Chiral symmetry and B decays in the SCET

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Outline

• Effective field theories: HQET, heavy hadron chiral perturbation theory
• The Soft-Collinear Effective Theory
• Combining the SCET and chiral perturbation theory
• Applications
• Conclusions and outlook
Energy scales in B physics

Heavy quark (b) interacting with soft quarks and gluons

Two relevant scales: $\Lambda \sim 500$ MeV, $m_b \sim 4.6$ GeV

1. Systematic expansion in $\Lambda/m_b \sim 0.1$

2. Heavy-quark spin and flavor symmetry at leading order in $\Lambda/m_b$

HQET - the appropriate effective theory
HQET - symmetries

Heavy meson fields $B, B^*$ form a spin doublet

Superfield

$$H_a = \frac{1 + \frac{\mu}{2}}{2} [B^*_\mu \gamma^\mu - B \gamma_5]$$

Constructing operators using only $H$ automatically incorporates heavy quark symmetry

Example: semileptonic $B \rightarrow D(*)$ decay

$$\langle D^{(*)} | \bar{c} \Gamma b | \bar{B} \rangle = \xi (v.v') \text{Tr} [\bar{H}_{\nu'}^{(c)} \Gamma H_{\nu}] + O(\Lambda/m_c)$$

Isgur-Wise function
Chiral symmetry

Massless QCD $m_q = 0$ has a chiral symmetry

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$$

broken down spontaneously to the flavor group

Goldstone bosons

$$M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\
K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta
\end{pmatrix}$$

$$\xi = e^{iM/f} \quad \Sigma = \xi^2$$

Transformations:

$$\Sigma \rightarrow L\Sigma R^\dagger$$

$$\xi \rightarrow L\xi U^\dagger = U\xi R^\dagger$$
Write down the most general Lagrangian invariant under the chiral transformations

Organize the Lagrangian in an expansion in \((p_\pi, m_q)\)  

\[
\mathcal{L} = \frac{f^2}{8} \text{Tr} \partial^\mu \Sigma \partial_\mu \Sigma + \frac{f^2 B_0}{4} \text{Tr} (m_q \Sigma + m_q \Sigma^\dagger) + \cdots
\]

Can be extended to include also e.m. interactions

\[
\begin{array}{c}
\pi \\
\pi \\
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\pi \\
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\quad
\begin{array}{c}
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\]

Gasser, Leutwyler
Heavy hadron chiral perturbation theory

Wise, Burdman+Donoghue

Transformation under the heavy quark spin rotations and chiral symmetry

\[
H \rightarrow SH
\]

\[
H \rightarrow HU^\dagger
\]

\[
\mathcal{L} = -\text{Tr} \bar{H}_a i\nu \cdot D_{ba} H_b + g\text{Tr} \bar{H}_a H_b \gamma_\mu \gamma_5 A^\mu_{ba} + \cdots
\]

\[
B^{(*)}
\]

\[
B
\]

\[
\pi^a
\]

\[
B^*
\]

\[
A^\mu_{ab} = \frac{i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)
\]

\[
V^\mu_{ab} = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)
\]

\[
D^\mu_{ab} = \delta_{ab} \partial^\mu - V^\mu_{ab}
\]
Application

Semileptonic $B \to \pi$ decays with a soft pion

$$\langle \pi(p')|\bar{u}\gamma_\mu P_L b|\bar{B}(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

$$J^\mu_a = \bar{q}^a \gamma^\nu P_L b$$ transforms as $$(\bar{3}_L \otimes 1_R)$$

matches in the chiral perturbation theory as

$$J^\mu_a \to \frac{1}{2} i f_B m_B \text{Tr}[\gamma^\mu P_L H_b \xi^\dagger_{ba}] + \cdots$$
Result at LO in the chiral expansion

\[ f_+ (E_\pi) = g \frac{f_B m_B}{2 f_\pi} \frac{1}{E_\pi + \Delta} \]

Valid for not too energetic pions

\[ \Delta = m_{B^*} - m_B \sim 50 \text{ MeV} \]

\[ E_\pi < 4\pi f_\pi \sim 1 \text{ GeV} \]

\[ q^2 > 17 \text{ GeV}^2 \]
Energetic hadrons

Construct a heavy-quark expansion for B processes involving both soft and energetic light particles

Example: $B \rightarrow \pi \ell \bar{\nu}$ decays $(E_\pi \sim m_B/2 \sim 2.2 \text{ GeV})$

Energy scales:
- Soft $\Lambda \sim 500 \text{ MeV}$,
- Hard $m_b \sim 4.6 \text{ GeV}$
- Hard-collinear $\sqrt{\Lambda m_b} \sim 1.4 \text{ GeV}$
The soft-collinear effective theory (SCET)

