Measuring $\alpha$ with $B \rightarrow \rho\rho$ and $\rho\pi$

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May 14, 2005
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Workshop on Flavor Physics and QCD
Minimal introduction

- $\alpha = \arg[V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$ with $b \rightarrow u$ and mixing
- penguin pollution with $/V_{tb}^*V_{td}$ is a problem

$B \rightarrow \rho\rho$

- proved to be successful due to small pollution and $\sim 100\%$ $CP$-even
- isospin analysis like in $B \rightarrow \pi\pi$ (Gronau/London)
  $B \rightarrow \rho^0\rho^0$ is the key

$B \rightarrow \rho\pi$

- proved to be successful with Dalitz analysis (interference)
- quasi-two-body isospin analysis is not feasible
Part I: $B \to \rho \rho$ Decays

\[ \alpha_T \cdot T + \alpha_C \cdot C + \alpha_P \cdot P \]

"Tree"

\[ u, d \]

\[ \bar{b} \]

\[ W^+ \]

\[ u \]

\[ \bar{d} \]

\[ \bar{u} \]

\[ \bar{d} \]

\[ \bar{u} \]

\[ u, d \]

"Penguin"

\[ u, d \]

\[ \bar{b} \]

\[ W^+ \]

\[ q \]

\[ g \]

\[ q \]

\[ u, d \]

\[ \omega \rho^+ \]

\[ \omega \rho^0 \]

\[ B \text{ decay} \]

\[ \alpha_T \]

\[ \alpha_C \]

\[ \alpha_P \]

\[ B (10^{-6}) \]

\[ f_L = |A_0|^2 / \Sigma |A_m|^2 \]

\[ N_{\bar{B}B} \]

| $B$ decay | $\alpha_T$ | $\alpha_C$ | $\alpha_P$ | $B (10^{-6})$ | $f_L = |A_0|^2 / \Sigma |A_m|^2$ | $N_{\bar{B}B}$ |
|-----------|-----------|-----------|-----------|----------------|----------------|-----------------|
| $\rho^0 \rho^0$ | 0 | 1 | -1 | \(< 1.1 \ (90\%)\) | – | BABAR \((230 \cdot 10^6)\) |
| $\rho^- \rho^+$ | $\sqrt{2}$ | 0 | $\sqrt{2}$ | \(30 \pm 4 \pm 5\) | $0.978 \pm 0.014^{+0.021}_{-0.029}$ | BABAR \((89/230 \cdot 10^6)\) |
| $\rho^0 \rho^+$ | 1 | 1 | 0 | \(23^{+6}_{-5} \pm 6\) \(32 \pm 7^{+4}_{-7}\) | \(0.97^{+0.03}_{-0.07} \pm 0.04\) \(0.95 \pm 0.11 \pm 0.02\) | BABAR \((89 \cdot 10^6)\) \(\text{BELLE} \((85 \cdot 10^6)\) |
| $\omega \rho^+$ | -1 | -1 | 2 | \(12.6^{+3.7}_{-3.3} \pm 1.6\) | \(0.88^{+0.12}_{-0.15} \pm 0.03\) | BABAR \((89 \cdot 10^6)\) |
| $\omega \rho^0$ | 0 | 0 | $-\sqrt{2}$ | \(< 3.3 \ (90\%)\) | – | BABAR \((89 \cdot 10^6)\) |

• $\rho^- \rho^+, \rho^0 \rho^+, \omega \rho^+$: large decay rate with "tree" (compared to $\pi \pi$)
• Confirm $f_L \sim 1 \implies CP$-even eigenstate $\rho^- \rho^+$
• $\rho^0 \rho^0, \omega \rho^0$: small "penguin" $\implies$ great for Unitarity Triangle $\alpha$
Issue 1: Isospin Triangles have to close

- Enforce isospin relation in toy MC generated with measured values:

<table>
<thead>
<tr>
<th></th>
<th>input ((10^{-6}))</th>
<th>output ((10^{-6}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B^{00})</td>
<td>0.6 ± 0.3</td>
<td>0.8 ± 0.3</td>
</tr>
<tr>
<td>(B^{+-})</td>
<td>30.0 ± 6.4</td>
<td>34.0 ± 5.6</td>
</tr>
<tr>
<td>(B^{+0})</td>
<td>26.0 ± 6.0</td>
<td>19.9 ± 3.9</td>
</tr>
</tbody>
</table>

\[
A(\rho^0\rho^0) + A(\rho^+\rho^-)/\sqrt{2} = A(\rho^0\rho^+)
\]

\[
B^0 \rightarrow \rho^0\rho^0
\]

\[
B^0 \rightarrow \rho^+\rho^-
\]

\[
B^+ \rightarrow \rho^+\rho^0
\]

