Moment Analysis in $B \to X_c \ell \nu$

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and

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• Could some scheme be better than others?

  Yes

  The difference can be just “a scheme” (renormalization), or lie in the approach itself

  A HQP $xx$ is $xx = \ldots \pm \ldots$ So what?

  Depends on the scheme!

• How useful are $b \to c$ moments for $b \to u$ decays?

  Very useful! (For a model-independent approach)

• More technical issues:

  the error estimates, the error treatment

• My question: are only $V_{cb}$, $V_{ub}$ or other UT elements with as many digits as possible, the goal?
• Inclusive semileptonic $b \to c \ell \nu$ distributions can and should be further explored

• We have good control over inclusive $B \to X_s + \gamma$ and $B \to X_u \ell \nu$ decay characteristics with the today's experimental capabilities, if fully use the potential of the OPE, in particular utilizing OPE fit results from $B \to X_c \ell \nu$

• Inclusive studies shed light on $B$ physics in general, including the dynamics driving their specific decays

**************************************************************************
All available clv moments plus the BELLE bsg moments at 1.8 GeV and bsg CLEO at 2.0 and BABAR moments at 1.9 GeV
ALL BSG moments are fitted with BIAS correction "active"
**************************************************************************

FCN= 18.23169 FROM MINOS STATUS=SUCCESSFUL 921 CALLS 1464 TOTAL
EDM= 0.17E-07 STRATEGY= 1 ERROR MATRIX

ACCURATE

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Why moments? Has been partially addressed by E.B. the rest – in the talk

The decay probability is not truly short-distance near the free parton kinematics where $M^2_X \approx M^2_D$, near the end-point of lepton spectrum, etc., the nonperturbative effects blow up. Nevertheless, the polynomial integrals are expressed through local operators – hence the moments

Now when the 1% threshold for $V_{cb}$ has been met, we must work hard to justify the accuracy

Interesting physics lies not only in $V_{cb}$

Precision values of $m_b, m_c$ may eventually test FSM* as precisely as CKM does SM

Any correct treatment in the consistent scheme would yield the same numerical result for $V_{cb}$ within its accuracy

Q: Can we do without adopting a scheme at all?

A: Not while doing calculations. Can display the results in the ‘observable through observable’ form, but in the era of computers this hardly saves time. But we would loose lots of physics cf. end of the talk

*Future Standard Model
Why ‘kinetic scheme’?

Calculations have been done correctly, including (but not limited to) perturbative corrections do not need to call $\alpha_s^2$ terms $\alpha_s^1$, etc.

Theoretically sound /completely formulated

OPE is done according to Wilson

reduced number of new HQ parameters

$m_b(\mu), \mu_\pi^2(\mu), \rho_3^D(\mu) \ldots$ are well-defined and have direct physics

meaning \{ $\implies$ their values are constrained e.g. $\mu_\pi^2 \geq \mu_G^2$

$\implies$ lead to informative relations see later

Do not need to impose mass constraints on $m_b-m_c$ (though this always remains an option)

Free from expanding in $1/m_c$, even if it is $\frac{8}{m_cm_b}$

All this helps making substantiated error estimates

many alternative suggestions are difficult to accept

Calculations are quite stable, show mild cut-dependence; minimal scale-dependence little time-dependence
Running coupling resummation (BLM) is not a problem:

With the IR piece cut off according to Wilson we can work for precision!

A similar analysis may not be possible in alternative approaches

So, calculate all the observables in terms of

\[ m_b(1 \text{ GeV}), m_c(1 \text{ GeV}), \mu_\pi^2(1 \text{ GeV}), \mu_G^2, \rho_D^3, \rho_{LS}^3, \ldots \]

and confront with the experimental data

Aquila, P.G., Ridolfi, N.U. hep-ph/0503083

Alternative analyses: Bauer et al., hep-ph/0210027, /0408202

Opinion: Fits should be left to professionals (experiments) theorists provide only logistic support
A comprehensive fit including all moment measurements: 
(by the professionals)

Experimental fit to all these data:

\[ V_{cb} = (4.144 \pm 0.043) \cdot 10^{-3} \]

\[ (4.156 \pm 0.047) \quad \text{only} \]

Not all the sources of theory errors are possibly included
• SL decays yielded accurate $m_b$ itself not obvious a priori

The combination $m_b - 0.74m_c$ is determined with only a 15 MeV error bar!

