Exclusive $B \rightarrow X_u \ell \nu$ Decays

Vera Lüth
SLAC

- What is the status of measurements of $BF$, $FF$, and $|V_{ub}|$?
- How can we improve these measurements in the future?
- What theoretical input is needed?
- What else can we learn from s.l. decays?
- How can we relate the FF to other B decays?

N. B. This is not a review of all results currently available!
Rates for $B \to \pi (\rho, \omega) l \nu$ Decays at Y(4S)

Tagging a BB event by full reconstruction of one B is a powerful tool

- hadronic decays: $B \to D^{(*)} + (\pi^\pm, \pi^0, K^\pm, K^0)$
- semileptonic decays, full reconstruction of $D^* l \nu$ or partial $(D)\pi_S l \nu$
  - separate the 2 B decays: tag and recoil - remove combinatorics/cross feed
  - determine B charge, flavor, momentum
  - reduce non-BB bg - fewer additional kinematic cuts

- less variation of efficiency in kinematic variables
- BUT - sizable reduction is rate

<table>
<thead>
<tr>
<th></th>
<th>Breco tag</th>
<th>$D^{(*)} l \nu$ tag</th>
<th>No tag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4,000 evt fb</td>
<td>10,000 evt fb</td>
<td>1,100,000 evt fb</td>
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</table>

<table>
<thead>
<tr>
<th>Decay</th>
<th>evts fb</th>
<th>S/B</th>
<th>evts fb</th>
<th>S/B</th>
<th>evts fb</th>
<th>S/B</th>
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</thead>
<tbody>
<tr>
<td>$\pi^- l^+ \nu$</td>
<td>0.13</td>
<td>10</td>
<td>0.5</td>
<td>3</td>
<td>4.4</td>
<td>1</td>
</tr>
<tr>
<td>$\rho^- l^+ \nu$</td>
<td>0.22</td>
<td>4</td>
<td>0.4</td>
<td>1</td>
<td>2.4</td>
<td>1</td>
</tr>
<tr>
<td>$\omega l^+ \nu$</td>
<td>0.1</td>
<td>3</td>
<td></td>
<td></td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>
TAG $B_{\text{had}}$ fully reconstructed: $B \to \pi^+ (\rho \quad \omega \quad \eta') \ell \nu$

<table>
<thead>
<tr>
<th>Decay</th>
<th>No. events</th>
<th>$\text{BF} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ \ell \nu$</td>
<td>$11.0 \pm 3.9$</td>
<td>$0.89 \pm 0.34 \pm 0.12$</td>
</tr>
<tr>
<td>$\pi^0 \ell \nu$</td>
<td>$15.5 \pm 4.1$</td>
<td>$0.91 \pm 0.28 \pm 0.14$</td>
</tr>
<tr>
<td>$\rho^+ \ell \nu$</td>
<td>$20.1 \pm 5.1$</td>
<td>$3.5 \pm 1.1 \pm 0.07$</td>
</tr>
<tr>
<td>$\rho^0 \ell \nu$</td>
<td>$15.7 \pm 4.3$</td>
<td>$1.04 \pm 0.39 \pm 0.16$</td>
</tr>
<tr>
<td>$\omega^0 \ell \nu$</td>
<td>$9.3 \pm 3.3$</td>
<td>$1.25 \pm 0.55 \pm 0.24$</td>
</tr>
<tr>
<td>$\eta' \ell \nu$</td>
<td>$13.9 \pm 4.2$</td>
<td>$2.7 \pm 1.2 \pm 0.05$</td>
</tr>
</tbody>
</table>

- Clean sample
- Full kinematics
- Full $q^2$ acceptance
- Very low rate

$\sigma_{\text{stat}} \sim 30\%$
TAG $B_{\text{lept}}$ fully reconstructed: $B \to \pi (\rho) \ell \nu$

- Two s.l. decays/event Tag: $B \to D^{(*)} \ell \nu$, with various $D$ decays
- extract $B$ direction, back-to-back, from “OC Fit”
- Recoil: Signal decay, veto additional particles

\[ \cos^2 \phi_B = \frac{\cos^2 \theta_{BY} + \cos^2 \theta_{B\pi\ell} + 2 \cos \theta_{BY} \cos \theta_{B\pi\ell} \cos \gamma}{\sin^2 \gamma}, \]

Belle: $x_B^2 = 1 - \cos^2 \phi_B$
TAG $B_{\text{lept}}$ fully reconstructed: $B \to \pi (\rho) \ell \nu$

