Feasibility of $\sin(2\beta+\gamma)$ Measurement From Time Distribution of $B^0 \rightarrow D K_S$ Decay

Vivek Sharma
University of California
San Diego
\[\sin(2\beta+\gamma)\] From TD analysis of \(B^0/\bar{B}^0 \to D^{(*)0}K^{(*)0}\) Decays

- **CPV due to Interference between decay and mixing as in \(B \to D^{(*)}\pi\)**
  - **Advantages:**
    - Expect large (40\%) CPV since two processes of similar strength
    - Time-dependent measurement with \(K^0 \to K_S\)
    - Probe \(r_B\) in self-tagging final state \(B \to DK^{*0}\) with \(K^{*0} \to K^+\pi^-\)
  - **Disadvantages:**
    - Color suppressed decays, \(Br\ (B \to DK_S) \approx \lambda^2\ Br(B \to \psi K_S)\)
      - Smaller decay rates (x 100) than \(B \to D^{(*)}\pi\)
    - Possible competing effects from Doubly-Cabibbo-suppressed \(D^0\) decays
  - **“Overall requires x3 less \(B\) sample to measure \(\sin^2(2\beta+\gamma)\) than \(B \to D\pi\)“ – Kayser & London, PRD 61, 116103, 2000
$B^0/\bar{B}^0 \rightarrow D^{(*)0}K^{(*)0}$

Decay Rate Measurements By Belle & BaBar (2004)
\( \mathcal{B}^0/\overline{\mathcal{B}}^0 \rightarrow D^{(*)0}K_s \) Rates: What We Know Now (BaBar)

hep-ex/0408052

Cannot distinguish \( \mathcal{B}^0 \) from \( \overline{\mathcal{B}}^0 \)

Hidden strangeness with \( K_s \) in final state

124 million \( B\overline{B} \)

<table>
<thead>
<tr>
<th>Mode</th>
<th>BF (10^{-5})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^0 \rightarrow D^0\overline{K}^0 )</td>
<td>6.2 ( \pm 1.2 ) ( \pm 0.4 )</td>
</tr>
<tr>
<td>( \overline{B}^0 \rightarrow D^{*0}\overline{K}^0 )</td>
<td>4.5 ( \pm 1.9 ) ( \pm 0.5 )</td>
</tr>
</tbody>
</table>

Can measure \( \sin(2\beta+\gamma) \) with some precision with \( D^{(*)0} \) \( K_s \) in 500 fb\(^{-1}\) assuming \( r \sim 0.4 \) (?)
**$\bar{B}^0/B^0 \rightarrow D^{(*)0}K_S$ Rates: What We Know Now (Belle)**

**hep-ex/0408108**

274 million $B\bar{B}$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\Delta E$ yield</th>
<th>Efficiency ($10^{-3}$)</th>
<th>$B$ ($10^{-5}$)</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{B}^0 \rightarrow D^0\bar{K}^0$</td>
<td>$78.1 \pm 14.1$</td>
<td>$8.02$</td>
<td>$3.72 \pm 0.65 \pm 0.37$</td>
<td>$6.6\sigma$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow D^0\bar{K}^{*0}$</td>
<td>$77.7 \pm 14.6$</td>
<td>$9.98$</td>
<td>$3.08 \pm 0.56 \pm 0.31$</td>
<td>$6.3\sigma$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow D^{*0}\bar{K}^0$</td>
<td>$19.2^{+6.4}_{-5.8}$</td>
<td>$1.97$</td>
<td>$3.18^{+1.25}_{-1.12} \pm 0.32$</td>
<td>$3.2\sigma$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow D^{*0}\bar{K}^{*0}$</td>
<td>$12.3^{+7.5}_{-7.0}$</td>
<td>$2.54$</td>
<td>$2.34^{+1.24}_{-1.15}$ (&lt; 4.8) 90% CL</td>
<td>$2.1\sigma$</td>
</tr>
</tbody>
</table>
Isolating $V_{ub}$ and $V_{cb}$ Driven amplitudes in $B \rightarrow D^0 K^{*0}$