$V_{cb}$ depends on nearly the same combination of $m_b$ and $m_c$ – that is why it is so accurate

Running ‘kinetic’ mass is an observable and has no intrinsic limitation on precision

Theoretical expectation: $m_b(1 \text{ GeV}) = (4.57 \pm 0.06) \text{ GeV}$

Voloshin 1995–1996
Melnikov, Yelkhovsky 1998–1999
Beneke, Signer

$e^+e^- \rightarrow \gamma(1S, 2S, 3S, 4S, 5S)$
moments of $\sigma(e^+e^- \rightarrow b\bar{b})$
What is still missing:

- $\alpha_s$-corrections to the power-suppressed Wilson coefficients: the principal limiting factor

S. Gardner, 10/2001

- Is charm sufficiently heavy? we do not expand in $\frac{1}{m_c}$, yet

Effects of the nonperturbative four-quark expectation values with charm $\langle B|\bar{b}c\bar{c}b|B\rangle$ loosely referred to as ‘Intrinsic Charm’

Required in the consistent OPE

see Benson et al., hep-ph/0302262

Analysis (Zwicky, I.B. & N.U., to appear):

Some surprises in the $1/m$ expansion. The effect appears at the sub-% level in $\Gamma_{sl}$, is expected below 0.5% due to cancellations

Experiment directly constrains the effect at 1 to 2% level

Expect improvement down to 0.5% where it would not affect precision of $V_{cb}$
Theory ‘error bars’

No theoretical equation has absolute precision, in particular with truncated OPE
discarded $\mu_{\text{hadr}}^{4+k}$ terms

uncalculated non-BLM $O(\alpha_s^2)$ corrections

advantage of the kinetic scheme: suppressed perturbative corrections;
the bulk of non-BLM corrections can be readily accounted for

uncalculated $\alpha_s$-corrections to Wilson coefficients of
nonperturbative operators yielding terms $\frac{\alpha_s}{\pi} \mu_{\text{hadr}}^{2+k}$

No recipe for theory error bars can be completely correct; we try to get the principal things right

Guidelines:

• vary power-suppressed Wilson coefficients by
  $\pm 20\%$ for $\mu_\pi^2$ and $\mu_G^2$, by $\pm 30\%$ for $\rho_D^3$ and $\rho_{LS}^3$

  example: Schwinger anomalous magnetic moment, $1 + \frac{\alpha}{2\pi}$
  anomalous dimension of ‘$\mu_G^2$’ in QED vanishes, so using only scale variation would mislead

• assume the additive ‘uncertainty’ in $m_b$ and in $\mu_\pi^2$,
  $\delta m_b \approx 20$ MeV, $\delta \mu_\pi^2 \approx 0.02$ GeV$^2$

  dictated by the non-BLM running between $\mu = 0.5$ GeV and 1.5 GeV
• For higher moments, assume no particular correlations in the central moments (with respect to average); this implies strong correlation for the usual moments!

  Physically transparent on example of $b \rightarrow s + \gamma$

• additional uncertainty for biases at high cuts at and above 1.5 GeV

Differ significantly from Bauer et al. The prescribed counting yields unrealistically small theory uncertainties, typically by more than an order of magnitude. However, since central moments involve significant cancellations, the estimated accuracy often appears similar in size.

Many limitations:

some theory uncertainties are correlated and therefore the predictions for the relations between different moments may be more accurate

Corrections to Wilson coefficients in general depend not only on the moment, but also on the cut in lepton energy. Clearly cannot be independent at close cuts, but vary for very different cuts

Open question: how these uncertainties are treated in practical fits?

Some way to fit may actually reduce the uncertainty (‘measuring’ Wilson coefficients). This not always would be justified
Bauer et al., hep-ph/0408202:

Do not reproduce our predictions for hadronic moments
Claim large ‘scale-dependence’ in our Wilsonian scheme
Claim we underestimate theory uncertainty by up to 10 times

For other moments $\mu$-dependence is even less significant

Suppressed $\mu$-dependence ($\propto \alpha_s^2$) is a cross-check of algebra...

Power counting by Bauer et al. do not respect Wilsonian scale-independence holds in our scheme

‘1S scheme’: Use ‘$m_b(\eta_b)$’ instead of ‘$m_b(1S)$’, nothing changes yet $\delta m_b \simeq -25 \text{ MeV}$ much stronger scale dependence!

