \[ |p_+/t_+| = 0.10^{+0.06}_{-0.04} \text{ and } |p_-/t_-| = 0.10^{+0.09}_{-0.05} \]

Beneke, Neubert, 03
\[ \alpha \] from quasy 2-body

- assumed \( \arg(t_-/t_+) \) small to resolve ambiguities
  \[ \alpha = (94 \pm 4 \pm 15)^\circ \]
  \( \alpha - \alpha_{\text{eff}} \) +SU(3) breaking

- no interference information used (BaBar+Belle)
- bound on penguins used, not fit
- annihilation like topologies neglected

- in future can use unconstrained fit to obtain \( \alpha \)
- one solution for \( \alpha \), all ambiguities resolved
- SU(3) breaking on extracted \( \alpha \) are small, of order \( p_\pm^2/t_\pm^2 \) \( \Rightarrow \) MC study with up to 30\% SU(3) breaking on penguins gives \( \sqrt{\langle (\alpha^{\text{out}} - \alpha^{\text{in}})^2 \rangle} \sim 2^\circ \)
Conclusions

- calculable isospin breaking effects in $\alpha$ extraction are of expected size
- uncalculated isospin breaking effects in $B \rightarrow \rho\pi$ are suppressed by $P/T$ and
- an alternative of extracting $\alpha$ from $B \rightarrow \rho^{\pm}\pi^{\mp}$ alone has an ultimate theory error of a few degrees but does not use info from tails of BW functions
\[ B \rightarrow \pi \pi \]

- completely general isospin decomposition

\[
A_{+-} = \langle \pi^+ \pi^- | H | B^0 \rangle = -A_{1/2} + \frac{1}{\sqrt{2}} A_{3/2} - \frac{1}{\sqrt{2}} A_{5/2}
\]

\[
A_{00} = \langle \pi^0 \pi^0 | H | B^0 \rangle = \frac{1}{\sqrt{2}} A_{1/2} + A_{3/2} - A_{5/2}
\]

\[
A_{+0} = \langle \pi^+ \pi^0 | H | B^+ \rangle = \frac{3}{2} A_{3/2} + A_{5/2}
\]

- neglecting \( A_{5/2} \sim \alpha A_{1/2} \) (i.e. \( \sim 1\% \) correction)

\[
A_{+-} + \sqrt{2} A_{00} = \sqrt{2} A_{+0}
\]

\[\bar{A}_{+-} + \sqrt{2} \bar{A}_{00} = \sqrt{2} \bar{A}_{+0}\]

- neglecting EWP \( \Rightarrow A_{+0} \) only tree contribs.

\[
e^{i\gamma} A_{+0} = e^{-i\gamma} \bar{A}_{+0} \Rightarrow |A_{+0}| = |\bar{A}_{+0}|\]

Gronau, London (1990)

J. Zupan Isospin breaking effects... INT2005 – p. 23
Using data

- use SU(3) decomposition for $A_{0 \eta(')}$, $A_{+ \eta(')}$
  + neglect annihilation-like contributions

\[
A_{+-} = t + p \quad \xleftrightarrow{SU(2)} \quad A_{33} = \frac{1}{\sqrt{2}}(c - p) \quad \xleftrightarrow{SU(2)} \quad A_{+3} = \frac{1}{\sqrt{2}}(t + c)
\]

\[
\uparrow \quad SU(3) \quad \downarrow
\]

\[
A_{3\eta} = \frac{1}{\sqrt{6}}(2p + s) \quad A_{3\eta'} = \frac{1}{\sqrt{3}}(p + 2s)
\]

\[
A_{+\eta} = \frac{1}{\sqrt{3}}(t + c + 2p + s) \quad A_{+\eta'} = \frac{1}{\sqrt{6}}(t + c + 2p + 4s)
\]
Using data II

triangle relation is modified only slightly

\[ A_{+-} + \sqrt{2}A_{00} - \sqrt{2}A_{+0}(1 - e_0) = 0 \]

where \( e_0 = \sqrt{\frac{2}{3}}\epsilon + \sqrt{\frac{1}{3}}\epsilon' = 0.016 \pm 0.003 \)