- Systematic power counting in $\Lambda/m_b$ implemented at the level of momenta, fields, operators
- Construct effective Lagrangians for strong and weak interactions expanded in powers of $\Lambda/m_b$
- New symmetries (gauge, reparameterization invariance)
- Nonlocal operators and Lagrangians
Degrees of freedom

Introduce distinct fields for each relevant degree of freedom, with well defined momentum scaling

<table>
<thead>
<tr>
<th>modes</th>
<th>field</th>
<th>$p_\mu \sim (+, -, \perp)$</th>
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<tbody>
<tr>
<td>hard</td>
<td></td>
<td>(Q, Q, Q)</td>
</tr>
<tr>
<td>hard-collinear</td>
<td>$A_{n,q}, \xi_{n,p}$</td>
<td>($\Lambda, Q, \sqrt{Q\Lambda}$)</td>
</tr>
<tr>
<td>collinear</td>
<td>$A_{n,q}, \xi_{n,p}$</td>
<td>($\Lambda^2/Q, Q, \Lambda$)</td>
</tr>
<tr>
<td>soft/ultrasoft</td>
<td>$A_s, q, b_v$</td>
<td>($\Lambda, \Lambda, \Lambda$)</td>
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Integrate out successively the hard scales $Q^2 \rightarrow Q\Lambda$
QCD $\rightarrow$ SCET$_I$ $\rightarrow$ SCET$_{II}$

Integrate out hard collinear modes $p_{hc}^2 \sim Q\Lambda$

SCET$_I$ $\rightarrow$ SCET$_{II}$

SCET$_{II}$ = theory containing only soft and collinear modes $p_s^2, p_c^2 \sim \Lambda^2$

Soft-collinear decoupling at LO

$\mathcal{L}_{SCET_{II}} = \mathcal{L}_S^{(0)} + \mathcal{L}_C^{(0)} + \cdots$

$\mathcal{O} = \mathcal{O}_S \times \mathcal{O}_C$

$\langle \mathcal{O} \rangle = \langle \mathcal{O}_S \rangle \times \langle \mathcal{O}_C \rangle$

factorization
Combining SCET with xPT

Objective: construct an effective theory describing energetic hadrons and soft Goldstone bosons

Relevant scales:
- hard scale $Q$
- hard-collinear scale $\sqrt{Q\Lambda}$
- soft scale $\Lambda$
- pion momenta $|\vec{p}_\pi| \sim m_\pi$

Hierarchy: $Q \gg \sqrt{Q\Lambda} \gg \Lambda \sim p_\pi \sim m_\pi$

Use SCET for soft-collinear factorization and apply xPT to the soft part
B decays without collinear hadrons in the final state

Factorization in semileptonic radiative decays $B \rightarrow \pi S \gamma \ell \bar{\nu}_\ell$

Cirigliano, DP
Factorization in $B \rightarrow \gamma \ell \bar{\nu}_\ell$

Simplest exclusive B decay, proceeding through weak annihilation of the b quark with the light spectator Lunghi, DP, Wyler; Becher, Hill, Lange, Neubert

$$e^{iq \cdot x} T \{ j_{\mu}^{em}(x), V_\nu(0) \} \rightarrow H(E_\gamma) \int d\lambda e^{i\lambda n \cdot x} J(k_+) [\bar{q}(\lambda n) Y_n(\lambda, 0) \Gamma b(0)]$$

hard $\otimes$ jet $\otimes$ soft
Factorization relation

\[ A(B \rightarrow \gamma e\nu) \sim C^{(v)}(2E_\gamma, \mu) \int dk_+ J(k_+) \phi_B^+(k_+) \]

- Wilson coeff - hard momenta \( C_1(\omega) = 1 + O(\alpha_s(Q)) \)
- Jet function - hard-collinear scales

\[ J(k_+) = \frac{\pi \alpha_s(\sqrt{Q\Lambda}) C_F}{k_+} \left( 1 + O(\alpha_s(\sqrt{Q\Lambda})) \right) \]

- B light-cone wave function

\[
\langle 0|\bar{q}(\lambda n)Y_n(\lambda, 0)\Gamma b(0)|\bar{B}\rangle = \int dk_+ e^{-i\lambda k_+} \text{Tr} \left[ \frac{1 + \gamma^5}{2} (\bar{\phi} \gamma^5 \phi_B^+(k_+) + \bar{\phi} \gamma^5 \phi_B^-(k_-)) \gamma_5 \Gamma \right]
\]
Operator matching

\[ e^{iq \cdot x} T \{ j_\mu^{em}(x), V_\nu(0) \} \rightarrow H(E_\gamma) \int d\lambda e^{i\lambda n \cdot x} J(k_+) [\bar{q}(\lambda n)Y_n(\lambda, 0)\Gamma_b(0)] \]

Independent on the soft state \((B \rightarrow \text{vacuum})\)