Varying $\mu$ is a useful probe for the potential size of higher orders in $\alpha_s$ (non-BLM). In this case it probably underestimates them

Introducing different normalization scales for different masses and/or nonperturbative operators yields a somewhat larger variation more realistically representing the actual higher-order effects
More on ‘scale-dependence’ in our scheme

Higher lepton energy moments:

\[ \Delta = \sqrt{(\delta m_b M_{m_b}')^2 + (\delta \mu_{\pi}^2 M_{\mu_{\pi}}')^2 + (\delta \rho_D^3 M_{\rho_D}^3)^2} \]

\[ \delta m_b = 10 \text{ MeV} \quad \delta \mu_{\pi}^2 = 0.02 \text{ GeV}^2 \quad \delta \rho_D^3 = 0.03 \text{ GeV}^3 \]

Higher hadron mass squared moments:
No apparent problem with $\langle M_X^2 \rangle$ vs. $E_{\text{cut}}^\ell$

Robust OPE approach à la Wilson, $\mu = 1$ GeV:

![Graph showing data and expectations as of July 2003.](image)

Parameters fixed from the BaBar fit

hep-ex/0404017

Good agreement where the right theory is used right

OPE seems to work even where may be expected to break down
Have an accurate and reliable determination of many HQ parameters from experiment

Extracting $|V_{cb}|$ from $\Gamma_{\text{s1}}(B)$ has good accuracy and solid grounds

Have precision checks of the OPE at the nonperturbative level

Overall there are many remarkable agreements with predictions

I think the most impressive is good consistency between $\langle M_X^2 \rangle$ and $\langle E_\ell \rangle$: A sensitive check of the nonperturbative sum rule for $M_B - m_b$

Important: the HQ values emerge in accord with the theoretical expectations: $m_b, \mu_\pi^2 > \mu_G^2, \ldots$

the right scale for $\rho_D^3$

Theory seems to work too well? ‘Theoretical correlations’

Need to check in a different environment:

consider $b \rightarrow$ light $q$ decays

Local duality and its violations

Validity of the OPE itself was questioned under ‘duality’ term

Sometimes is regarded as an experiment-only accessible issue

then one should not forget about a successful prediction

it must be very small in inclusive rates at low cut
Using \( b \to c \ell \nu \) information for \( b \to u \ell \nu \)?

Contrary to naive nonrelativistic quark models, the heavy quark distribution function in QCD entering \( b \to c \ell \nu \) is different from the one describing \( b \to u \ell \nu \) or \( b \to s + \gamma \) inclusive decays.

Nevertheless, the OPE relates the most important lowest moments, in particular those associated with \( m_b \) and \( \mu_\pi^2 \). This may look not obvious when perturbative corrections are superimposed onto ‘primordial’ Fermi motion, but remains true.

The achieved precision in \( b \to c \) is higher, and it can be utilized to make predictions for \( b \to u \) constrained rates benefit from the values of \( m_b, \mu_\pi^2, \ldots \); eliminating them in favor of the light-cone distribution function alone counterparts skepticism about the OPE relations.
Soft gluons $|k_\mu| \lesssim \mu_{\text{hadr}}$ are included into HQ distribution function $F(x)$ (Fermi motion). Other, hard gluons are in the Wilson coefficients (kernel)

$$dW^{\text{pert}} = \int \frac{d\omega}{\omega} \int \frac{dk_\perp^2}{k_\perp^2} C_F \frac{\alpha_s(k_\perp^2)}{\pi} \ dW_{\text{born}},$$

However, ‘hard gluons’ may be energetic yet highly collinear, then $k_\perp \lesssim \mu_{\text{hadr}}$, in this case they are in fact nonperturbative

$$k_+ \sim \frac{k_\perp^2}{|\vec{k}|}$$

Nevertheless, the integer moments of $M_X^2$ or $E_\gamma$ are not affected by strong-coupling domain. They still are given by local heavy quark operators, plus genuinely short-distance perturbative corrections

N.U. hep-ph/0407359

Physics behind: growth of $\alpha_s$ is due to final state interactions

Yet the nonperturbative physics of an effective theory without hard collinear modes is different from actual QCD
● \( b \to s + \gamma \) moments?

Problems were faced when relied on relations \textit{imprecise} with a high cut on \( E_\gamma \)

\[
\langle E_\gamma \rangle = \frac{m_b}{2} + ... \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = \frac{\mu_2^2}{12} + ...
\]

A good way to accurately measure HQ parameters...