- **$V_{cb}$ contribution:**
  - $B^0 \rightarrow D^{0}\bar{D} K^{*0}$
    - $D^{0}\bar{D} \rightarrow K^+ \pi^- \pi^- \pi^0, 3\pi$
    - $K^{*0} \rightarrow K^+ \pi^-$
  - Same sign kaons

- **$V_{ub}$ contribution:**
  - $B^0 \rightarrow D^0 K^{*0}$
    - $D^0 \rightarrow K^- \pi^+, \pi^+ \pi^0, 3\pi$
    - $K^{*0} \rightarrow K^+ \pi^-$
  - Kaons with opposite sign

- Determine $r_B$ by measuring the 2 branching fractions

- Different sources of background because of charge correlation
  - Treat them as 2 different decay modes
What We don’t Know: Limit on $r_B$ from Self-Tagging $\bar{B}^0 \rightarrow D^0 \bar{K}^*$

Charge correlation to separate $B^0$ decay from $\bar{B}^0$

$\bar{B}^0 \rightarrow D^0 \bar{K}^*$
$D^0 \rightarrow K^- X^+$
$\bar{K}^* \rightarrow K^- \pi^+$

$B^0 \rightarrow \bar{D}^0 \bar{K}^*$
$\bar{D}^0 \rightarrow K^+ X^+$
$\bar{K}^* \rightarrow K^- \pi^+$

No Signal in $V_{ub}$ mediated Decay

**Preliminary**

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</tr>
<tr>
<td>$\bar{B}^0 \rightarrow D^0 K^*$</td>
<td>$&lt; 4.1 @ 90%$ CL</td>
</tr>
</tbody>
</table>

$V_{cb}$ transition
$\bar{B}^0 \rightarrow D^0 \bar{K}^*$
$N = 45 \pm 9$

$V_{ub}$ driven
$B^0 \rightarrow D^0 K^* ; K^* \rightarrow K^+ \pi^-$

Need large $V_{ub}$ driven amplitude for measurement of $\gamma$ !

$r_B < 0.8$ @ 90% C.L. ; more sensitive analysis on way.
What We don’t Know Now: Limit on $r_B$ from Self-Tagging $\bar{B}^0 \rightarrow D^0 \bar{K}^{*0}$

Belle 274 million $\bar{B}\bar{B}$

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<tr>
<td>$\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0$</td>
<td>$0.4^{+3.6}_{-3.1}$</td>
<td>6.52</td>
<td>$&lt;0.51$ 90% CL</td>
<td>—</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow \bar{D}^{*0} \bar{K}^0$</td>
<td>$3.3^{+2.7}_{-2.1}$</td>
<td>1.72</td>
<td>$&lt;1.9$ 90% CL</td>
<td>—</td>
</tr>
</tbody>
</table>

$\Rightarrow r_B < 0.39$ @ 90% CL
Toy Study of $B \rightarrow D K_s$
Sensitivity to $\sin(2\beta+\gamma)$

(Shahram Rahatlou @ CKM Angles Workshop 2003)
Time-Dependent Decay Distributions

\[
f(B^0_{\text{tag}}, K^- X^+; \Delta t) = \frac{e^{-|\Delta t|}}{8\tau} \left( 1 + \frac{1}{1 + r^2} \cos \Delta m \Delta t - \frac{2r}{1 + r^2} \sin(2\beta + \gamma - \delta) \sin \Delta m \Delta t \right)
\]

\[
f(B^0_{\text{tag}}, K^+ X^-; \Delta t) = \frac{e^{-|\Delta t|}}{8\tau} \left( 1 - \frac{1}{1 + r^2} \cos \Delta m \Delta t - \frac{2r}{1 + r^2} \sin(2\beta + \gamma + \delta) \sin \Delta m \Delta t \right)
\]

\[
f(\overline{B}^0_{\text{tag}}, K^- X^+; \Delta t) = \frac{e^{-|\Delta t|}}{8\tau} \left( 1 - \frac{1}{1 + r^2} \cos \Delta m \Delta t + \frac{2r}{1 + r^2} \sin(2\beta + \gamma - \delta) \sin \Delta m \Delta t \right)
\]