\( A_{+0} \) is a sum of pure \( \Delta I = 3/2 \) amplitude \( A_{+3} \) with weak phase \( \gamma \) and isospin-breaking terms

\[ A_{+0} = A_{+3}(1 + e_0) + \sqrt{2}\epsilon A_{0\eta} + \sqrt{2}\epsilon' A_{0\eta'} . \]

while \( e^{i\gamma}A_{+3} = e^{-i\gamma}\bar{A}_{+3} \) no longer \( e^{i\gamma}A_{+0} = e^{-i\gamma}\bar{A}_{+0} \)

also \( |A_{+0}| \neq |\bar{A}_{+0}| \iff \) exp. check
Using data III

- varying the phases of $A_{0\eta'}$, $\bar{A}_{0\eta'}$ gives bound

$$|\Delta \alpha_{\pi-\eta-\eta}| \leq \sqrt{2} \frac{\tau_+}{\tau_0} \left( \epsilon \sqrt{\frac{B_{0\eta}}{B_{+0}}} + \epsilon' \sqrt{\frac{B_{0\eta'}}{B_{+0}}} \right)$$

- at 90% CL using WA values

$$|\Delta \alpha_{\pi-\eta-\eta'}| < 1.05\epsilon + 1.28\epsilon' = 1.6^\circ$$

- the bound can be improved using the SU(3) relations

$$A_{+\eta'} = \frac{\sqrt{2}}{\sqrt{3}} A_{+0} + \sqrt{2} A_{0\eta'}$$

leading to

$$|\Delta \alpha_{\pi-\eta-\eta'}| < 1.4^\circ$$
SU(3) decompos. for $\pi\pi$, interm. result

\[ A_{+0} = A_{+3} + \epsilon A_{+\eta} + \epsilon' A_{+\eta'} \]

\[ = \frac{1}{\sqrt{2}}(t + c)(1 + e_0) + \frac{1}{\sqrt{3}}\epsilon(2p + s) + \frac{\sqrt{2}}{\sqrt{3}}\epsilon'(p + 2s) \]

\[ A_{00} = A_{33} + \sqrt{2}\epsilon A_{3\eta} + \sqrt{2}\epsilon' A_{3\eta'} \]

\[ = \frac{1}{\sqrt{2}}(c - p) + \frac{1}{\sqrt{3}}\epsilon(2p + s) + \frac{\sqrt{2}}{\sqrt{3}}\epsilon'(p + 2s) \]
\[ g(\rho^+ \to \pi^+\pi^0) \]

- \( \pi - \eta - \eta' \) mixing induces

\[
g(\rho^+ \to \pi^+\pi^0) = g(\rho^+ \to \pi^+\pi_3) + \epsilon g(\rho^+ \to \pi^+\eta) + \epsilon' g(\rho^+ \to \pi^+\eta')
\]

- at (84\% CL)

\[
\left| \frac{g(\rho^+ \to \pi^+\eta)}{g(\rho^+ \to \pi^+\pi_3)} \right| = \left[ \left( 1 - \frac{m_{\eta}^2}{m_{\rho}^2} \right) \frac{Br(\rho^+ \to \pi^+\eta)}{Br(\rho^+ \to \pi^+\pi_3)} \right]^{1/2} \leq 0.055
\]
First bound $B \rightarrow \pi\pi$

$$(\Delta\alpha - \Delta\alpha_0)_{\pi-\eta-\eta'} \equiv \frac{1}{2} \text{Arg}(e^{2i\gamma} \bar{A}_0 A_{+0}^*)$$

$$= \frac{1}{\sqrt{2}|A_{+0}|} \left[ \epsilon \left(|\bar{A}_0\eta| \sin \bar{\psi}_\eta - |A_0\eta| \sin \psi_\eta\right) + \epsilon' \left(|\bar{A}_0\eta'| \sin \bar{\psi}_{\eta'} - |A_0\eta'| \sin \psi_{\eta'}\right) \right]$$

extreme at $\bar{\psi}_{\eta'}(\eta) = -\psi_{\eta'}(\eta) = \pi/2$

$$|\Delta\alpha - \Delta\alpha_0|_{\pi-\eta-\eta'} \leq \epsilon \left(\frac{|A_0\eta| + |\bar{A}_0\eta|}{\sqrt{2}|A_{+0}|}\right) + \epsilon' \left(\frac{|A_0\eta'| + |\bar{A}_0\eta'|}{\sqrt{2}|A_{+0}|}\right)$$

experimental input

$$-\frac{\tau_+}{\tau_0} = 1.081 \pm 0.015 \quad , \quad B_{+0} = (5.5 \pm 0.6) \times 10^{-6}$$

$$B_{0\eta} < 2.5 \times 10^{-6} \quad (90\% \text{ CL}) \quad B_{0\eta'} < 3.7 \times 10^{-6} \quad (90\% \text{ CL})$$
Improved bound $\pi \pi$

