Open Heavy-Flavour Production
in Hadron and Photon Collisions

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Effective Field Theory, QCD, and Heavy Hadrons
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1 Theory of inclusive single-hadron production

(a) QCD-improved parton model

QCD Lagrangian,

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + \sum_q \bar{q}(i\not{D} - m_q)q, \]

\[ F^{(a)}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f_{abc} A^b_\mu A^c_\nu, \]

\[ D_\mu = \partial_\mu - \frac{i}{2} g_s \lambda^a A^a_\mu, \]

contains quarks \( q \) and gluons \( A^a_\mu \) as elementary fields. Asymptotic states (particle beams, targets and produced particles) correspond to hadrons. QCD-improved parton model reconciles QCD with confinement of colour.

- Initial state: Parton density functions (PDF’s) \( F^A_a(x, M^2) \), where \( p_a = xp_A \) and \( M \) is factorization scale; nonperturbative input.
- Hard scattering: Partonic cross sections; amenable to perturbative QCD.
- Final state:
  - Inclusive jet production: Partons are clustered according to jet algorithm (parton-hadron duality).
  - Inclusive particle production: FF’s \( D^h_c(x, M^2) \), where \( p_c = p_h/x \) and \( M \) is factorization scale; nonperturbative
input if $m_h \lesssim \Lambda_{QCD}$, calculable in NRQCD if $m_h \gg \Lambda_{QCD}$.

![diagram](image)

**Figure 1**: Inclusive photoproduction of single hadrons.

**Invariant cross section:**

$$\frac{d^3\sigma}{dy\, d^2p_T}(\gamma p \rightarrow h + X) \propto \sum_{b,c} \int dx_p \frac{dx_h}{x_h^2} F^p_b(x_p, M^2_p) \times D^h_c(x_h, M^2_h) \frac{d\sigma_{b \to c}}{dt}(\mu^2, M^2_p, M^2_h),$$

where $p_T$ is transverse momentum and $y$ is rapidity of $h$. 
Photoproduction

Incoming electrons (or positrons) act as source of quasi-real, high-energetic photons. Flux of transverse photons with energy fraction \( x \) and virtuality \( Q^2 \) is

\[
f^e_\gamma(x, Q^2) = \frac{\alpha}{2\pi Q^2} \left[ \frac{1 + (1 - x)^2}{x} - \frac{2m_e^2 x}{Q^2} \right].
\]

Deep inelastic scattering is greatly suppressed. Weizsäcker-Williams approximation:

\[
F^e_\gamma(x, Q^2_{\text{max}}) = \int_{Q^2_{\text{min}}}^{Q^2_{\text{max}}} dQ^2 f^e_\gamma(x, Q^2)
\]

\[
= \frac{\alpha}{2\pi} \left[ \frac{1 + (1 - x)^2}{x} \ln \frac{Q^2_{\text{max}}}{Q^2_{\text{min}}} + 2m_e^2 x \left( \frac{1}{Q^2_{\text{max}}} - \frac{1}{Q^2_{\text{min}}} \right) \right],
\]

where \( Q^2_{\text{min}} = m_e^2 x^2 / (1 - x) \).

- Tagged events: \( Q^2_{\text{max}} \approx 0.01 \text{ GeV}^2 \).
- Untagged events: \( Q^2_{\text{max}} \approx 4 \text{ GeV}^2 \).

Longitudinal flux \( \lesssim 2\% \) of the transverse flux.
Direct photoproduction: \[
\frac{d\sigma}{dt}(\gamma b \rightarrow c + X) \propto \alpha^2 \frac{\ln M^2}{\Lambda^2_{\text{QCD}}} \]

Mostly backward events with large \( p_T \).

