Nuclear Shell Model with a Realistic Self-Consistent Auxiliary Potential

Luigi Coraggio

Istituto Nazionale di Fisica Nucleare - Sezione di Napoli
Summary

- The nuclear shell model
- The self-consistent Hartree-Fock potential
- Realistic low-momentum interactions
- The derivation of the shell-model effective interaction
- An application: calculations for $p$-shell nuclei
- Perspectives
The nuclear shell model

A brief introduction

Magic numbers: 2, 8, 20, 28, 50, ...

\[ H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V_{ij} = T + V = \]

\[ = (T + U) + (V - U) = H_0 + H_1 \]
The Hartree-Fock potential

\[ H = \sum_{\alpha, \beta} T_{\alpha \beta} a_{\alpha}^\dagger a_{\beta} + \frac{1}{2} \sum_{\alpha, \beta, \gamma, \delta} V_{\alpha \beta \gamma \delta} a_{\alpha}^\dagger a_{\beta}^\dagger a_{\delta} a_{\gamma} \]

Hartree-Fock theory: the ground state wavefunction is represented by a single Slater determinant \(|\Phi\rangle\)

\[ |\Phi\rangle = \prod_{\mu=1}^{A} a_{\mu}^\dagger |0\rangle; \quad |0\rangle \text{ vacuum state} \]

The “best” \(|\Phi\rangle\) is the one corresponding to a minimum of \(\langle E\rangle\)

\[ \delta\langle \Phi | H | \Phi \rangle = 0 \]
The variational constraint is

\[ \langle \delta \Phi | H | \Phi \rangle = 0 \]

\[ \downarrow \]

\[ |\delta \Phi \rangle = \eta a^\dagger \sigma a^\lambda |\Phi \rangle \]

Now, the variational constraint can be written as

\[ \langle \Phi | a^\dagger \lambda a^\sigma H | \Phi \rangle = 0 \]
The Hartree-Fock potential

Solve by an iterative procedure the secular equation

\[ T_{\alpha\beta} + \sum_{\mu=1}^{A} V_{\alpha\mu\beta\mu} = \epsilon_{\alpha} \delta_{\alpha\beta} \]

Hartree-Fock self-consistent one-body potential
The Hartree-Fock potential

**Problem:** Realistic $NN$ interactions contain a strongly repulsive short-range component

**Solution:** Brueckner-Hartree-Fock approach

**Shortcoming:** The $G$-matrix is energy dependent $\rightarrow$ the BHF one-body potential for states above the Fermi surface is not unique

One needs to renormalize $V_{NN}$ in different
Low-momentum $NN$ potentials

The deuteron case:

In the full space

$$\langle k | (T + V_{NN}) | k' \rangle \langle k' | \Psi_n \rangle = E_n \langle k | \Psi_n \rangle \quad 0 < k, k' < \infty$$

In a reduced model space $P = \sum_{k<\Lambda} |k\rangle \langle k|$,

$$\langle k | (T + V_{\text{eff}}) | k' \rangle \langle k' | \Phi_m \rangle = \tilde{E}_m \langle k | \Phi_m \rangle \quad 0 < k, k' < \Lambda$$

Fundamental constraint: $\tilde{E}_m \in \{ E_n \}$

How to calculate $\langle k | H_{\text{eff}} | k' \rangle$?
We are looking for an operator $\Omega$, whose inverse $\Omega^{-1}$ exists, so that the transformed matrix

$$\mathcal{H} = \Omega^{-1} H \Omega,$$

has the same eigenvalues of the matrix $H$. 
Andreozzi-Lee-Suzuki method

In order to switch the eigenvalue problem in the full space into two separate eigenproblems ($P$-space and $Q$-space), we require that:

$$QHP = 0$$

As supplementary conditions on $\Omega$ (without any loss of generality), we choose:

$$P\Omega P = I_p, \quad P\Omega Q = 0$$
$$Q\Omega P = \omega, \quad Q\Omega Q = I_q$$
Finally, the decoupling equation is:

$$\omega PHQ\omega + \omega PHP - QHQ\omega - QHP = 0$$

To solve it:

Iterative techniques: Lee-Suzuki and Krenciglowa-Kuo
### Deuteron binding energy

#### CD-BONN POTENTIAL

<table>
<thead>
<tr>
<th>$\Lambda$ (in fm$^{-1}$)</th>
<th>$PV_{\text{eff}}$ $P$ (in MeV)</th>
<th>$V_{NN}$ (in MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>-2.224</td>
<td>-2.224</td>
</tr>
<tr>
<td>1.8</td>
<td>-2.224</td>
<td>-2.224</td>
</tr>
<tr>
<td>2.0</td>
<td>-2.224</td>
<td>-2.224</td>
</tr>
<tr>
<td>2.2</td>
<td>-2.224</td>
<td>-2.224</td>
</tr>
</tbody>
</table>
Phase-shift calculations

Lipmann-Schwinger equation

\[ \langle p' | T(\epsilon_p) | p \rangle = \langle p' | V_{NN} | p \rangle + \int_0^{\infty} k^2 dk \frac{\langle p' | V_{NN} | k \rangle \langle k | T(\epsilon_p) | p \rangle}{\epsilon_p - \epsilon_k} \]

\[ \langle p' | T_{\text{eff}}(\epsilon_p) | p \rangle = \langle p' | V_{\text{eff}} | p \rangle + \int_0^{\Lambda} k^2 dk \frac{\langle p' | V_{\text{eff}} | k \rangle \langle k | T_{\text{eff}}(\epsilon_p) | p \rangle}{\epsilon_p - \epsilon_k} \]
Low-momentum $NN$ potentials

$\sqrt{S_0}$

- $\Lambda = 2.0$ fm$^{-1}$
- CD–Bonn NN potential

Phase-shifts [degree]

E laboratory [MeV]

$\pm10$

$\pm20$
### Low-momentum $NN$ potentials

<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>CD-Bonn</th>
<th>$V_{\text{low-k}}$</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62.1</td>
<td>62.1</td>
<td>62.1</td>
</tr>
<tr>
<td>10</td>
<td>60.0</td>
<td>60.0</td>
<td>60.0</td>
</tr>
<tr>
<td>25</td>
<td>50.9</td>
<td>50.9</td>
<td>50.9</td>
</tr>
<tr>
<td>50</td>
<td>40.5</td>
<td>40.5</td>
<td>40.5</td>
</tr>
<tr>
<td>100</td>
<td>26.4</td>
<td>26.4</td>
<td>26.8</td>
</tr>
<tr>
<td>150</td>
<td>16.3</td>
<td>16.3</td>
<td>16.9</td>
</tr>
<tr>
<td>200</td>
<td>8.3</td>
<td>8.3</td>
<td>8.9</td>
</tr>
<tr>
<td>250</td>
<td>1.6</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>300</td>
<td>-4.3</td>
<td>-4.3</td>
<td>-4.5</td>
</tr>
</tbody>
</table>

### $^1S_0$ channel phase-shifts (degrees)
$G$ matrix vs $V_{\text{low-}k}$

**$G$ matrix**
- Energy dependent
- No direct connection to the original $V_{NN}$ potential

**$V_{\text{low-}k}$**
- No energy-dependence
- In the $k$-space it reproduces all the two-body problem data - it is a real effective potential
Cut-off momentum $\Lambda$

General criterion

- small enough so to give a reasonably smooth potential suitable to be used directly in a perturbative scheme
- large enough so that $V_{\text{low-k}}$ reproduces the same phase-shifts of the original $V_{NN}$ up to the anelastic threshold ($E_{\text{lab}} \approx 350$ MeV)

$$E_{\text{lab}} \leq \frac{2\hbar^2 \Lambda^2}{M} \rightarrow \Lambda \approx 2.0 \text{fm}^{-1}$$
The derivation of the effective interaction

Schrödinger equation for $A$-nucleon system:

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

Rewrite the above equation in terms of the wave operator $\Omega$

$$\Omega^{-1} H \Omega^{-1} |\Psi_\nu\rangle = \mathcal{H} \Omega^{-1} |\Psi_\nu\rangle = E_\nu \Omega^{-1} |\Psi_\nu\rangle$$

We want $\Omega$ so that

$$\mathcal{H}|\Psi_i^D\rangle = E_i |\Psi_i^D\rangle$$

$|\Psi_i^D\rangle$ eigenfunctions of the model space $D$ \quad $E_i \in \{E_\nu\}$
The derivation of the effective interaction