\[
f(\overline{B}^0_{\text{tag}}, K^+ X^-; \Delta t) = \frac{e^{-|\Delta t|}}{8\tau} \left( 1 + \frac{1}{1 + r^2} \cos \Delta m \Delta t + \frac{2r}{1 + r^2} \sin(2\beta + \gamma + \delta) \sin \Delta m \Delta t \right)
\]

\[
\sin^2(2\beta + \gamma) = \frac{1}{2} \left[ 1 + S^+ S^- \pm \sqrt{(1 - S^{+2})(1 - S^{-2})} \right]
\]

\* Similar to B\(\rightarrow\)D*\(\pi\) time-distribution
- But \(r\) expected to be much larger (x 10-20)
  - Linear dependency on \(r\): can it be measured in the fit (?)
- Tag-side DCS effects are small compared to signal amplitude
- But there are other potential complications due to DCS decays on reco side (20%)

**fit for \(r_B\) and \(\sin(2\beta+\gamma-\delta)\) and \(\sin(2\beta+\gamma+\delta)\)**
Assumptions Used In This Toy Study

• Signal Yield: ≈ 0.3 events / fb^{-1} → 160 reconstructed events (B\to D^0Ks only for now)
  – Yields can be 50% higher (Br. Ratio knowledge, better event selection)
  – Additional modes can increase signal yield but won't be as clean

• No combinatorial or peaking background
  – Signal asymmetries expected to be weakly correlated with background parameters
    ➢ β=23°±3°, γ=59°±19° → 2β+γ=105°±20° (1.83 rad)
  – Use 3 values in toys: 0.9, 1.88, and 2.8 rad

• No Knowledge of strong phase
  – Use different values δ = 0.0, 0.8, 2.4 rad

• Realistic BaBar Flavor Tagging circa '03
  – 105 tagged events in 500 fb^{-1}

• Vertexing: realistic resolution function
  – 10% tail and 0.2% outliers with category dependent bias
  – Parameters fixed in the fit

<table>
<thead>
<tr>
<th>Category</th>
<th>Efficiency</th>
<th>Dilution</th>
<th>Dil. Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>10.3%</td>
<td>0.933</td>
<td>0.028</td>
</tr>
<tr>
<td>Kaon 1</td>
<td>17%</td>
<td>0.801</td>
<td>0.022</td>
</tr>
<tr>
<td>Kaon 2</td>
<td>19.4%</td>
<td>0.582</td>
<td>0.084</td>
</tr>
<tr>
<td>Other</td>
<td>19.9%</td>
<td>0.368</td>
<td>0.058</td>
</tr>
</tbody>
</table>
Summary of Signal efficiencies and yields circa 2003

Yields for 500 fb\(^{-1}\) with BF = 4 x 10\(^{-5}\)

<table>
<thead>
<tr>
<th></th>
<th>Kpi</th>
<th>Kpipi0</th>
<th>K3pi</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D0Ks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Produced</td>
<td>263</td>
<td>908</td>
<td>520</td>
<td>1691</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.23</td>
<td>0.06</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Reco</td>
<td>61</td>
<td>54</td>
<td>51</td>
<td>167</td>
</tr>
<tr>
<td><strong>D0K*0(K+pi-)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Produced</td>
<td>527</td>
<td>1816</td>
<td>1040</td>
<td>3382</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.15</td>
<td>0.04</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Reco</td>
<td>81</td>
<td>80</td>
<td>79</td>
<td>240</td>
</tr>
<tr>
<td><strong>D*0Ks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Produced</td>
<td>163</td>
<td>562</td>
<td>322</td>
<td>1047</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.12</td>
<td>0.03</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Reco</td>
<td>19</td>
<td>17</td>
<td>16</td>
<td>52</td>
</tr>
<tr>
<td><strong>D<em>0K</em>0(K+pi-)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Produced</td>
<td>326</td>
<td>1124</td>
<td>643</td>
<td>2093</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.08</td>
<td>0.02</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Reco</td>
<td>25</td>
<td>25</td>
<td>24</td>
<td>74</td>
</tr>
</tbody>
</table>