using

$$A_{+\eta(\prime)} = \frac{\sqrt{2}}{\sqrt{3}} A_0 + \sqrt{2} A_{0\eta(\prime)}$$

and ignoring info from CP asymmetries gives a bound

$$|(\Delta \alpha - \Delta \alpha_0)_{\pi - \eta - \eta}| \leq \sqrt{2 \frac{\tau_+}{\tau_0}} \left( \epsilon \sqrt{\frac{B_{0\eta}}{B_{+0}}} (1 - r_{\eta}) + \epsilon' \sqrt{\frac{B_{0\eta'}}{B_{+0}}} (1 - r_{\eta'}) \right)$$

with

$$r_{\eta} = \frac{3}{16} \frac{\left[ \sqrt{\frac{\tau_0}{\tau_+}} (B_{+\eta} - \frac{2}{3} B_{+0}) - 2 \sqrt{\frac{\tau_+}{\tau_0}} B_{0\eta} \right]^2}{B_{+0} B_{0\eta}}$$

and

$$r_{\eta'} = \frac{3}{8} \frac{\left[ \sqrt{\frac{\tau_0}{\tau_+}} (B_{+\eta'} - \frac{1}{3} B_{+0}) - 2 \sqrt{\frac{\tau_+}{\tau_0}} B_{0\eta'} \right]^2}{B_{+0} B_{0\eta'}}$$
Generalities for $B \rightarrow \rho\rho$

Isospin analysis in the same spirit as for $B \rightarrow \pi\pi$

- 3 separate isospin relations (for each polarization)
- almost completely longitudinally polarized
- since $\Gamma_\rho \neq 0 \Rightarrow I = 1$ contributions possible

$O(\Gamma^2_\rho/m^2_\rho)$ effect

possible to constrain experimentally

Falk, Ligeti, Nir, Quinn (2003)
Effect of $\rho - \omega$ mixing

- only relevant for $B^+ \rightarrow \rho^+ \rho^0$ and $B^0 \rightarrow \rho^0 \rho^0$
- the effect of $\rho - \omega$ can be determined by fits to $m_{\pi^+\pi^-}$ distributions
- focusing on $B^+ \rightarrow \rho^+ \rho^0$

$$A_{+0}(s_{12}, s_{34}) = g_cg_I \left[ A(B^+ \rightarrow \rho^+ \rho_I)D_{\rho\rho}(s_{34}) 
+ A(B^+ \rightarrow \rho^+ \omega_I)\tilde{D}_{\rho\omega}(s_{34}) \right]$$

- the scalar part of $\rho$ propagator near the pole
  $$D_{\rho\rho}(s) = \frac{1}{s - m_{\rho}^2 + i m_{\rho} \Gamma_{\rho}}$$
- the mixed $\rho\omega$ contribution has a double pole
  $$\tilde{D}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(s)D_{\rho\rho}(s)D_{\omega\omega}(s)$$
\[ \pi - \eta^{(\prime)} \text{ in } B \to \rho\pi \]

- \[ A_+ + A_- + 2A_0 = Te^{-i\alpha} + P_{EW} + \epsilon P_{\rho\eta} + \epsilon' P_{\rho\eta'} \]

- **SU(3) decomposition**

  \[ |T| = |t_P + t_V + c_P + c_V| \]

  \[ P_{\rho\eta} = \frac{1}{\sqrt{6}}(-p_P - p_V - s_V) \]

  \[ P_{\rho\eta'} = \frac{1}{2\sqrt{3}}(p_P + p_V + 4s_V) \]

  using \[ |T| \geq |t_V|, \frac{|p_V|}{|t_V|} = 0.2 \]

  \[ |P_{\rho\eta}| \simeq \frac{1}{\sqrt{6}}|s_V| \leq \frac{0.3}{\sqrt{6}}|p_V| \]

  \[ |P_{\rho\eta'}| \simeq \frac{2}{\sqrt{3}}|s_V| \leq \frac{0.6}{\sqrt{3}}|p_V| \]

  \[ |\Delta \alpha_{\pi-\eta-\eta'}| = \frac{|\epsilon P_{\rho\eta} + \epsilon' P_{\rho\eta'}|}{|T|} \leq 0.024\epsilon + 0.069\epsilon' \leq 0.1^\circ \]