\textbf{Bottle neck: ‘Hardness’ \( Q \) often gets too low with the cuts}

\( Q \simeq m_b - m_c \) for total widths, but \( Q \) is below 1 GeV for \( E_\ell > 1.7 \) GeV

A complementary consideration suggests the expansion for \( M_X^2 \) loses sense for \( E_{\text{cut}} \geq 1.7 \) GeV

Terms appear \( \propto e^{-\frac{Q}{\mu_{\text{hadr}}}} \)

\textbf{In} \( b \to s + \gamma \) \( Q \simeq M_B - 2E_{\text{min}} \simeq 1.2 \) GeV

if the cut is at \( E_\gamma = 2 \) GeV

\[
\tilde{\delta m_b} \quad \tilde{\delta \mu_2^2}
\]

\[
GeV \quad GeV^2
\]

\[
1.6 \quad 1.7 \quad 1.8 \quad 1.9 \quad 2 \quad 2.1 \quad 2.2
\]

Accounting for these biases yielded a good agreement between all measurements
Perturbative corrections with the explicit Wilsonian cutoff have been calculated including all orders in BLM
Benson, Bigi, N.U. hep-ph/0410080

Breaking news: complete $\alpha_s^2$ spectrum!
Melnikov, Mitov 05/2005

BELLE 2004: With $E_\gamma > 1.8$ GeV cut biases are not that much an issue

\[
\langle E_\gamma \rangle = 2.292 \pm 0.026_{\text{stat}} \pm 0.034_{\text{sys}} \text{ GeV} \\
\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0305 \pm 0.0073_{\text{stat}} \pm 0.0063_{\text{sys}} \text{ GeV}^2
\]

For the extracted HQ values we would get

\[
\langle E_\gamma \rangle = 2.315 \text{ GeV} \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0325 \text{ GeV}^2
\]

CLEO 2001: $E_{\text{cut}} = 2$ GeV

\[
\langle E_\gamma \rangle = 2.346 \pm 0.032_{\text{stat}} \pm 0.011_{\text{sys}} \text{ GeV} \\
\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0226 \pm 0.0066_{\text{stat}} \pm 0.0020_{\text{sys}} \text{ GeV}^2
\]

vs.

\[
\langle E_\gamma \rangle = 2.345 \text{ GeV} \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.022 \text{ GeV}^2
\]

Quite consistent!

most recent (available), BaBar 2005: $E_{\text{cut}} = 1.9$ GeV

\[
\langle E_\gamma \rangle = 2.343 \pm 0.053_{\text{stat}} \pm 0.053_{\text{sys}} \text{ GeV} \\
\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0325 \pm 0.016_{\text{stat}} \pm 0.011_{\text{sys}} \text{ GeV}^2
\]

vs.

\[
\langle E_\gamma \rangle = 2.327 \text{ GeV} \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0275 \text{ GeV}^2
\]
\textbf{NB}: The new standard for charm mass is needed. For the $\overline{\text{MS}}$ quark mass the normalization around 1.2 GeV is manifestly too low for precision physics. Even light quarks are nowadays normalized at 2 GeV.

\[ \bar{m}_c(\bar{m}_c) \text{ is not a stable quantity} \]

A spectrum of possibilities includes $\bar{m}_c(2\bar{m}_c)$, $\bar{m}_c(m_b)$, $\bar{m}_c(2 \text{ GeV})$, $\bar{m}_c(3 \text{ GeV})$, $\bar{m}_c(5 \text{ GeV})$, ...

What should we recommend to use?

My suggestions would be a fixed-scale

\[ \bar{m}_c(2.5 \text{ GeV}) \text{ or } \bar{m}_c(3 \text{ GeV}) \]
Besides $V_{cb}$ and $V_{ub}$, how do we benefit from knowing the heavy quark parameters?

Precise values of $m_b$, $m_c$ – for textures (future)

Today’s use: $B$ nonperturbative dynamics

OPE operating in terms of the universal QCD operators has comprehensive applications not limited to only inclusive decay rates

For instance, heavy quark sum rules – particularly constraining with the HQP values coming from experiment

advantage of the ‘SV’ (‘kinetic’) renormalization scheme – all bounds are most apparent

are not applicable for poorly defined objects

Good example: bound $\varrho^2 > \frac{3}{4}$

N.U. 2000

If $\mu_\pi^2$ is close to $\mu_G^2$ there is also a strong upper bound for the IW slope!