2-dimensional fit to $(M_x, x_B^2)$ distribution:
- extract normalization for signal $\pi \ell \nu$ and $\rho \ell \nu$, $X_u \ell \nu$ and BB background
- PDF from MC, constraint on total $B (B \to X_u \ell \nu)$
- extract yield from $M_x$ distribution for 3 different $q^2$ bins

$\pi / \nu (82 \pm 13)$
$\rho / \nu (65 \pm 20)$
other $X_u / \nu$

$m_x \text{ GeV}/c^2$

$x_B^2$ dist. for $\pi$ mass region (I)

$x_B^2$ dist. for $\rho$ mass region (II)

$B(\pi^- \ell^+ \nu) = (1.75 \pm 0.28_{\text{stat}} \pm 0.20_{\text{syst}} \pm 0.03_{\text{FF}}) \times 10^{-4}$

$B(\rho^- \ell^+ \nu) = (2.54 \pm 0.78_{\text{stat}} \pm 0.85_{\text{syst}} \pm 0.30_{\text{FF}}) \times 10^{-4}$

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Signal and background normalization via binned $\chi^2$ fit to $\cos^2\phi_B$ in 3 bins of $q^2$

- $q^2 = (p_B - p_\pi)^2$ with $p_B \approx 0 \; \sigma_{q^2} \approx 0.7 \text{ GeV}^2$

Mainly $B^0\bar{B}^0$

$B(\pi\ell\nu) = (1.03 \pm 0.25_{\text{stat}} \pm 0.13_{\text{syst}}) \times 10^{-4}$
No TAG: $\nu$ Reconstruction: $\mathbf{B} \rightarrow \pi(\rho)\ell\nu$

- Reconstruct neutrino from full event + ensure good reconstruction quality (as first introduced by CLEO)

$$(\vec{p}_{\text{miss}}, E_{\text{miss}}) = (\vec{p}_{\text{beams}}, E_{\text{beams}}) - \left( \sum_i \vec{p}_i, \sum_i E_i \right)$$

- 3-dimensional Max-LH Fit to $\Delta E$, $m_{ES}$, and $q^2$

extract relative normalization of signal and BG

$$m_{ES} = \sqrt{s / 4 - |\vec{p}_B^*|^2}$$

$$\Delta E = E_B^* - \sqrt{s / 2}$$

$$q^2 = (p_{\ell}^2 + p_{\nu}^2)$$

- Neutrino momentum scaled so $\Delta E = 0$ → improved $q^2$ resolution

- Assume isospin relations

\[
\begin{align*}
\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu) &= 2\Gamma(B^+ \rightarrow \pi^0 \ell^+ \nu) \\
\Gamma(B^0 \rightarrow \rho^- \ell^+ \nu) &= 2\Gamma(B^+ \rightarrow \rho^0 \ell^+ \nu)
\end{align*}
\]

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SLAC/INT Workshop, May 2005
No TAG: Fitted $\Delta E$ and $m_{ES}$: $B \rightarrow \rho \ell \nu$

Sum: 95 $\rho^+$ and 98 $\rho^0$
No TAG: Fitted $\Delta E$ and $m_{ES}$

$B \rightarrow \pi l\nu$

Sum of $396 \pi^+$ and $137 \pi^0$

BABAR Preliminary

76/fb
No TAG: Systematic Uncertainties

<table>
<thead>
<tr>
<th>$q^2$ Range (GeV$^2$)</th>
<th>$\delta B_\pi / B_\pi$ (%)</th>
<th>$\delta B_\rho / B_\rho$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5</td>
<td>8.3 7.0 11.4 3.7 11.0</td>
<td>8.1 21.0 12.7 18.2 17.0</td>
</tr>
<tr>
<td>5–10</td>
<td>4.2 1.4 1.8 3.7 3.6</td>
<td>2.9 11.9 2.7 2.8 5.9</td>
</tr>
<tr>
<td>10–15</td>
<td>5.0 3.9 3.9 4.3 3.6</td>
<td>4.2 7.2 1.8 3.8 4.2</td>
</tr>
<tr>
<td>15–20</td>
<td>0.7 1.6 0.8 3.2 7.7</td>
<td>2.8 10.9 9.0 21.2 12.9</td>
</tr>
<tr>
<td>20–25</td>
<td>13.5 2.4 1.0 2.2 7.8</td>
<td>5.6 11.2 0.9 1.6 4.6</td>
</tr>
<tr>
<td>Total Error</td>
<td>17.4 9.2 12.6 8.3 16.8</td>
<td>12.8 30.5 16.8 28.6 24.7</td>
</tr>
</tbody>
</table>