Because of properties of $P$ and $Q$ operators

$$QHP = 0$$

decoupling equation

The effective hamiltonian is:

$$H_{\text{eff}} = PHP = P\Omega^{-1}H\Omega P$$
The derivation of the effective interaction

Rewrite the wave operator $\Omega$ as

$$\Omega = 1 + \chi \quad (\Omega^{-1} = 1 - \chi)$$

$\chi$ correlation operator

$$P\Omega P = P \Rightarrow P\chi P = 0, \quad Q\Omega P = Q\chi P = \chi P$$

Without loss of generality we can write:

$$Q\chi Q = 0, \quad P\chi Q = 0$$

So we choose:

$$\chi = Q\chi P$$
The perturbative approach

Now the form of the decoupling equation is:

\[ QHP - \chi HP + QH\chi - \chi H\chi = 0 \]

In a degenerate model space we have

\[ PH_0P = E_M P \quad (H = (T + U) + (V - U) = H_0 + H_1) \]

The decoupling equation becomes:

\[ (E_M - QH_0Q - QH_1Q)\chi = QH_1P - \chi PH_1P - \chi PH_1Q\chi \]
The perturbative approach

The equation for the operator $\chi$ is

$$\chi = \frac{1}{E_M - QHQ}QH_1 P - \frac{1}{E_M - QHQ} \chi (PH_1 P + PH_1 Q\chi P)$$

$H_{\text{eff}}$ has to be written in terms of $H_0$ and $H_1$:

$$H_{\text{eff}} = PHP + PH\chi = PH_0 P + V_{\text{eff}}(\chi)$$

where $V_{\text{eff}}(\chi) = PH_1 P + PH_1 \chi$
The perturbative approach

The equation $\chi$ in terms of $V_{\text{eff}}$:

$$\chi = \frac{1}{E_M - QHQ} QH_1 P - \frac{1}{E_M - QHQ} \chi V_{\text{eff}}(\chi)$$

We introduce the so-called $\hat{Q}$-box as

$$\hat{Q}(\omega) = PH_1 P + PH_1 Q \frac{1}{\omega - QHQ} QH_1 P$$
The perturbative approach

Multiplying on both sides the equation for $\chi$ by $PH_1$ and adding $PH_1P$, such equation becomes

$$PH_1P + PH_1\chi = PH_1P + PH_1Q \frac{1}{EM - QHQ} QH_1P -$$

$$- PH_1 \frac{1}{EM - QHQ} \chi V_{\text{eff}}(\chi)$$

So the effective interaction $V_{\text{eff}}(\chi)$ can be expressed in terms of the $\hat{Q}$-box

$$V_{\text{eff}} = \hat{Q}(EM) - PH_1 \frac{1}{EM - QHQ} \chi V_{\text{eff}}(\chi)$$
The perturbative approach

In a perturbative approach $\hat{Q}$-box is composed by a collection of valence-linked two-body diagrams.
The perturbative approach

...and even more!

The perturbative approach

The effective interaction can be schematically written in operator form as:

\[ V_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots , \]

where the integral sign represents a generalized folding operation \( \hat{Q}' \) is obtained from \( \hat{Q} \) by removing terms of 1st order in \( H_1 \)

How to solve this equation?
The perturbative approach


\[ V_{\text{eff}} = \sum_{i=0}^{\infty} F_i \]

where

\[ F_0 = \hat{Q} \]
\[ F_1 = \hat{Q}_1 \hat{Q} \]
\[ F_2 = \hat{Q}_2 \hat{Q}_1 \hat{Q} \]

\[ \hat{Q}_m = \frac{1}{m!} \frac{d^m \hat{Q}(\omega)}{d\omega^m} \]
The perturbative approach


Iterative technique:

\[ V_{\text{eff}}(\chi_n) = \left[ 1 - \hat{Q}_1 - \sum_{m=2}^{n-1} \hat{Q}_n \prod_{k=n-m+1}^{n-1} V_{\text{eff}}(\chi_k) \right]^{-1} \hat{Q} \]
The perturbative approach

\( H_0 \) non-degenerate (Hartree-Fock one-body potential)

\[ \Rightarrow \text{things get more complicated} \]