Can describe equally well \(B \rightarrow \pi, B \rightarrow \pi\pi, \cdots\)
Find the chiral representation of the soft light cone operator

\[ O^a_L(k_+) = \int d\lambda e^{-i\lambda k_+} \bar{q}_L^a(\lambda n)Y_n(\lambda, 0)\Gamma_b(0) \]

Transforms like \((3_L, 0_R)\) under chiral \(SU(3)_L \times SU(3)_R\)

Most general chiral LO operators

\[ O^a_L(k_+) \rightarrow \text{Tr} \left[ \hat{\alpha}(k_+) \Gamma H_b \xi^\dagger_{ba} \right] + \cdots \]

\[ \hat{\alpha}(k_+) = a_1 + \varphi a_2 + \varphi a_3 + [\varphi, \varphi] a_4 \]

Heavy quark symmetry fixes \(a_1-a_4\) in terms of the B light-cone wave functions
All $B \rightarrow$ Goldstone boson matrix elements of $\langle O_L \rangle$ are fixed by heavy quark and chiral symmetry

\[ B^{(*)} \]

No need for additional low-energy constants!

Previous work on chiral representation of nonlocal operators:

Twist-2 DIS and DVCS structure functions

\[ J.W.\text{Chen}, \text{M. Savage} \]

SU(3) breaking in $\pi, K$ wave functions

\[ J.W.\text{Chen}, \text{I. Stewart} \]
Application: $B \rightarrow \pi \gamma \ell \bar{\nu}$

In the kinematical region with an energetic photon and one soft pion, SCET+xPT give a factorization theorem

**Validity region**

$E_\gamma \gg \Lambda$

$E_\pi < \Lambda \chi_{SB}$

Important for a precise measurement of the $B \rightarrow \pi \ell \nu$ branching fractions.
$B \rightarrow \pi \gamma \ell \bar{\nu}$ from SCET + xPT

Leading order diagrams

Factorization relation (schematic)

$$A(B \rightarrow \pi \gamma \ell \nu) \sim C(2E_\gamma) \int dk_+ J(k_+) \phi_B^+(k_+) S(P_\pi)$$
Results

The $B(p) \rightarrow \pi(p')\gamma(q, \varepsilon)\ell\bar{\nu}$ amplitude is given by

$$T_{\mu\nu}^J = i \int d^4 x e^{i q \cdot x} \langle \pi(p') | j_\mu^{\text{em}}(x), J_\nu(0) | \bar{B}(p) \rangle$$

Form-factor parameterization compatible with the Ward identity $q^\mu V_{\mu\nu} = F_\nu \equiv \langle \pi | V_\nu | B \rangle$

$$\varepsilon^{*\mu} V_{\mu\nu} = V_1 (\varepsilon^*_\nu - \frac{\varepsilon^* \cdot W}{q \cdot W} q_\nu)$$

$$+ (p' \cdot \varepsilon^* - \frac{(p' \cdot q)(\varepsilon^* \cdot W)}{q \cdot W}) (V_2 q_\nu + V_3 W_\nu + V_4 p'_\nu)$$

$$+ \varepsilon^* \cdot W \frac{1}{q \cdot W} F_\nu(p, p')$$

$W = p - p' - q$
Factorization relations for $B \rightarrow \pi \gamma \ell \bar{\nu}$

$V_1 - 4$ are functions of $(E_\pi, E_\gamma, W^2)$

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<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1 = C_1^{(v)}(2E_\gamma) S_L(p_\pi) I(E_\gamma, \mu)$</td>
<td>$V_2 = \frac{1}{2E_\gamma} (C_2^{(v)}(2E_\gamma) + 2C_3^{(v)}(2E_\gamma)) S_R(p_\pi) I(E_\gamma, \mu)$</td>
<td>$V_3 = \frac{1}{n \cdot W} C_2^{(v)}(2E_\gamma) S_R(p_\pi) I(E_\gamma, \mu)$</td>
<td>$V_4 = O(\Lambda/Q)$</td>
</tr>
</tbody>
</table>

$SL, SR =$ calculable functions of the pion momentum

$I(E_\gamma, \mu) = \int dk_+ J(k_+, \mu) \phi_B^+(k_+)$

Analogous results for the axial current
Decays with one collinear hadron in the final state

Forward–backward asymmetry in $B \to K_n \pi_S \ell^+ \ell^-$

Grinstein, DP
**SCET\textsubscript{II} matching**

Matching a current onto SCET\textsubscript{II} produces both factorizable and nonfactorizable operators

\[ \bar{q} \Gamma_{\mu}^\perp b \rightarrow c_1(\omega) \bar{q}_n,\omega \gamma_\mu^\perp P_L b_v \]

\[ + b_{1L} \otimes J_\perp \otimes (\bar{q}_n \gamma_\mu^\perp \gamma_\lambda^\perp P_R b_v) \otimes (\bar{q}_n,\omega_1 \gamma_\lambda^\perp q_n,\omega_2) \]

\[ + b_{1R} \otimes J_\parallel \otimes (\bar{q}_n \gamma_\mu^\perp P_R b_v) \otimes (\bar{q}_n,\omega_1 \gamma_\lambda^\perp P_L q_n,\omega_2) \]
Factorization relations for form factors