$\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.38 \pm 0.10 \pm 0.17 \pm 0.08) \times 10^{-4}$

$\mathcal{B}(B^0 \rightarrow \rho^- \ell^+ \nu) = (2.14 \pm 0.21 \pm 0.51 \pm 0.28) \times 10^{-4}$

Dominant systematic errors from $\nu$ reconstruction (track losses, $\gamma$ bg, KL), and non-BB background at low $q^2$,

- BUT $B \rightarrow X_u l \nu$ simulation a la DeFazio Neubert, and unknown FF for $B \rightarrow \rho l \nu$
Form Factors for $B \rightarrow \pi \ell \nu$ Decays

- **Calculations:**
  - **Light-Cone Sum Rules**
    - Ball/Zwicky (hep-ph/0406232) quote 10-13% uncertainty at $q^2=0$
    - Validity: $q^2 < 14 \text{ GeV}^2$
  - **Lattice QCD**
    - Unquenched calculations from HPQCD (hep-lat/0408019) and FNAL (hep-lat/0409116) quote 11% uncertainty at high $q^2$
    - Validity: $q^2 > 15 \text{ GeV}^2$
  - **ISGW2**
    - quark model
    - No uncertainty quoted

\[
\frac{d\Gamma(B^0 \rightarrow \pi^- \ell^+\nu)}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 p_\pi^3 |f_+(q^2)|^2
\]

\[
q^2 = m_{l\nu}^2 = (p_l + p_\nu)^2
\]
Extrapolation beyond the range of calculations by pole-ansatz of Becirevic-Kaidalov (BK):

\[ f_+(q^2) = \frac{c_B(1-\alpha)}{(1-q^2/m_B^2)(1-\alpha q^2/m_{B*}^2)} \]

One shape parameter:
- HPQCD: \( \alpha = 0.42 \pm 0.05 \)
- FNAL: \( \alpha = 0.62 \pm 0.07 \)

Fit to BABAR Data
\( \alpha = 0.60 \pm 0.14 \)

But normalization still has large uncertainties!

There are other, mostly older quark model predictions, ISGW2, Melikov/Stech, …Are they passé?

Q: What can we learn from SCET?
Current Measurements of $\mathcal{B}(B\to\pi l\nu)$ as function of $q^2$

Clearly:
We need more data
Recent LQCD and LCSR calculations agree well with each other and with data.

ISGW2 shows marginal agreement for $\pi\ell\nu$

Errors for $\rho\ell\nu$ still too large to study 3 form factors.

<table>
<thead>
<tr>
<th>$\pi$ FF Model</th>
<th>ISGW II</th>
<th>BALL’04</th>
<th>FNAL’04</th>
<th>HPQCD’04</th>
<th>BK Fit to Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob($\chi^2$) (%)</td>
<td>3</td>
<td>52</td>
<td>49</td>
<td>39</td>
<td>51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho$ FF Model</th>
<th>ISGW II</th>
<th>BALL’98</th>
<th>BALL’05</th>
<th>Fit to Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob($\chi^2$) (%)</td>
<td>52</td>
<td>26</td>
<td>48</td>
<td>68</td>
</tr>
</tbody>
</table>
Extraction of $|V_{ub}|$ from $B \to \pi \ell \nu$ Decays

Extraction of $|V_{ub}|$ relies on FF normalization available in distinct $q^2$ ranges:

- LCSR $q^2<15$ GeV$^2$, LQCD $q^2>15$ GeV$^2$,
- Extrapolation to whole $q^2$ range

\[ |V_{ub}| = \sqrt{\frac{\Delta B}{\tau_B \Gamma_{thy}}} \]

\[ \tilde{\Gamma}_{thy} = \frac{G_F^2}{24\pi^3} \int_{q_{min}^2}^{q_{max}^2} |f_+(q^2)|^2 p_\pi^3 dq^2. \]

Currently theoretical error on FF normalization dominates, $\sim 15\%$.

