OPEN QUESTIONS IN JETS IN SCET

INT SEMINAR
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"SCET was made for jets!"
-M. Wise

(Alternative talk title: Why M. Wise no longer works on SCET...)


Outline:

I KINEMATICS
II FACTORIZATION OF DECAY RATE OF $\pi \rightarrow JETS$
III JET ENERGY DISTRIBUTION
IV THRUST AND OTHER DISTRIBUTIONS
V UNIVERSAL NP EFFECTS?
VI CONCLUSION AND OPEN QUESTIONS

I KINEMATICS
$e^+ e^- \rightarrow \text{hadrons}$

or $Z \rightarrow \text{hadrons}$ (focus on 2-jet-like events)

$E_{\text{hadrons}} < E_{\text{Max}}$

$E, S, \lambda = \sqrt{E^2 - m^2}$

Steven Weinberg (1972): jet outrun:
All but $E_{\text{hadrons}}$ of the total energy goes into two cones of angle $< S$. 
II \ DECAY RATES AND FACTORIZATION

recall total hadronic decay rate of $Z$ goes due

$$\Gamma(Z \rightarrow \text{hadrons}) = \Gamma_{\text{pt.}}(Z \rightarrow \text{partons}) + \Delta\Gamma_{\text{n.p.}}$$

in perturbation theory leading nonperturbative correction given by

$$\sim \langle 0\mid \text{Tr}(\text{C}_{\mu \nu}) \mid 0 \rangle$$

$\Delta\Gamma_{\text{n.p.}}$ suppressed by $(\frac{\Lambda}{M_Z})^4$ relative to $\Gamma_{\text{pt.}}$.

$\rightarrow$ Strategy: look for larger NP effects in less inclusive variables

Start with:

$$d\Gamma = \frac{1}{2M_Z} \sum \sum c_x \left| \langle x \mid j^\mu, c_\mu \mid 0 \rangle \right|^2 \left( 2\pi \right)^n \delta^n(p_e - p_f)$$

$Q^2 \Gamma_{\mu\nu}^q$  $P_M^\mu = q_x y^\mu + q_e y^\mu$

match

$\text{SEC}_{T_q} \left[ \delta_{\text{Whitney}} Y_n \Gamma_{\mu\nu}^{T_q} C(p, p^\prime) Y^\dagger \right] Y_{n^\dagger} [W^\dagger \delta_q^\dagger]$  

New collinear and ultra-light sectors are decoupled

$\rightarrow$ split final state into $J_n, J_{\bar{n}}, X_u, X_s$ sectors

$$d\Gamma = \frac{1}{2M_Z} \sum \sum c_x \left| \langle x \mid J_n, X_u \rangle \delta_{\text{Whitney}} Y_n \Gamma_{\mu\nu}^{T_q} C(p, p^\prime) Y^\dagger \right|^2 \left( 2\pi \right)^n \delta^n(p_e - p_f - p_{\bar{n}} - p_{\mu\nu})$$

$\sqrt{\text{factorize}}$
\[ d\Gamma = \frac{1}{6m_e^2} \sum_{\text{F, S}} \sum_{\text{F, S}} |\langle J_\text{F, S} | T^{\text{F, S}}_0 \rangle|^2 \langle W^+_0 | \tilde{W}^-_0 \rangle^2 \]

\[ \times |\langle X_{\text{us}} | Y_{\text{us}} Y_{\text{us}}^+ | 0 \rangle|^2 \] 

\[ = \frac{1}{(2\pi)^9} 8^4 \delta^4 (p_1 - p_2 - p_3 - p_4) \]

NP effects come from here

**Master Formula**

**Question:**
Is it valid to split to color rigid state |X\rangle into the (colored) partonic states |F_\text{us} J_0 | X_{\text{us}} \rangle? Are the sums over hadronic or partonic states equivalent?

Answer is probably "no," so how much error is introduced into final result?

Return to these (?)'s in Sec. VI

### III Jet Energy Distribution

Measure \( E_J \), energy of one of the jets.

Work to leading order in \( \alpha_s \) → just make \( q_\tilde{\tau}, \tilde{q}_\text{in} + \tilde{X}_{\text{us}} \) in final state.

Insert \( dE_J S(E_J - p_\tilde{\tau}^0) \) in master formula.

**Question:** Why not include \( u \bar{u} \) particles inside jet?

Answer in Sec. VI

\[ \Rightarrow \frac{d\Gamma}{dE_J} = \frac{1}{6m_e^2} \sum_{\text{F, S}} \sum_{\text{F, S}} \left| \frac{d^3p_\tilde{\tau}}{d^3p_{\tilde{\tau}}} \right|^2 \left| \frac{d^3p_{\tilde{\tau}}}{d^3p_{\tilde{\tau}}} \right|^2 \langle \tilde{q}_{\text{in}} \tilde{q}_{\text{in}} | \tilde{W}^-_0 \rangle^2 \]

\[ \times \sum_{\text{F, S}} \left| \langle X_{\text{us}} | Y_{\text{us}} Y_{\text{us}}^+ | 0 \rangle \right|^2 \] 

\[ \times \frac{1}{(2\pi)^9} 8^4 \delta^4 (p_1 - p_2 - p_3 - p_4) S(E_J - p_\tilde{\tau}^0) \]
To perform phase space integrals, align $p_0$ along $\overline{n}$ such that:
\[
\begin{align*}
\bar{p}_t^+ &= \bar{p}_x^+ + k_s^+ \\
\bar{p}_x^+ &= k_x^+ \\
\bar{p}_x^- &= 0 \\
\bar{p}_x^- &= \bar{p}_x^- + k_s^- \\
\bar{p}_x^- &= 0 \\
\bar{p}_x^- &= \bar{p}_x^- + k_s^- \\
\end{align*}
\]