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Measuring $\alpha$ with $\rho\rho$ like with $\pi\pi$

- $A_{\rho^-\rho^+}(t) = C_L^{+-} \cos(\Delta m_B t) - S_L^{+-} \sin(\Delta m_B t)$

\[
\sin(2\alpha_{\text{eff}}^+-) = \text{Im}(\frac{q}{p} \frac{A_L^{+-}}{A_{L^0}^-})/|\lambda| = S_L^{+-} / \sqrt{1 - C_L^{+-}^2}
\]

\[
2\Delta\alpha^{+-} = \text{arg}(\frac{A_L^{+-}}{A_{L^0}^-}) - \text{arg}(\frac{A_L^{+-}}{A_{L^0}^-})
\]

$\alpha = \alpha_{\text{eff}}^+- + \Delta\alpha^{+-}$

$B_{\text{BABAR}}$:

$\alpha_{\text{eff}}^+- = (100 \pm 9)^\circ$

$\Delta\alpha^{+-} = (0 \pm 11)^\circ$

$\alpha = (100 \pm 14)^\circ$
Issue 2: Penguin and Tree Size

- Extract Penguin (P) and Color-suppressed tree (C) amplitudes
  - t-convention: dominant t-penguin $V_{tb}^*V_{td}$, $P/T$ weak phase $\alpha$
  - “correct” $\alpha \sim 90^\circ$: T dominates, $|P| \sim |C|$

- “wrong” $\alpha \sim 180^\circ$: possible large $|C|/|T|$ (could use for constraints)
- phase assumption could be used (e.g. Gronau/Lunghi/Wyler)
Issue 3: What is the true branching of $B \rightarrow \rho^0 \rho^0$

1. set only limits on $\rho^0 \rho^0 \Rightarrow$ straight-forward but unlikely
2. $\mathcal{B}(\rho^0 \rho^0) \sim 0.8 \times 10^{-6}$

not resolved ambiguities ($8/180^\circ$)
each solution to few$^\circ$ at $\sim$10ab$^{-1}$

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Measuring $\alpha$ with $\rho^0 \rho^0$

- Unique to $B \to \rho \rho$ (vs. $\pi \pi$): $B^0 \to \rho^0 \rho^0 \to \pi^+ \pi^- \pi^+ \pi^-$
  - measure $\alpha_{00}^{\text{eff}}$ with $A_{\rho^0 \rho^0}(t) \Rightarrow$ need high statistics
  - $\Delta \alpha^{00}$ from the same triangle

- Limited $\alpha$ resolution due to “long” sides systematics in $B^{+-}$ and $B^{0+}$
  (assume $\sim 7\%$ here)

- Real advantage:
  resolving triangle ambiguities

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Resolving $\alpha$ Ambiguities with $\rho^0\rho^0$ and $\rho^+\rho^-$

- Triangle orientations should match for $\Delta\alpha^{00}$ and $\Delta\alpha^{+-}$

![](A) opposite orientations

![](B) the same orientations

- Could resolve $\alpha$ ambiguities with $\rho^0\rho^0$
  - could completely resolve if branching systematics $\sim 1\%$
Projection to Higher Luminosity

- Strongly depends on true values
Conclusion on $\rho \rho$ and Open Issues

- $B \to \rho^0 \rho^0$ unique, may allow $\alpha$ precision few$^{\circ}$ together with $\rho^+ \rho^-^0$

- Challenges:
  - broad $\rho \Rightarrow l=1$ contribution, isospin-breaking, interference
  - high background and $B$ cross-feed
  - branching $B^{+0}$ and $B^{+-}$ systematics down to few $0\%$
  - even relative $\Upsilon(4S) \to B^+ B^-$ vs. $B^0 \bar{B}^0$ matters for $B^{+0}/B^{+-}$
  - assume $f_L(\rho^0 \rho^0) = 1$, lesson of $\phi K^*$: other contributions to $\rho \rho$?