BABAR's Choice

\[ |V_{ub}| = (3.82 \pm 0.14_{stat} \pm 0.24_{syst} \pm 0.11_{FF-0.52LQCD}) \times 10^{-3} \]
Extrapolation to 500x10^6 BB Events: B \to \pi^- \ell^+ \nu

Experimental errors will be reduced

- improved track and neutral reconstruction (\nu reco !)
- more measurements of B \to Xc\ell\nu and B \to Xu\ell\nu (res & non-res) BFs, FFs
- Extrapolation of BF errors to 500x10^6 BB pairs

<table>
<thead>
<tr>
<th>Event Selection</th>
<th>Yield [Evt/500x10^6BB]</th>
<th>S/B</th>
<th>\sigma_{stat}</th>
<th>\sigma_{syst}</th>
<th>\sigma_{exp}</th>
</tr>
</thead>
<tbody>
<tr>
<td>hadronic tags</td>
<td>60</td>
<td>11</td>
<td>13 %</td>
<td>4%</td>
<td>13%</td>
</tr>
<tr>
<td>D(*) \ell \nu tags</td>
<td>240</td>
<td>2.5</td>
<td>6 %</td>
<td>6%</td>
<td>10%</td>
</tr>
<tr>
<td>(D)\pi_S \ell. tags</td>
<td>320</td>
<td>1.5</td>
<td>5 %</td>
<td>12%?</td>
<td>13% ?</td>
</tr>
<tr>
<td>No tags</td>
<td>2,400</td>
<td>1.0</td>
<td>3 %</td>
<td>8%</td>
<td>9%</td>
</tr>
</tbody>
</table>

Theoretical input is needed for translation of BF to |V_{ub}|:

- Progress will depend on reliable theoretical estimates (incl. errors) of FF normalization
- Even for \pi\ell\nu the \rho\ell\nu FF are important, because bg is hard to remove at large q^2.
- We use deFazio-Neubert with JETSET fragmentations for decays to heavy mesons X_u. This also is a major systematic uncertainty!
Form Factors for $B \to \rho \ (\omega) \ \ell \ \nu$ – What else can we learn?

Q: What can we learn from measuring other decay modes?
- pseudo scalars? $\eta, \eta'$
- vector mesons? $\rho, \omega, \ldots$

Q: If $\rho$ is too wide, does $\omega$ work for lattice?
Q: Can we expect similar predictions and ansatz' for shape of FF for decays to $\rho, \omega$??

$$F(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_{Rt}^2}$$
for V, $A_0$, $A_1$

Some guidance might help get shape from data and possibly extrapolate to full $q^2$ range?

Q: How do s.l. FF relate to $B \to s\gamma$, $\to s\ell^+\ell^-$
Q: What can SCET tell us from $B \to \pi\pi$?
Q: Should we worry about rad. corrections beyond Ginsberg (1968)?? (PHOTOS)
Summary of $|V_{ub}|$ extraction from $B \rightarrow \pi l \nu$

- Ball-Zwicky
- HPQCD
- FNAL

CLEO v reco.
BABAR v reco.
Belle s.l. tag
BABAR s.l. tag
Average

CLEO v reco.
BABAR v reco.
Belle s.l. tag
BABAR s.l. tag
Average

CLEO v reco.
BABAR v reco.
Belle s.l. tag
BABAR s.l. tag
Average

$|V_{ub}| (10^{-3})$
$|V_{ub}|$ determined from $B \to \pi^- \ell^+ \nu$ with lattice QCD.

w/ quenched LQCD [FNAL/JLQCD/APE/UKQCD] 

Average $\Gamma_{\text{thy}} = 1.92^{+0.32}_{-0.12} \pm 0.47$

$\Rightarrow$ $(3.90 \pm 0.71 \pm 0.23^{+0.62}_{-0.48}) \times 10^{-3}$

w/ unquenched LQCD [FNAL/HPQCD]

Preliminary results reported at Lattice’04.

FNAL’04 $\Gamma_{\text{thy}} = 1.96 \pm 0.51 \pm 0.39$

$\Rightarrow$ $(3.87 \pm 0.70 \pm 0.22^{+0.85}_{-0.51}) \times 10^{-3}$

HPQCD $\Gamma_{\text{thy}} = 1.31 \pm 0.33$

$\Rightarrow$ $(4.73 \pm 0.85 \pm 0.27^{+0.74}_{-0.50}) \times 10^{-3}$

140fb-1, preliminary 4th error from $\Gamma_{\text{thy}}$

FF-dep. in $\text{Br}$ is small for $\pi \ell \nu$ data

Vera Lüth
SLAC/INT Workshop, May 2005