Then momentum-conserving delta function enforces:
\[
\begin{align*}
\delta(E_3 - \bar{p}_t^0) &= \delta(E_3 - \bar{p}_t^x) \\
&= \delta(E_3 - \bar{p}_x^x + k_s^x) \\
&= \delta(E_3 - M_x^2 + k_s^x) \\
&= \delta(E_3 - M_x^2 + k_s^x/2) \\
\end{align*}
\]

Thus we obtain, to leading-order in $x_s$,
\[
\frac{d\Gamma}{dE_3} = \Gamma^{10}(z \rightarrow q \bar{q}) \delta(M_x^2 - E_3)
\]

in pert. th.

where
\[
\delta(M_x^2 - E_3) = \sum_{X_{us}} \delta(M_x^2 - E_3 - k_s^x/2) \frac{1}{N_e} \langle 0 | \gamma \gamma_{\mu} Y_{\nu} | X_{us} \rangle
\]

Consider screening distribution over a region
\[
\Lambda_{\alpha0} \ll \Lambda \ll M_x
\]
\( S(\frac{M^2}{2} - E_3) = S(\frac{M^2}{2} - E_3) - S'(\frac{M^2}{2} - E_3) \frac{\langle k_{us}^+ \rangle}{2} + \ldots \)

where \( \langle k_{us}^+ \rangle = \frac{1}{N_c} \sum_{k_{us}} \langle \theta_1 \theta_2 \cdots \theta_N | k_{us}^+ | 0 \rangle \)

\( \Rightarrow \) leading NP correction to smeared distribution is \( \delta \left( \frac{N_{c_0}}{\Delta} \right) \)

II OTHER VARIABLES

Hemisphere \( T = \frac{1}{M_2} \max_i \sum_{i} |\vec{p}_i \cdot \hat{t}| \quad (2 \text{ back-to-back jets } \Rightarrow T=1) \)

\( \Rightarrow \) sum over all particles in final state

\( ^{\text{a}} \) vs. \( ^{\text{b}} \) in hemispheres

jet masses \( M_a^2, M_b^2 \) (total invariant mass in two hemispheres defined by the hemi-axes)

\( M_2^2 = M_a^2 + M_b^2 \)

other:

\( C \) parameter, jet broadening, etc.
UNIVERSALITY IN NP EFFECTS?

Suppose derivation in Sec. III for $T$, $M_s^2$, etc.

$$\frac{dP}{dT} = \Gamma_{\mu \nu}^{\mu \nu \tau} S_{\tau}(1-T)$$

$$= S(1-T) - S'(1-T) \frac{1}{M_2} \langle k^{(a)}_{\mu} + k^{(b)}_{\mu} \rangle$$

Different components in $a$, $b$ hemispheres

$\Rightarrow$ not same as for $E_3$ dist.

$$\frac{dP}{dM_s^2} = \Gamma_{\mu \nu}^{\mu \nu \tau} \left[ S(M_s^2) - M_2 S'(M_s^2) \langle k^{(a)}_{\mu} + k^{(b)}_{\mu} \rangle \right]$$

$\Rightarrow$ same as for $T$!

Other distributions do not have universal NP constraints, only $T$ and $M_s^2$

$\Rightarrow$ consistent with data from DELPHI (2003)
VI. Conclusions and Questions

Answer to Q #2: in jet energy distribution, insert instead:

\[ \delta \left( E_3 - p_T^2 \right) \]

which becomes

\[ \delta \left( E_3 - \frac{M_2^2}{2} + \frac{k_{\perp}^2}{2} - \frac{k_{\perp}\left( \cos(\phi) + k_{\perp} \cos(\phi) \right)}{2} \right) \]

\[ = \delta \left( E_3 - \frac{M_2^2}{2} + \frac{k_{\perp}^2 (\cos(\phi))}{2} - \frac{k_{\perp} \cos(\phi)}{2} \right) \]

\[ \downarrow \text{ limit } \lambda \to 0 \text{, or c.m. any } s \to 0 \]

\[ \delta \left( E_3 - \frac{M_2^2}{2} + \frac{k_{\perp}^2}{2} \right) \quad \text{same as before} \]

Mumbling about Q #1:

how much do color recombination effects shift above distributions?

\[ q_{\bar{n}} \quad \text{our jets are colored!} \]

\[ \leftrightarrow \quad \text{any correlations between } n, \bar{n} \text{ jets are suppressed in SCET} \]

We assume these do not shift distributions \( \frac{dN}{dE_3} \), etc.

by as large an amount as \( \frac{N_{\text{Born}}}{\Delta} \) effects.

(cf. Steffen-Karchenby, 1999)

Started with \[ \sum_x \langle x | j_{\mu} e^{-10} \rangle \] (hadronic states)

\[ \sum_x \sum_{p'} \langle x | p \rangle \langle p | j_{\mu} e^{-10} \rangle \] (hadronic states)

would like to quantify size of effects from color recombination/jet conditions

ignored these effects