Form factors can be written as combinations of soft and hard scattering terms

\[ f_i^{B \rightarrow M} (E) = c_i \zeta^{BM} + b_i J \otimes \phi_B^+ \otimes \phi_M \]

\(c_i, b_i\) are Wilson coefficients in SCET-1
Predictions for transverse helicity amplitudes at LO in 1/mb for B $\to$ M decays

$$H_+(B \to V) = c_10 + b_1R0$$

$$H_-(B \to V) = c_1\zeta_\perp + b_1L \otimes J_\perp \otimes \phi_B^+ \otimes \phi_\perp^V$$

1. The right-handed amplitude vanishes exactly at LO in 1/mb

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<table>
<thead>
<tr>
<th>Symmetry relations</th>
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<tbody>
<tr>
<td>$\frac{m_B}{m_B + m_V} V(E) = \frac{m_B + m_V}{2E} A_1(E)$</td>
</tr>
<tr>
<td>$T_1(E) = \frac{m_B}{2E} T_2(E)$</td>
</tr>
</tbody>
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2. Prediction independent on the hadron spin
Multibody decays

LO predictions for $B \rightarrow M \pi$ w/ soft pion

$H_+(B \rightarrow M_n\pi) = c_1 0 + b_{1R} J_{||} \otimes \langle \pi | O_S | \bar{B} \rangle \otimes \phi_{M}^{||}$

$H_-(B \rightarrow M_n\pi) = c_1 \zeta_{\perp}^{BM\pi} + b_{1L} J_{\perp} \otimes \langle \pi | O_S | \bar{B} \rangle \otimes \phi_{M}^{\perp}$

1. The right-handed amplitude is non-vanishing!

Completely factorizable $\rightarrow$ calculable if the soft matrix element is known

2. New soft functions contributing to the left-handed amplitude
Factorization in $B \rightarrow K\pi\ell^+\ell^-$

Use chiral perturbation theory to compute the soft matrix element $\langle \pi | O_S | \bar{B} \rangle$

\[
\langle \pi | O_S | \bar{B} \rangle = \zeta_{\perp} B K \pi
\]

New soft function $\zeta_{\perp} B K \pi$
Factorization relations

Right-handed amplitude

\[ H_+(B \to K\pi) = \frac{1}{2} m_B^2 \left( \frac{g}{f_\pi} \frac{\varepsilon_+^* \cdot p_\pi}{v \cdot p_\pi + \Delta} \right) \int dz b_1 R(z) \zeta_J^{BK}(z) \]

Same convolution as that appearing in B->K form factors

\[ \zeta_J^{BK}(z) = \frac{f_B f_K}{m_B} \int dx dk_+ J(x, z, k_+) \phi_+^B(k_+) \phi_K(x) \]
FB asymmetry

New contribution to the FBA of the lepton momentum distribution

\[
A_{FB}(q^2) = \frac{\Gamma(\theta_+ > 0) - \Gamma(\theta_+ < 0)}{\Gamma(\theta_+ > 0) + \Gamma(\theta_+ < 0)}
\]

The FB asymmetry measures the interference of the 2 transverse helicity amplitudes

\[
A_{FB}(q^2) \sim \text{Re} \left[ H_-(A)^* H_-(V) - H_+(A)^* H_+(V) \right]
\]
Results for the zero

\[
q_0^2 | M_{K\pi} = M_K + M_\pi = 3.69^{+0.42}_{-0.53} \text{GeV}^2
\]

For given \( M(K+\pi) \), predict a zero at \( q_0[M(K+\pi)] \).

The effect depends on the size of the factorizable m.e. \( \zeta_{BK} \).
Other applications

Tests of the Standard Model with nonresonant modes: the photon helicity in \( B \rightarrow K S \pi^0 \gamma \)

Ligeti et al.

Semileptonic and radiative B decays into multibody states (one collinear + multiple soft pions)

e.g. \( B \rightarrow \pi_n \pi_S \ell \nu \) \( B \rightarrow K_n \pi_S \ell^+ \ell^- \)

Larger branching ratios, more observables
Rich phenomenology

- 20-odd pages in the PDG on nonleptonic and semileptonic B decays
- SCET helps reduce the number of independent hadronic parameters
- Model-independent analyses of exclusive semileptonic and nonleptonic B decays are possible
- The SCET+xPT combination extends these methods to multibody decays