Resolved photoproduction: \[
F^\gamma_a \times \frac{d\sigma}{dt}(ab \rightarrow c + X) \propto \frac{\alpha}{2\pi} \ln \frac{M^2}{\Lambda^2_{\text{QCD}}} \]

Mostly forward events with small \( p_T \).
NLO formalism (massless case)

\( \gamma p \rightarrow h + X \) via resolved photoproduction:

\[
\frac{d^3 \sigma}{dy d^2 p_T} = \frac{1}{\pi} \sum_{a,b,c} \int dx_\gamma dx_p \frac{dx_h}{x_h^2} \ F_a^\gamma(x_\gamma, M_\gamma^2) F_b^p(x_p, M_p^2) \\
\times D_c^h(x_h, M_h^2) \left[ \frac{d\sigma^{0}_{ab\rightarrow c}}{dt}(s, t, \mu^2) \delta \left( 1 + \frac{t + u}{s} \right) \right.
\]

\[
+ \frac{\alpha_s(\mu^2)}{2\pi} K_{ab\rightarrow c}(s, t, u, \mu^2, M_\gamma^2, M_p^2, M_h^2) \theta \left( 1 + \frac{t + u}{s} \right) \right],
\]

where \( s = (p_a + p_b)^2 \), \( t = (p_a - p_c)^2 \) and \( u = (p_b - p_c)^2 \). NLO corrections\(^1\) \( K_{ab\rightarrow c} \) comprise 16 channels and their \( t \leftrightarrow u \) crossed counterparts.

---

\( \gamma p \rightarrow h + X \) via direct photoproduction:

\[
F_a^\gamma(x_\gamma, M_\gamma^2) \rightarrow \delta(1 - x_\gamma),
\]

\[
d\sigma_{ab\rightarrow c}^0 \rightarrow d\sigma_{\gamma b\rightarrow c}^0,
\]

\[
K_{ab\rightarrow c} \rightarrow K_{\gamma b\rightarrow c}.
\]

At LO, only Bethe-Heitler process \( \gamma g \rightarrow q\bar{q} \) and Compton process \( \gamma q \rightarrow gq \) contribute. NLO corrections\(^2\) \( K_{\gamma b\rightarrow c} \) comprise 8 channels.

Effect of NLO corrections:

- Reduction of dependence on \( \mu, M_\gamma, M_p, M_h \) (theoretical uncertainty).
- Sizeable shift in cross section in certain regions of phase space (\( K \) factor).

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Treatment of divergences

Ultraviolet, infrared and collinear divergences are extracted using dimensional regularization \((n = 4 - 2\epsilon)\).

\[
\begin{align*}
\delta\sigma_{ab\rightarrow c}^0 & \quad \text{CT} \\
+ & \quad \text{UV div. } \frac{1}{\epsilon} \\
K_{ab\rightarrow cd}^0 \delta \left(1 + \frac{t + u}{s}\right) & \quad \text{virt. RC} \\
+ & \quad \text{IR div. } \frac{1}{\epsilon^2} \\
K_{ab\rightarrow cde}^0 \theta \left(1 + \frac{t + u}{s}\right) & \quad \text{real RC} \\
\times & \quad \text{coll. div. } \frac{1}{\epsilon} \\
F_{a\gamma}^0(x_\gamma) F_{b\gamma}^{p0}(x_p) D_{c\gamma}^{h0}(x_h) & \quad \text{bare PDF, FF}
\end{align*}
\]

\[
= K_{ab\rightarrow c}(\mu^2, M_\gamma^2, M_p^2, M_h^2) \\
\times F_{a\gamma}(x_\gamma, M_\gamma^2) F_{b\gamma}^{p}(x_p, M_p^2) D_{c\gamma}^{h}(x_h, M_h^2).
\]
Factorization

