\gamma\text{ from } B \rightarrow DK \text{ Dalitz decays}

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Technion
Introduction

- $B \rightarrow DK$ can be used to get $\gamma$, basically with no hadronic uncertainties
- It is modular: we can use theoretical assumptions as needed
- The problem is statistics
- We need to use all possible modes
Use interference between $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$

Interference between

$B^+ \rightarrow DK^+$ follows by $D \rightarrow f$

$B^+ \rightarrow \overline{D}K^+$ follows by $\overline{D} \rightarrow f$

$f$ can be any common final state to $D$ and $\overline{D}$
In order to get a common final state we need either DCS or $K - \overline{K}$ mixing

- **Double Cabibbo Suppression (DCS)**
  - $D \rightarrow K^+ \pi^-$ \hspace{1cm} $\overline{D} \rightarrow K^+ \pi^-$
  - $D \rightarrow K^+ K^-$ \hspace{1cm} $\overline{D} \rightarrow K^+ K^-$

- **$K - \overline{K}$ mixing**
  - $D \rightarrow K_S \pi^0$ \hspace{1cm} $\overline{D} \rightarrow K_S \pi^0$
  - $D \rightarrow K_S \pi^+ \pi^-$ \hspace{1cm} $\overline{D} \rightarrow K_S \pi^+ \pi^-$
Hadronic parameters

The theoretical problem is how to get rid of unknown hadronic parameters

\[ N(\text{observables}) \geq N(\text{hadronic parameters}) + 1 \]

By now there are many methods where the above condition is satisfied

- The problem with $\gamma$ is statistics

- We would like to include new modes which introduce more observables than hadronic parameters
Example: Two body $D$ decay

\[ B^+ \to f_D K^+ \quad f_D = K^+\pi^-, K^{*+}\bar{K}^-,... \]

Hadronic parameters

\[
\frac{A(B^- \to \overline{D}K^-)}{A(B^- \to DK^-)} \equiv r_B e^{i(\delta_B + \gamma)} \quad \frac{A(D \to K^+\pi^-)}{A(\overline{D} \to K^+\pi^-)} \equiv r_D e^{i\delta_D}
\]

Get $\gamma$ from the interference

\[
\Gamma(B^+ \to f_D K^+) \propto r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D + \gamma)
\]

- 4 rates: $B^- (\gamma \to -\gamma)$ and $\overline{f} (r_D \to r_D^{-1})$
- We get $r_D$ from the large tagged $D$ decay data sample
- Take $k$ different final states. we have $k + 3$ unknowns $(\delta_D^f, r_B, \delta_B, \gamma)$ and $4k$ measurements ⇒ $\gamma$
Three body $D$ decay

- Each point in the Dalitz plot is like another two body mode.
- In practice we have to integrate over parts of the Dalitz plot.
- The integration results in $\delta_f$ not a pure phase.

$$
\int ds A_f(s) \bar{A}_f(s)^* \equiv \sqrt{\Gamma_f \bar{\Gamma}_f} e^{i\delta_f} e^{-\xi_f}
$$

- Still, we can win if we fit also $\xi_f$: $2k + 3$ unknowns and $4k$ observables.
Beside the $B^+ \to DK^-$ rate we need the Dalitz plot of

$$B^\pm \to (K_S \pi^- \pi^+)_{DK}^\pm$$

$B$ hadronic parameters

$$A(B^+ \to \overline{DK})$$

$$A(B^+ \to DK)$$

$$\equiv r_B e^{i(\delta_B + \gamma)}$$

$D$ hadronic parameters

$$A_D(s_{12}, s_{13}) = A_{12,13} e^{i\delta_{12,13}} \equiv A(D \to K_S(p_1)\pi^-(p_2)\pi^+(p_3))$$

$$= A(\overline{D} \to K_S(p_1)\pi^+(p_2)\pi^-(p_3))$$
The method - definitions

- Partition the Dalitz plot to $2^k$ bins
- Label bins below symmetry axis $i$, above axis $\bar{i}$

\begin{align*}
s_{12} &= m_{K_S\pi^-}^2 \quad \text{and} \quad s_{13} = m_{K_S\pi^+}^2
\end{align*}

\begin{align*}
c_i &\equiv \int dp \ A_{12,13} \ A_{13,12} \cos(\delta_{12,13} - \delta_{13,12}) \\
s_i &\equiv \int dp \ A_{12,13} \ A_{13,12} \sin(\delta_{12,13} - \delta_{13,12}) \\
T_i &\equiv \int dp \ A_{12,13}^2
\end{align*}

unknowns

$\bar{c}_i = c_i$, $\bar{s}_i = -s_i$
Determining $\gamma$

- A set of $4k$ equations
- The $k$ equations for the $i$ bins

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^- \rightarrow (K_S\pi^-\pi^+)_D K^-) =$$

$$T_i + r_B^2 T_i^- + 2r_B [\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i]$$

- The fact that $\delta_f$ is not a pure phase is

$$c_i^2 + s_i^2 < T_i^2$$

- $2k + 3$ unknowns: $c_i$, $s_i$, $r_B$, $\delta_B$, $\gamma$
- Solvable for $k \geq 2$
Some variations

- If the strong phase is known from $D$ data the fit is much simpler
  - We can model the Dalitz plot with a Breit-Wigner (or maybe something better?)
  - We can measure the strong phase in an entangled $D$ states at a charm factory

- The method can be extended to
  - Other multi-body $D$ decay,
  - Multi-body $B$ decay
  - Untagged $B^0$ and $B_s$ decays

- How to optimize the bins? The smaller the better, but not too small since then $D^*$ data introduce errors
\( D - \bar{D} \) mixing

In all \( B \to DK \) type methods to get \( \gamma \), the largest theoretical uncertainty in the SM is from \( D - \bar{D} \) mixing.

- In the SM we know that the mixing is CP conserving

\[
x \equiv \Delta m / \Gamma, \quad y \equiv \Delta \Gamma / 2 \Gamma, \quad x \sim y \lesssim 10^{-2}
\]

- Consider a DCS mode and ignoring \( D - \bar{D} \) mixing. This introduces large error in the determination of \( r_D \)

\[
\tilde{r}_D = r_D[1 + O(x/r_D)] \lesssim r_D[1 + O(20\%)]
\]

- Yet, the error on \( \gamma \) is much smaller

\[
\Delta \gamma \sim \frac{1}{8} \frac{x^2}{r_D^2}
\]
We can take $D - \bar{D}$ mixing into account by performing time integration over the $D$ decay.

Its effect is that $\delta_f$ is not a pure phase. Just like the integration over parts of the Dalitz plot:

$$\int dt A_f(t) \bar{A}_f(t)^* \equiv \sqrt{\Gamma_f \Gamma_\tilde{f}} e^{i\delta_f} e^{-\epsilon_f}$$

In methods where we already take care of this dilution, $D - \bar{D}$ mixing has no effect at all.

In cases we do not take care of it, we introduce an error of the order of the change in the “length” of $\delta_f$, which is quadratic in the small deviation.

CP violating $D - \bar{D}$ mixing implies $\Delta \gamma \sim O(x/r_D)$.
Conclusions

- $B \rightarrow DK$ can be used to get $\gamma$, basically with no hadronic uncertainties.

- It is modular: we can use theoretical assumptions as needed. In particular this is true for the Dalitz $D$ decay.

- Neglecting SM $D - \bar{D}$ mixing is justified.