- From global SU(3) fits
  - \[ \arg t_P \sim \arg t_V \]
  - \( c_P, c_V \) smaller
  - \[ |p_P/t_P| \sim |p_V/t_V| \sim 0.2, \]
  - \( p_V \sim -p_P, s_V \) smaller
$B \rightarrow \pi\pi$ in SCET

$$A = \frac{G_F m_B^2}{\sqrt{2}} \left\{ f_{M_1} \int_0^1 du dz T_{1J}(u, z) \zeta^{B M_2}_J(z) \phi^{M_1}(u) ight. + f_{M_1} \zeta^{B M_2} \int_0^1 T_{1\zeta}(u) \phi^{M_1}(u) \left. \right\} + \left\{ 1 \leftrightarrow 2 \right\} + \lambda_c^{(f)} A_{c\bar{c}}^{M_1, M_2}.$$  

• **Isospin breaking** in $T_{iJ}(u), T_{i\zeta}(u)$ is $1/m_B$ suppressed

• **Remaining isospin violation** is encoded in $\zeta^{BM}, \zeta^B_J(z), f_M \phi^M(u)$, and $A_{c\bar{c}}^{M_1, M_2}$, with $M_{1,2}$ isospin eigenstates
Bounds on penguin

this leads to the following ranges at (90% CL)

\[
0.14 \leq |p_+/t_+| \leq 0.25
\]

\[
0.14 \leq |p_-/t_-| \leq 0.34
\]

the ranges because of unknown strong phases

\[\arg(p_+/t_+)\] that need to be varied

QCD factorization

\[|p_+/t_+| = 0.10^{+0.06}_{-0.04}\text{ and }|p_-/t_-| = 0.10^{+0.09}_{-0.05}\]

Beneke, Neubert, 03
\( \alpha_{\text{eff}} \)

- following \( B(t) \rightarrow \pi^+ \pi^- \) define (not observables (!))

\[
\alpha_{\text{eff}}^\pm \equiv \frac{1}{2} \arg \left( e^{-2i\beta A} A^* \right) = \alpha + O(r) 
\]

- the observables are

\[
2\alpha_{\text{eff}}^\pm \hat{\delta} \equiv \arg \left( e^{-2i\beta A} A^* \right) = \arcsin \left( \frac{S \pm \Delta S}{\sqrt{1 - (C \pm \Delta C)^2}} \right) 
\]

with \( \hat{\delta} = \delta_t + O(r) \)

- the average we define to be

\[
\alpha_{\text{eff}} \equiv \frac{1}{2} \left( \alpha_{\text{eff}}^+ + \alpha_{\text{eff}}^- \right) = \alpha + O(r) 
\]
Bound on $\alpha_{\text{eff}}$

- the difference bounded by SU(3), at 90% CL
  \[ |\alpha_{\text{eff}}^+ - \alpha| \leq 7.3^\circ - 11.7^\circ, \quad |\alpha_{\text{eff}}^- - \alpha| \leq 15.4^\circ \]

- combining the two bounds
  \[ |\alpha_{\text{eff}} - \alpha| \leq 11.3^\circ - 13.5^\circ \]

- assume that $|\delta_t| \ll 90^\circ$ (QCD fact.: $\delta_t = (1 \pm 3)^\circ$)

- this distinguishes ambiguous solutions for $2\alpha_{\text{eff}}^\pm \pm \hat{\delta}$

  \[ (2\alpha_{\text{eff}}^+ + \hat{\delta}) - (2\alpha_{\text{eff}}^- - \hat{\delta}) = 2\delta_t + O(r_{\pm}) \]

- in $(0^\circ, 180^\circ)$ range
  \[ \alpha_{\text{eff}} = \{94^\circ \pm 4^\circ, 175^\circ \pm 4^\circ\} \]
\( \alpha \) from quasy 2-body

- assumed \( \arg(t_-/t_+) \) small to resolve ambiguities

\[
\alpha = (94 \pm 4 \pm 15)^\circ
\]

- no interference information used (BaBar+Belle)
- bound on penguins used, not fit
- annihilation like topologies neglected
- in future can use unconstrained fit to obtain \( \alpha \)
- one solution for \( \alpha \), all ambiguities resolved
- SU(3) breaking on extracted \( \alpha \) are small, of order

\[
p^2_-/t^2_+ \Rightarrow MC\ study\ with\ up\ to\ 30\%\ SU(3)\ breaking\ on\ penguins\ gives\ \sqrt{\langle (\alpha^{\text{out}} - \alpha^{\text{in}})^2 \rangle} \sim 2^\circ
\]