After elimination of UV and IR divergences, subtract collinear ones in the following way:

\[
K_{0}^{ab \to c}(s, t, u) - H_{a \to i}^{(S)}(x, M_{\gamma}^{2}) \frac{d\sigma_{ib \to c}^{0}}{dt'}(s', t')
\left|_{p_i = xpa}^{p_i = xp_b} \right.
- H_{b \to i}^{(S)}(x, M_{p}^{2}) \frac{d\sigma_{ai \to c}^{0}}{dt'}(s', t')
\left|_{p_i = xpb}^{p_i = pc/x} \right.
- H_{i \to c}^{(T)}(x, M_{h}^{2}) \frac{d\sigma_{ab \to i}^{0}}{dt'}(s', t')
\]

with universal factorization kernels

\[
H_{a \to i}^{(S)}(x, M_{\gamma}^{2}) = -\frac{1}{\bar{\epsilon}} \left( \frac{\mu^2}{M_{\gamma}^2} \right) \bar{\epsilon} P_{a \to i}^{(S)}(x) + \frac{\alpha_s(\mu^2)}{2\pi} f_{a \to i}(x),
\]

\[
H_{i \to c}^{(T)}(x, M_{h}^{2}) = -\frac{1}{\bar{\epsilon}} \left( \frac{\mu^2}{M_{h}^2} \right) \bar{\epsilon} P_{i \to c}^{(T)}(x) + \frac{\alpha_s(\mu^2)}{2\pi} d_{i \to c}(x),
\]

where \(1/\bar{\epsilon} = 1/\epsilon + \ln(4\pi) - \gamma_E\) and \(P_{a \to b}^{(S,T)}(x)\) are spacelike/timelike Altarelli-Parisi splitting functions.

Form of collinear divergence just depends on splitting \(a \to b\), but not on process \(AB \to h + X\) (factorization theorem). Thus,
PDF’s and FF’s are universal, and QCD-improved parton model is predictive.

\( f_{a\to i}(x) \) and \( d_{i\to c}(x) \) define factorization scheme.

- \( \overline{\text{MS}} \) scheme: \( f_{a\to i}(x) = d_{i\to c}(x) = 0 \).
- DIS scheme: NLO structure function \( F_2(x, Q^2) \) of \( \nu N \) DIS has same form as in naïve parton model, i.e.

\[
    f_{g\to q} = \frac{1}{2} \left\{ \left[ x^2 + (1 - x)^2 \right] \ln \frac{1 - x}{x} + 8x(1 - x) - 1 \right\},
\]

etc.
- DIS\( \gamma \) scheme:

\[
    f_{\gamma\to q}(x) = 2N_c e_q^2 \frac{\alpha}{\alpha_s} f_{g\to q}(x).
\]

We adopt \( \overline{\text{MS}} \) scheme.
DGLAP equations

The $\mu^2$ evolution of $D^h_a(x, \mu^2)$ is ruled by the Altarelli-Parisi equations,

$$\frac{\mu^2 d}{d\mu^2} D^h_a(x, \mu^2) = \sum_b \int_x^1 \frac{dz}{z} P^{(T)}_{a\rightarrow b} \left( \frac{x}{z}, \alpha_s(\mu^2) \right) D^h_b(x, \mu^2),$$

with timelike $a \rightarrow b$ splitting functions

$$P^{(T)}_{a\rightarrow b} \left( x, \alpha_s(\mu^2) \right) = \frac{\alpha_s(\mu^2)}{2\pi} P^{(0,T)}_{a\rightarrow b}(x) + \left( \frac{\alpha_s(\mu^2)}{2\pi} \right)^2 P^{(1,T)}_{a\rightarrow b}(x) + O(\alpha_s^3).$$

Numerical solution methods:

- Brute-force evolution in $x$ space.
- Mellin transform technique, i.e.

$$F(n) = \int_0^1 dx \, x^{n-1} f(x) \quad (n = 1, 2, \ldots),$$

$$f(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} dn \, x^{-n} F(n),$$

where $\gamma$ is such that integration path is right of all singularities. Renders convolutions to products.
(b) Fragmentation functions