Assuming D\(^{*-0}\) reco. efficiency of 50%

- \(K^{*0}\rightarrow K_s\pi^0\):
  - Expected events: \(N(K^+\pi^-)\times 0.5\times0.5\times0.75\times(Ks\text{ eff})\sim 50\) events
  - But more background from \(\pi^0\)

- \(D^0\rightarrow K_s\pi\pi\): done but not included for technical reason. Fewer events than \(D^0\rightarrow K\pi\) mode
- \(D^0\rightarrow KK, \pi\pi\): Branching fraction \(~10\) smaller than BF(K\pi)

- \(B^0\rightarrow D^{**0}K^{(*)0}\): similar to \(B^0\rightarrow D^0K^{(*)0}\) but reduced by intermediate branching fraction

\[\]
Fit Validation with 50 ab$^{-1}$: $r_B$

- Validate fit with 500 experiments of 50 ab$^{-1}$
  
  - $r_B(\text{gen}) = 0.4$, $2\beta+\gamma = 1.88$, $\delta = 0.0$

Slightly better errors for larger $r_B$

No bias in the fitted values for any parameter
First ToyFit Attempt for 500 fb\(^{-1}\) sample: fit 3 parameters \(r_B\), \(\sin(2\beta+\gamma \pm \delta)\)

- Problem with fit convergence

<table>
<thead>
<tr>
<th>(2\beta+\gamma)</th>
<th>(\delta=0.0)</th>
<th>(\delta=0.8)</th>
<th>(\delta=2.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>3.0%</td>
<td>2.4%</td>
<td>3.6%</td>
</tr>
<tr>
<td>1.88</td>
<td>2.6%</td>
<td>2.2%</td>
<td>3.4%</td>
</tr>
<tr>
<td>2.80</td>
<td>3.8%</td>
<td>2.4%</td>
<td>3.0%</td>
</tr>
</tbody>
</table>

- Small values of \(r_B\) are problematic
  - \(r_B\) constrained to be positive in the fit

- \(\sim 10\%\) of fits with \(r_B\) close to the limit

![Histogram](image1.png)

![Histogram](image2.png)
Plan B ➡️ Sensitivity for $\sin(2\beta + \gamma \pm \delta)$ with $r_B = 0.4$ (fixed)

- All fits successful!
  - Problems with MINOS errors in some fits
- Under investigation

<table>
<thead>
<tr>
<th>Residue $\sin(2\beta + \gamma + \Delta)$ RMS</th>
<th>$\delta = 0.0$</th>
<th>$\delta = 0.8$</th>
<th>$\delta = 2.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\beta + \gamma = 0.90$</td>
<td>0.63</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>$2\beta + \gamma = 1.88$</td>
<td>0.64</td>
<td>0.69</td>
<td>0.60</td>
</tr>
<tr>
<td>$2\beta + \gamma = 2.80$</td>
<td>0.65</td>
<td>0.63</td>
<td>0.56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Residue $\sin(2\beta + \gamma - \Delta)$ RMS</th>
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<th>$\delta = 0.8$</th>
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<td>0.65</td>
<td>0.64</td>
<td>0.66</td>
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<tr>
<td>$2\beta + \gamma = 1.88$</td>
<td>0.64</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>$2\beta + \gamma = 2.80$</td>
<td>0.60</td>
<td>0.61</td>
<td>0.59</td>
</tr>
</tbody>
</table>
Summary of Preliminary Toy Study

• **Sensitivity with 0.5 ab⁻¹?**
  
  - Simultaneous determination of \( r_B \) and \( \sin(2\beta+\gamma\pm\delta) \) probably not be feasible with current method with \( \approx 500 \text{ fb}^{-1} \) samples \( \rightarrow \) (too few data)
  
  - With \( 500 \text{ fb}^{-1} \), expected uncertainty on \( \sin(2\beta+\gamma\pm\delta) \) \( \sim 0.6-0.7 \) provided \( r_B \) known from elsewhere:
    - *ignoring DCS effects which at ~22% is small compared to expected errors and where CLEO-c can help?*

• **Sensitivity with 5 ab⁻¹?**
  
  - No problem measuring \( r_B \) and \( \sin(2\beta+\gamma\pm\delta) \)
  
  - All simultaneous fits successful
    
    - *Uncertainty on \( r_B \): \( \sim 0.04-0.05 \)
    - *Uncertainty on \( \sin(2\beta+\gamma) \): ~0.15-0.20*
    - The errors now scale with luminosity


Learning Today From Data on $B \to DK$ Family of Decays

$BR(B^+ \to D^+ K^0) < 5.0 \times 10^{-6}$ (@ 90% Prob.)