Introducing multi-energy \( \hat{Q} \)-boxes

\[ \hat{Q}(\epsilon_\alpha) = P H_1 P + P H_1 Q \frac{1}{\epsilon_\alpha - Q H Q} Q H_1 P \]

Generalized multi-energy \( \hat{Q} \)-box:

\[ (-1)^n P H_1 Q \frac{1}{\epsilon_1 - Q H Q} \frac{1}{\epsilon_2 - Q H Q} \cdots \frac{1}{\epsilon_{n+1} - Q H Q} Q H_1 P \]

Calculations for \( p \)-shell nuclei

We expand the HF SP states \( |\alpha\rangle \) in terms of oscillator wave functions \( |\mu\rangle \) (restricted HF)

\[
|\alpha\rangle = \sum_{\mu} C_{\mu}^{\alpha} |\mu\rangle
\]

\( C_{\mu}^{\alpha} \) are determined by solving self–consistently the HF equations

\[
\sum_{\mu'} \langle \mu | t + U | \mu' \rangle C_{\mu'}^{\alpha} = \epsilon_{\alpha} C_{\mu}^{\alpha}
\]

with

\[
\langle \mu | U | \mu' \rangle = \sum_{\alpha_h} \langle \mu \alpha_h | V_{\text{low-k}} | \mu' \alpha_h \rangle
\]
Calculations for \( p \)-shell nuclei

We remove the spurious center-of-mass kinetic energy writing the kinetic energy operator \( T \) as

\[
T = \frac{1}{2Am} \sum_{i<j} (p_i - p_j)^2
\]

So, the hamiltonian can be re-written as

\[
H = \left(1 - \frac{1}{A}\right) \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} \left(V_{ij} - \frac{p_i \cdot p_j}{mA}\right)
\]
Calculations for $p$-shell nuclei

\[ E \pm \omega \] 

\begin{align*}
\text{N}=2 & : -15 \\
\text{N}=3 & : -16
\end{align*}

$^4\text{He}$
4\text{He Results}

- CD-Bonn $NN$ potential $\Lambda = 2.1 \text{ fm}^{-1}$
- $\hbar \omega = 45 A^{-1/3} - 25 A^{-2/3} = 18 \text{ MeV}$
- Goldstone series summed up to third order contributions in $V_{\text{low-k}}$
- $^4\text{He}$ ground-state energy

<table>
<thead>
<tr>
<th></th>
<th>HF</th>
<th>HF+2nd</th>
<th>HF+2nd+3rd</th>
<th>FY</th>
<th>Expt</th>
</tr>
</thead>
</table>

\begin{align*}
\gamma_1 + \gamma_2 &= \gamma_3 + \gamma_4 \\
\gamma_5 + \gamma_6 &= \gamma_7 + \gamma_8 \\
\gamma_9 + \gamma_{10} &= \gamma_{11} + \gamma_{12}
\end{align*}
Calculations for $p$-shell nuclei

Experimental and theoretical SP energies (referred to $^4$He closed-shell core) calculated including up to second- and third-order diagrams in $V_{\text{low-}}k$. The experimental widths of the states are also given. Energies are in MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>HF</th>
<th>2nd order</th>
<th>3rd order</th>
<th>Expt.</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi 0p_{3/2}$</td>
<td>4.217</td>
<td>1.109</td>
<td>1.355</td>
<td>1.97</td>
<td>1.23</td>
</tr>
<tr>
<td>$\pi 0p_{1/2}$</td>
<td>7.500</td>
<td>4.800</td>
<td>4.970</td>
<td>3.46</td>
<td>6.60</td>
</tr>
<tr>
<td>$\nu 0p_{3/2}$</td>
<td>3.106</td>
<td>-0.081</td>
<td>0.202</td>
<td>0.89</td>
<td>0.65</td>
</tr>
<tr>
<td>$\nu 0p_{1/2}$</td>
<td>6.577</td>
<td>3.796</td>
<td>3.984</td>
<td>2.16</td>
<td>5.57</td>
</tr>
</tbody>
</table>
Calculations for $p$-shell nuclei