General strategy for determination

- Make ansatz for $D_{a}^{h}(x, \mu_{0}^{2})$ at starting scale $\mu_{0}$.
- Determine input parameters by fitting two sets of $e^{+}e^{-}$ data at respective scales $\mu = \sqrt{s}$.
- Determine also $\Lambda_{QCD}$ from scaling violation.
- Test scaling violation (Altarelli-Parisi equations) through comparison with $e^{+}e^{-}$ data at a third scale $\mu = \sqrt{s}$.
- Test universality of FF’s (factorization theorem) through comparisons with data of $ep, p\bar{p}, \gamma\gamma$ etc. scattering at scale $\mu = p_{T}$.
**FF’s for D\(^{\bullet\pm}\) mesons**

Determine realistic \(C \rightarrow D^{\bullet\pm}\) FF’s by fitting high-\(s\) data on \(e^+e^- \rightarrow D^{\bullet\pm} + X.\)\(^3\) Must also consider \(B \rightarrow D^{\bullet\pm}\). Adopt \(\Lambda^{(5)}_{\text{MS}}\) from charged-hadron analysis.

- **Standard set (S), with 6 fit parameters \((Q = C, B)\):**

\[
D_Q^{D^{\bullet\pm}}(x, \mu_0^2) = N x^\alpha (1 - x)^\beta.
\]

- **Mixed set (M), with 5 fit parameters:**

\[
D_C^{D^{\bullet\pm}}(x, \mu_0^2) = \frac{N}{x[1 - 1/x - \epsilon/(1 - x)]^2},
\]
\[
D_B^{D^{\bullet\pm}}(x, \mu_0^2) = N x^\alpha (1 - x)^\beta.
\]

Table 1: \(\chi^2_{\text{DF}}\) of fits to LEP1 data on \(e^+e^- \rightarrow D^{\bullet\pm} + X.\)

<table>
<thead>
<tr>
<th>set</th>
<th>total</th>
<th>ALEPH</th>
<th>OPAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>sum</td>
<td>c</td>
</tr>
<tr>
<td>LO S</td>
<td>0.81</td>
<td>0.73</td>
<td>1.29</td>
</tr>
<tr>
<td>NLO S</td>
<td>0.79</td>
<td>0.78</td>
<td>1.31</td>
</tr>
<tr>
<td>LO M</td>
<td>1.03</td>
<td>1.05</td>
<td>2.11</td>
</tr>
<tr>
<td>NLO M</td>
<td>0.95</td>
<td>1.10</td>
<td>2.05</td>
</tr>
</tbody>
</table>

Figure 2: Comparison of the LEP1 data on \(e^+e^- \rightarrow D^{*\pm} + X\) with the LO S (dashed) and NLO S (solid) calculations.
\[ \epsilon = \begin{cases} 0.0926 & \text{LO M}, \\ 0.0674 & \text{NLO M}. \end{cases} \]

Cf. \( \epsilon = 0.06^{+0.01}_{-0.02} \text{(stat.)}^{+0.01}_{-0.02} \text{(syst.)} \) from analysis\(^4\) of PEP and PETRA data with Lund Monte Carlo.

Figure 3: Comparison of the ARGUS, HRS, TASSO and OPAL data on \( e^+e^- \rightarrow D^{*\pm} + X \) with the LO S (dashed) and NLO S (solid) calculations.

(c) Schemes

- Focus on inclusive heavy-meson production.
- Appropriate scheme depending on $p_T/m_Q$?
- Heavy-quark fragmentation?