$BR(B^+ \to D^0 K^+) = (3.7 \pm 0.6) \times 10^{-4}$

$r_{D^0 K^-} = 0.10 \pm 0.04$ (GLW, ADS, Dalitz)

$r_{D^0 K^0} = \frac{|A(B^0 \to D^0 K^0)|}{|A(B^0 \to D^- K^0)|} = \frac{|C_s + A|}{T + C_s}$

$BR(B^0 \to D^0 K^0) = (5.0 \pm 1.4) \times 10^{-5}$

...and the same for the $D^*$
With present exp. values, we obtain for the ratios $r_{D^0K^0}$ and $r_{D^{*0}K^0}$

$$r_{D^0K^0} = 0.26 \pm 0.16 \text{ @ 68\% Prob}$$

$$r_{D^{*0}K^0} = 0.29 \pm 0.21 \text{ @ 68\% Prob}$$

Where

$\frac{1}{2}$ of the error on $r_{D^0K^0}$ comes from $r_{D^0K^+}$

$\frac{1}{2}$ of the error on $r_{D^0K^0}$ comes from $BR(B^* \rightarrow D^+ K^0)$

Important to improve this measurement!
Method works better if annihilation is small.

Viola Sordini, Universita' di Roma La Sapienza, INFN
Error on $\sin(2\beta + \gamma)$ for different values of $r_{D^0K^0}$ and $r_{D^{*0}K^0}$ and for different luminosities using the errors

<table>
<thead>
<tr>
<th>Integrated Luminosity</th>
<th>500 fb$^{-1}$</th>
<th>1 ab$^{-1}$</th>
<th>2 ab$^{-1}$</th>
</tr>
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<tbody>
<tr>
<td>Error on $r(D^0K^0)$</td>
<td>0.12</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>Error on $r(D^{*0}K^0)$</td>
<td>0.20</td>
<td>0.19</td>
<td>0.18</td>
</tr>
</tbody>
</table>

(for this exercise $\delta = 45^\circ$)

For $r_{D^0K^0} = r_{D^{*0}K^0} = 0.30$ and associated error with 500 fb$^{-1}$ and with errors on C and S:

$\delta(S) = \delta(\bar{S}) = 0.6$  $\delta(C) = 0.5$

(BaBar internal document BAD #884, by Shahram Rahatlou)
Like with most pursuits of $\gamma$, strength of the $b \rightarrow u$ amplitude is key

So far no observation, only limits on $r_B$ from $B^0 \rightarrow D(\ast)^0 K^\ast 0$ modes, $r_B < 0.39$ (Belle)

Playing with measured $B \rightarrow DK$ rates and constraining various contributing amplitudes suggests $r_{B \rightarrow DKs} = 0.26 \pm 0.16$. Is this approach theoretically kosher?

ToyMC based time-dependent CPV studies indicate that with 500 fb$^{-1}$ samples, mild information only on $\sin(2\beta+\gamma)$ provided $r_B$ obtained from elsewhere

More $B$ modes need to be added
  - Self tagging decay $B \rightarrow D^{\ast\ast} 0 K_S$ decays (poor Br. for self tagging modes)
  - $B \rightarrow DKs$, $D \rightarrow Ks \pi\pi$ mode not prolific (yield ~4x smaller than $D \rightarrow K\pi$ mode)

Precise measurements require $> ab^{-1}$ data samples

Can LHCb do such modes (Large $Ks$ decay lengths)

There are no shortcuts in clean measurements of $\gamma$!