![Diagram showing energy levels and quantum numbers for $^6\text{Li}$ nuclei for 2nd and 3rd order approximations compared to experiment.](image-url)
Calculations for $p$-shell nuclei

<table>
<thead>
<tr>
<th></th>
<th>6Li</th>
<th>2nd order</th>
<th>3rd order</th>
<th>Expt.</th>
<th>NCSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binding energy</td>
<td>32.198</td>
<td>31.501</td>
<td>31.955</td>
<td>29.34</td>
<td></td>
</tr>
<tr>
<td>$E_x(1^+_1 0)$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$E_x(3^+_0)$</td>
<td>1.757</td>
<td>1.912</td>
<td>2.186</td>
<td>2.841</td>
<td></td>
</tr>
<tr>
<td>$E_x(0^+_1)$</td>
<td>2.401</td>
<td>2.627</td>
<td>3.563</td>
<td>3.330</td>
<td></td>
</tr>
<tr>
<td>$E_x(2^+_0)$</td>
<td>5.170</td>
<td>5.215</td>
<td>4.312</td>
<td>4.610</td>
<td></td>
</tr>
<tr>
<td>$E_x(2^+_1)$</td>
<td>5.590</td>
<td>4.911</td>
<td>5.366</td>
<td>5.975</td>
<td></td>
</tr>
<tr>
<td>$E_x(1^+_2 0)$</td>
<td>7.305</td>
<td>7.825</td>
<td>5.65</td>
<td>6.544</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>6He</th>
<th>2nd order</th>
<th>3rd order</th>
<th>Expt.</th>
<th>NCSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binding energy</td>
<td>30.837</td>
<td>29.797</td>
<td>29.269</td>
<td>26.709</td>
<td></td>
</tr>
<tr>
<td>$E_x(0^+_1)$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$E_x(2^+_1)$</td>
<td>2.072</td>
<td>2.108</td>
<td>1.8</td>
<td>2.529</td>
<td></td>
</tr>
</tbody>
</table>
Calculations for $p$-shell nuclei

Experimental and calculated electromagnetic properties in $^6$Li. The reduced transition probabilities are expressed in W.u.

<table>
<thead>
<tr>
<th></th>
<th>$^6$Li</th>
<th>Calc.</th>
<th>Expt.</th>
<th>NCSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{gs}$ [e mb]</td>
<td></td>
<td>-0.435</td>
<td>-0.818(17)</td>
<td>-0.42</td>
</tr>
<tr>
<td>$\mu_{gs}$ [nm]</td>
<td></td>
<td>+0.857</td>
<td>+0.822</td>
<td>+0.847</td>
</tr>
<tr>
<td>$B(E2; 3^+_1 0 \rightarrow 1^+_0)$</td>
<td></td>
<td>6.57</td>
<td>16.5(1.3)</td>
<td>7.7</td>
</tr>
<tr>
<td>$B(E2; 2^+_1 0 \rightarrow 1^+_0)$</td>
<td></td>
<td>6.35</td>
<td>6.8(3.5)</td>
<td>6.9</td>
</tr>
<tr>
<td>$B(M1; 0^+_1 \rightarrow 1^+_0)$</td>
<td></td>
<td>9.01</td>
<td>8.62(18)</td>
<td>8.49</td>
</tr>
<tr>
<td>$B(M1; 2^+_1 1 \rightarrow 1^+_0)$</td>
<td></td>
<td>0.154</td>
<td>0.083(15)</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Calculations for $p$-shell nuclei

![Diagram showing energy levels for $p$-shell nuclei]

- $\frac{1}{2}^-$
- $\frac{3}{2}^-$
- $\frac{5}{2}^-$
- $\frac{3}{2}^+$
- $\frac{1}{2}^+$