Massive-$Q$ scheme: \( ab \to Q\overline{Q} + X \)

- \( n_l = n_f - 1 \) massless flavors \( a, b \) treated in \( \overline{\text{MS}} \) renormalization and factorization scheme, appear in PDFs.
- \( Q \) treated in OS scheme (OS mass and WFR CTs, decoupling in \( \alpha_s(\mu) \) for \( \mu \ll m_Q \)), not intrinsic.
- No collinear divergences related to outgoing \( Q \) line. \( \Rightarrow \) No factorization. \( \Rightarrow \) No conceptual necessity for FFs.
- Valid for \( 0 \leq p_T \lesssim \text{few} \times m_Q \). \( \Rightarrow \) \( \sigma_{\text{tot}} \) well defined.
- Appropriate for \( t \) (no fragmentation).
- Breaks down for \( p_T \gg m_Q \) due to would-be collinear divergences \( \propto \alpha_s \ln(p_T^2/m_Q^2) \).
- FFs introduced ad hoc to match \( D \) and \( B \) data. No AP evolution, no universality. \( \Rightarrow \) Different \( \epsilon_{\text{Peterson}} \) for different scales, types of experiment. \( \Rightarrow \) Global data analysis unfeasible.

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$n_f$-flavor $\overline{\text{MS}}$ scheme: $^{6} \; ab \rightarrow c + X \text{ with } c \rightarrow Q \text{ meson}$

- $n_f$ massless flavors $a, b, c$ treated in $\overline{\text{MS}}$ renormalization and factorization scheme, appear in PDFs.
- Collinear divergences related to outgoing $c$ line factorized into nonperturbative FFs.
  \[ \alpha_s^{n+1,n} \ln^n (p_T^2/m_Q^2) \] terms resummed by AP evolution. \( \sim \) Valid for $p_T \gtrsim \text{few} \times m_Q$.
- Scaling violation and universality of FFs guaranteed by factorization theorem. \( \sim \) Unique $\epsilon_{\text{Peterson}}$. \( \sim \) Global data analysis possible.
- $\; (m_Q/p_T)^n$ terms not included. \( \sim \) Breaks down for $p_T \lesssim m_Q$. \( \sim \) No $\sigma_{\text{tot}}$.

---

Perturbative FFs: \( ab \rightarrow c + X \) with \( c \rightarrow Q \)

- Match

\[
\frac{d\sigma_{e^+e^- \rightarrow Q\overline{Q}+X}}{dx}(x, s, m_Q^2) = \sum_c \int_x^1 \frac{dz}{z} D_c^Q \left( \frac{x}{z}, \mu^2, m_Q^2 \right) \times \frac{d\sigma_{e^+e^- \rightarrow c+X}}{dz}(z, s, \mu^2).
\]

- Can incorporate PFFs by change of scheme.
- Still need nonperturbative FFs to match data.

\( d\sigma_{e^+e^- \rightarrow Q\overline{Q}+X}/dx < 0 \) for \( x \gtrsim 0.9 \) ! \( \sim \) Low-quality fit.

\( \oplus \) Unsatisfactory perturbative stability.

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“FONLL”:\(^9\)

- \(\text{FONLL} = \text{FO} + G(m_Q, p_T) \times (\text{RS} - \text{FOM0})\)
  
  \text{FO}: massive-\(Q\) scheme; \(\alpha_s\) and evolution of gluon PDF in \(n_f\)-flavor \(\overline{\text{MS}}\) scheme; no intrinsic \(Q\).
  
  \text{RS}: \(n_f\)-flavor \(\overline{\text{MS}}\) scheme with PFFs.
  
  \text{FOM0}: \(m_Q \to 0\) limit of FO.
  
  \(G(m_Q, p_T)\): arbitrary function with \(G(m_Q, p_T) \to 1\) for \(m_Q/p_T \to 0\), e.g. \(G(m_Q, p_T) = p_T^2/(p_T^2 + 25m_Q^2)\).

- RS and FOM0 evaluated at \(p_T \to m_T = \sqrt{p_T^2 + m_Q^2}\).

\(\oplus\) \((\text{RS} - \text{FOM0})\) abnormally large.

\(\oplus\) For \(p_T \lesssim \text{few} \times m_Q\) problems of massive-\(Q\) scheme (non-universality of FFs).

\(\oplus\) For \(p_T \gtrsim \text{few} \times m_Q\) problems of PPFs (low-quality fit, unsatisfactory perturbative stability).