N.U. 2001

Assuming the spin sum rule is saturated at $\mu = 1 \text{ GeV}$ we have

$$\mu_\pi^2 - \mu_G^2 = 3 \tilde{\epsilon}^2 \cdot (\varrho^2 - \frac{3}{4})$$

Quite a constraint:

$$\left(\varrho^2 - \frac{3}{4}\right) = \frac{\mu_\pi^2 - \mu_G^2}{3 \tilde{\epsilon}^2} \lesssim 0.2 \ (0.4)$$

at $\mu_\pi^2 = 0.42 \ (0.5) \text{ GeV}^2$ since $\tilde{\epsilon} > 0.35 \text{ GeV}$

$\varrho^2$ is important in extrapolating the exclusive amplitudes to zero recoil
Eventually this prediction is being confirmed by experiment F. Simonetto @ CKM2005 (BaBar, to appear)

One of the miracles of the proximity to the ‘BPS’ regime

\[ \mu_\pi^2 \sim \mu_G^2 \] is a special point for \( B \) and \( D \) mesons!

In the strict limit \( \varrho^2 = \frac{3}{4} \)

Ultrarelativistic light cloud – antipode to NR quark models

Another application – \( B \rightarrow D \ell \nu \): expanding in \( \mu_\pi^2 - \mu_G^2 \) and using the analogue of the Ademollo-Gatto theorem which holds for the BPS expansion

\[
\frac{M_B + M_D}{2\sqrt{M_B M_D}} \ f_+(0) = 1.04 \pm 0.01 \pm 0.01
\]

All orders in \( 1/m \) in ‘BPS’, to \( 1/m^2 \cdot 1/\text{BPS}^2 \), \( \alpha_s^1 \)
A "$\frac{1}{2} > \frac{3}{2}$" puzzle

Heavy Quark Sum Rules + the known size of $\mu_G^2$, now also of $\mu_\pi^2$ give much information

Spin sum rules strongly suggest that $\frac{3}{2}$ $P$-wave states must dominate over $\frac{1}{2}$ states. This automatically happens in all quark models respecting QCD and Lorentz covariance

- Orsay quark models
- quark models on light front hep-ph/0310359

Theory:

The most natural solution of all HQSRs:

$\frac{3}{2}$ states at $\epsilon_3^2 \approx 450$ MeV and $\tau_3^2 \approx 0.3$ while $\tau_1^2 \approx 0.07 \div 0.12$ with $\epsilon_1^2 \approx 300 \div 500$ MeV

Why?

Average $P$-wave excitation mass gap:

$\bar{\epsilon}_P \simeq \frac{2\mu_\pi^2}{3\Lambda} \approx 0.45$ GeV \[\sqrt{\frac{\mu_\pi^2}{3(\epsilon^2 - \frac{1}{4})}} \approx 0.45\text{ GeV}\]

Typical $\tau^2$:

$\bar{\tau}^2 \simeq \frac{1}{3}(\epsilon^2 - \frac{1}{4}) \approx 0.25 \quad \frac{\Lambda}{6\bar{\epsilon}_P} \approx 0.25$

and $\tau_{1/2}^2 \ll \tau_{3/2}^2$ from the spin sum rules

Experiment: $\frac{3}{2}$ charm $P$-wave states are narrow and well identified, $\{D_1, D_2^*\}$
Nonleptonic decays $B \rightarrow D^{**} \pi$ assume factorization

While possibly consistent for $\tau_{3/2}$, they used to give definitely too large $\tau_{1/2} \sim 0.4$ (even at $q^2 = 0$), based on $B^- \rightarrow D^{**} \pi^-$

This would contradict spin sum rules

BELLE-CONF-0460: $B^0 \rightarrow D^{**} \pi^-$ modes witness smallness of $\tau_{1/2}$. The data are consistent with the decays only into $\frac{3}{2}$-states, at the right rate!

Theory predictions from the HQ sum rules seem to be confirmed

All the consequences can, probably, be formulated in terms of measurable moments and cross sections only, without recourse to quark masses, nonperturbative heavy quark expectation values, and even to $\alpha_s$.

What seems impossible is to come up with these nontrivial connections, at first glance (second either) referring to very different phenomena.
backup