Calc | Expt
Calculations for $p$-shell nuclei

<table>
<thead>
<tr>
<th></th>
<th>$^7\text{Li}$</th>
<th>Calc.</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binding energy</td>
<td></td>
<td>39.556</td>
<td>39.243</td>
</tr>
<tr>
<td>$Q_{gs}$ [e mb]</td>
<td></td>
<td>-24.4</td>
<td>-40.6(8)</td>
</tr>
<tr>
<td>$\mu_{gs}$ [nm]</td>
<td></td>
<td>+4.29</td>
<td>+3.256</td>
</tr>
<tr>
<td>$B(\text{E2}; \frac{1}{2}^- \rightarrow \frac{3}{2}^-)$</td>
<td>9.65</td>
<td>19.7(1.2)</td>
<td></td>
</tr>
<tr>
<td>$B(\text{E2}; \frac{7}{2}^- \rightarrow \frac{3}{2}^-)$</td>
<td>4.16</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>$B(\text{M1}; \frac{1}{2}^- \rightarrow \frac{3}{2}^-)$</td>
<td>2.40</td>
<td>2.75(14)</td>
<td></td>
</tr>
</tbody>
</table>
Calculations for $p$-shell nuclei
Calculations for $p$-shell nuclei

<table>
<thead>
<tr>
<th></th>
<th>$^7\text{Be}$</th>
<th>Calc.</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binding energy</td>
<td></td>
<td>37.751</td>
<td>37.600</td>
</tr>
<tr>
<td>$\mu_{\text{gs}}$ [nm]</td>
<td>-1.005</td>
<td></td>
<td>-1.398(15)</td>
</tr>
<tr>
<td>$B(\text{M1}; \frac{1}{2}^-<em>{2\text{1}} \rightarrow \frac{3}{2}^-</em>{2\text{1}})$</td>
<td>1.81</td>
<td>2.07(27)</td>
<td></td>
</tr>
</tbody>
</table>

Microscopic Nuclear Structure Theory – p.42
Calculations for $p$-shell nuclei

\begin{center}
\begin{tabular}{|c|c|}
\hline
E(MeV) & \text{Calc/Expt} \\
\hline
8 & $0^+, 2$ \\
6 & $2^+, 1$ \\
4 & $1^+, 1$ \\
2 & $2^+, 1$ \\
0 & $1^+, 1$ \\
\hline
\end{tabular}
\end{center}

$^8$Li $0^+, 2$

$4^+, 1$

$2^+, 1$

$1^+, 1$

$1^+, 1$

$2^+, 1$

$1^+, 1$

$2^+, 1$

$1^+, 1$
### Calculations for $p$-shell nuclei

<table>
<thead>
<tr>
<th></th>
<th>$^8$Li</th>
<th>Calc.</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binding energy</td>
<td>44.365</td>
<td>41.276</td>
<td></td>
</tr>
<tr>
<td>$Q_{gs}$ [e mb]</td>
<td>26.7</td>
<td>24(2)</td>
<td></td>
</tr>
<tr>
<td>$\mu_{gs}$ [nm]</td>
<td>+2.89</td>
<td>+1.653</td>
<td></td>
</tr>
<tr>
<td>$B(M1; 1^+; 1 \to 2^+; 1)$</td>
<td>2.64</td>
<td>2.8(9)</td>
<td></td>
</tr>
<tr>
<td>$B(M1; 3^+; 1 \to 2^+; 1)$</td>
<td>0.35</td>
<td>0.29(13)</td>
<td></td>
</tr>
</tbody>
</table>
Calculations for $p$-shell nuclei

![Diagram showing energy levels and states for $^8$B with calculated (Calc.) and experimental (Expt.) values.](image-url)
## Calculations for $p$-shell nuclei

<table>
<thead>
<tr>
<th></th>
<th>$^8\text{B}$</th>
<th>Calc.</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binding energy</td>
<td>40.429</td>
<td></td>
<td>37.74</td>
</tr>
<tr>
<td>$Q_{gs}$ [e mb]</td>
<td>40.1</td>
<td></td>
<td>64.6(1.5)</td>
</tr>
<tr>
<td>$\mu_{gs}$ [nm]</td>
<td>+1.73</td>
<td></td>
<td>+1.036</td>
</tr>
</tbody>
</table>
Calculations for $p$-shell nuclei

![Graph showing energy levels for $^8\text{Be}$]
Perspectives

- Better understanding of the role of cutoff $\Lambda$
- Improve the calculation of $V_{\text{eff}}$
- Approaching heavier-mass nuclei
- The role of Hartree-Fock potential in the convergence problem
SUNY - Stony Brook:
T. T. S. Kuo
University of Idaho - Moskow:
R. Machleidt
Napoli:
A. Covello
A. Gargano
N. Itaco
L. C.