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“NLO $n_f$-flavor scheme”:\textsuperscript{10}

- Massive-$Q$ scheme with would-be collinear singularities of form $\alpha_s \ln \left( \frac{p_T^2}{m_Q^2} \right)$ \textit{MS}-subtracted.
- Residual $\alpha_s \ln \left( \frac{p_T^2}{\mu_F^2} \right)$ terms small for $\mu_F \approx p_T$.
- $\alpha_s \ln \left( \frac{\mu_F^2}{m_Q^2} \right)$ terms absorbed into PDFs and FFs, and resummed by AP evolution.
- In $e^+e^-$ and direct $\gamma\gamma$ collisions, subtraction terms coincide with those generated by PPFs. Other processes?
  ⊕ Naturally interpolates between massive-$Q$ and $n_f$-flavor \textit{MS} schemes.
  ⊕ Factorization theorem\textsuperscript{11} in operation also for $p_T \lesssim \text{few} \times m_Q$.
  ≃ Universality of FFs.


(a) $D^{*\pm}$ mesons

$\gamma\gamma \rightarrow D^{*\pm} + X$ at LEP2:\textsuperscript{12}

- Massive-$Q$ and $n_f$-flavor $\overline{MS}$ schemes:

\begin{itemize}
  \item Massless-$Q$ and $nf$-flavor $\overline{MS}$ schemes:
    \begin{itemize}
      \item GRV, $m_c = 1.5$ GeV, $\mu_R = \mu_F / 2 = \xi m_T$
      \item $f(c \rightarrow D^*) = 0.267$, $\epsilon_c = 0.116$
    \end{itemize}
  \item Massive-$Q$ and $nf$-flavor $\overline{MS}$ schemes:
    \begin{itemize}
      \item GRS, $m_c = 1.5$ GeV, $\mu_R = \mu_F / 2 = m_T / 2$ (res)
      \item $f(c \rightarrow D^*) = 0.233$, $\epsilon_c = 0.035$
    \end{itemize}
\end{itemize}


B.A. Kniehl: Open Heavy-Flavour Production
• NLO 4-flavor scheme:

Massive (dash), massless (solid).


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B.A. Kniehl: Open Heavy-Flavour Production
$ep \rightarrow eD^{*\pm} j + X$ in photoproduction at HERA. 5-flavor $\overline{\text{MS}}$ scheme G. Heinrich, B.A. Kniehl, Phys. Rev. D70 (2004) 094035.

$^{14}$ZEUS, ICHEP’04.

B.A. Kniehl: Open Heavy-Flavour Production
\( \bar{p}p \rightarrow D^{*\pm} j + X \) in at the Tevatron.\(^{15}\) NLO 4-flavor scheme


(b) $B$ mesons

$p\bar{p} \rightarrow B^+ + X$ at the Tevatron.$^{16}$

Massive-$Q$ scheme

5-flavor $\overline{\text{MS}}$ scheme

FONLL, only $\langle x \rangle_{\text{LEP1}}$ used

3 Summary and outlook

- Massive-$Q$ scheme (with conventional $\epsilon_{\text{Peterson}}$) dramatically undershoots data of $B$ hadro-, lepto-, and photoproduction.
- Nonperturbative FFs crucial to describe $D^{*\pm}$ and $B$ data.
- AP evolution and universality of FFs requisite for global data analysis. Both lacking in massive-$Q$ scheme and for $p_T \lesssim \text{few} \times m_Q$ in FONLL!
- NLO $n_f$-flavor scheme introduces collinear factorization in massive-$Q$ framework.
- Implementation of $m_Q$ effects near threshold, kinematic constraints on threshold behaviour, $\text{ACOT}(\chi)$,\textsuperscript{17}...

Expect exciting new data from HERA-II, Tevatron Run II, $B$ factories, LHC, TESLA, . . . !