- We learn to deal with challenges as we accumulate statistics

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Resolving \((90^\circ - \alpha)\) Ambiguities

• Unresolved ambiguity \(\alpha \leftrightarrow (90^\circ - \alpha)\)
  
  – measure \(\sin(2\alpha)\) in individual channels \((\pi\pi, \rho\pi, \rho\rho, \text{etc})\)

• Angular-time analysis of \(B^0 \rightarrow \rho^+ \rho^-\) could solve:
  
  – \(\mathcal{I}m(A_{\|}(t)A_\perp(t)^*)\) and \(\mathcal{I}m(A_0(t)A_\perp(t)^*)\) \(\propto \cos(2\alpha) \sin(\Delta mt)\)
  
  – hopeless in near future given \(f_L \sim 98\%\)

• Dalitz-time analysis of \(B^0 \rightarrow \pi^+ \pi^- \pi^0\)
  
  – interference e.g. \(\rho^+ \pi^-\) and \(\rho^- \pi^+\)
  
  \[\Rightarrow \cos(2\alpha) \sin(\Delta mt)\]
  
  – proposed by Snyder/Quinn
  
  recently measured by \(B_{\Lambda B AR}\)
Part II: $B \to \rho \pi$ Decays

(1) How small is $B \to \rho^0 \pi^0$? ($\text{BABAR } 1.4 \pm 0.7$, BELLE $5.1 \pm 1.8$)

(2) $B \to \rho^\pm \pi^\mp$ not $CP$ eigenstate

<table>
<thead>
<tr>
<th>$B$ decay</th>
<th>HFAG $\mathcal{B}$ ($10^{-6}$)</th>
</tr>
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<tbody>
<tr>
<td>$\rho^0 \pi^0$</td>
<td>1.8 ± 0.6</td>
</tr>
<tr>
<td>$\rho^+ \pi^-$</td>
<td>24.0 ± 2.5</td>
</tr>
<tr>
<td>$\rho^- \pi^+$</td>
<td>12.0 ± 2.0</td>
</tr>
<tr>
<td>$\rho^0 \pi^+$</td>
<td>$9.1^{+1.4}_{-1.3}$</td>
</tr>
</tbody>
</table>

- Q2B isospin analysis unfruitful (12 params):

\[
\sin(2\alpha + \theta_{+-}), \sin(2\alpha + \theta_{-+}), \sin(2\alpha + \theta_{00})
\]

\[
\frac{A^{+-}}{\sqrt{2}} + \frac{A^{-+}}{\sqrt{2}} + \sqrt{2}A^{00} = A^{+0} + A^{0+} = T_1
\]

\[
A^{+-} - A^{-+} + \frac{A^{0+}}{\sqrt{2}} - \frac{A^{+0}}{\sqrt{2}} = T_2
\]

- better if very small $\rho^0 \pi^0$  

Lipkin/Nir/Quinn/Snyder, Gronau
Dalitz analysis of $B \to \rho \pi$

- Dalitz-time analysis of $B \to \pi^+ \pi^- \pi^0$ by $\text{BABAR}$ ($213 \times 10^6 B \bar{B}$)
  - key: time-dependence and interference of $\rho^+ \pi^-$ and $\rho^- \pi^+$

$$|A_{3\pi}^\pm(t)|^2 = \frac{e^{-|t|/\tau_{B^0}}}{4\tau_{B^0}} \left[ |A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2 \mp (|A_{3\pi}|^2 - |\bar{A}_{3\pi}|^2) \cos(\Delta m t) \right]$$

- 16/26 free parameters (incl. strong phase):

$$\delta_{+-} = \arg \frac{A(\rho^- \pi^+)}{A(\rho^+ \pi^-)} = (-67^{+28}_{-31} \pm 7)^\circ \quad \alpha = (113^{+27}_{-17} \pm 6)^\circ \text{ at } 68\% \text{ CL}$$

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Open issues in $B \to \rho\pi$

- Dalitz model is the key:
  - phenomenology of all resonances and phases
    - currently model $\rho(770)$, $\rho(1450)$, $\rho(1700)$, same P/T
    - absent S-wave (e.g. $f_0(600)$ or $\sigma$), etc
    - $B^0 \to \rho^0\pi^0$ ignore time-dependence at the moment

- Projection to higher statistics:
  - many more degrees of freedom than in $\rho\rho$
  - very different results depending on amplitude parameters
  - $\rho^0\pi^0$ rate alone is not as crucial as $\rho^0\rho^0$
Summary

- Best $\alpha$ constraints from $B \rightarrow \rho\rho$ and $B^0 \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$

- $B \rightarrow \rho\rho$
  - improvement depends on $\rho^0\rho^0$, understanding of $\rho^+\rho^-, \rho^+\rho^0$
  - current error $\sim 13^\circ$, ambiguities not solved

- $B \rightarrow \rho\pi$
  - Dalitz (interf.) and time of $\rho^+\pi^-$ and $\rho^-\pi^+$, some $\rho^0\pi^0$
  - current error $\sim 23^\circ$, ambiguities solved, likely improve