Stellar QCD

1. What hadrons exist.

2. How hadrons interact (2 opinions) + comparison with experiment (4 cases)

3. The goal: “$V_{BB'}(r)$”

4. SN-relevance: hadronic EOS for your proto neutron star (J.Stone, yesterday)
Nuclear and particle physics models

Exact solution of the Schroedinger equation (or its relativistic equivalent):

$$(T + V) |\phi> = E |\phi>$$

$V$ = nucleon-nucleon potential
$|\phi>$ = A-particle wave function

Today: How to calculate this.

Density dependence of parameters of these interactions
Models of supernova remnants require a realistic nuclear / strange matter EOS. This can be calculated from pairwise BB’ forces \((B=p, n, \Lambda, \Sigma,\ldots)\).

These BB’ interactions were traditionally calculated from t-channel meson exchange models in the NN system. However this is unphysical at short distances (baryon wfn overlap), and these forces presumably arise from quark-gluon interactions. These (OGE) work well for the short-ranged NN repulsion.

Remarkably, stellar models are still (only) using the meson exchange picture to describe forces between strange baryons.

**GOAL:** (no doubt to be redefined in future)

What are the quark model predictions for strange baryon forces? What is the resulting EOS?
1. QCD and Hadrons 101

\[
\frac{g^2}{4\pi} = \alpha_s \approx \frac{1}{2} (\text{light hadrons})
\]

\[
\frac{g^2}{4\pi} \times z \approx \frac{1}{137} (\beta_0)
\]

Small qq separation

Large qq separation
LGT simulation showing the QCD flux tube

\[ V_{\text{QCD}}(R) \]

The QCD flux tube
(LGT, G.Bali et al; hep-ph/010032)
“Naïve” physically allowed hadrons (color singlets)

Conventional quark model mesons and baryons.

100s of e.g.s

"exotica":

- glueballs: $g^2, g^3, \ldots$
- hybrids: $qag, q^3g, \ldots$
- multiquarks: $q^2q^2, q^4q, \ldots$

maybe 1 e.g.

maybe 1-3 e.g.s
The dangerous 1970s multiquark logic:
(which led to the multiquark fiasco)

The known hadron resonances, $qq$ and $qqq$ (and $qqq$) exist because they are color singlets.

Therefore all higher Fock space “multiquark” color singlet sectors will also possess hadron resonances.

$q^2 q^2$ “baryonia”
$q^6$ “dibaryons”
$q^4 q$ “Z*” for $q = s$ …
now “pentaquarks”

MANY theoretical predictions of a very rich spectrum of multiquark resonances followed in the 1970s/early 1980s.

(Bag model, potential models, QCD_SRs, color chemistry, …)
The simplest e.g. of had-had scat: \( I=2 \) \( \pi\pi \).

(A flavor-exotic 27 channel, no s-channel \( qq \) resonances, so no \( qq \) annihilation. Similar to the NN and BB' problems.)

\[ Q = +2 \text{ channel} \]
\[ \text{No } qq \text{ states.} \]

No \( I=2 \) \( qq \) resonance at 1.2 GeV.

(Bag model prediction, would give \( \Omega = +180 \text{ [deg]} \) there.)

Expt sees only repulsive \( \square \square \) scat.
Why are there no multiquark resonances?

“Fall-Apart Decay” (actually not a decay at all: no $H_1$)

Most multiquark models found that most channels showed short distance repulsion:

$$E(\text{cluster}) > M_1 + M_2.$$  

Thus no bound states.

Only $1+2$ repulsive scattering.

Exceptions:

1) nuclei and hypernuclei
   weak int-R attraction allows "molecules"

2) $E(\text{cluster}) < M_1 + M_2$.
   - bag model: $u^d\bar{s}$ H-dibaryon, $M_H - M_{\Lambda\Lambda} = -80$ MeV.
   - n.b. $\Lambda\Lambda$ hypernuclei exist, so this H was wrong.

3) Heavy-light $Q^2g^2 (Q=b, c?)$
**Post-fiasco physically allowed hadrons (color singlets)**

Naïve physically allowed hadrons (color singlets)

Post-fiasco physically allowed hadrons (color singlets)

Conventional quark model
mesons and baryons.

100s of e.g.s

Basis state mixing may be very important in some sectors.

"exotica":

(q³)ⁿ, (qq)(qq), (qq)(q³),…

nuclei / molecules

ca. 10⁶ e.g.s of (q³)ⁿ, maybe 1-3 others

(q²q²), (q⁴q),…

multiquarks

controversial
e.g. Θ(1542)?

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<table>
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<tr>
<td><strong>qq</strong></td>
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<tr>
<td><strong>g², g³,...</strong></td>
<td><strong>qag, q³g,...</strong></td>
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<td>glueballs</td>
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<td>maybe 1 e.g.</td>
<td>maybe 1-3 e.g.s</td>
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Beware of theorists predicting “quark stars”. These are the extreme limit of multiquark states, which fell apart.

(This warning may not be necessary.)

One problem was that the MIT bag model badly overestimated the binding energies of multiquark systems.
... back to conventional hadrons (qq meson) to illustrate forces.

The quark model treats conventional mesons as qq bound states.

Since each quark has spin-1/2, the total spin is

\[ S_{qq}^{\text{tot}} = \frac{1}{2} \times \frac{1}{2} = 1 + 0 \]

Combining this with orbital angular momentum \( L_{qq} \) gives states of total

\[ J_{qq} = L_{qq} \quad \text{spin singlets} \]

\[ J_{qq} = L_{qq} + 1, \ L_{qq}, \ L_{qq} - 1 \quad \text{spin triplets} \]
**qq mesons**

**quantum numbers**

Parity $P_{qq} = (-1)^{(L+1)}$

C-parity $C_{qq} = (-1)^{(L+S)}$

The resulting $qq$ NL states $N^{2S+1L_j}$ have $J^{PC} =$

1S: $^3S_1 1^{--}$; $^1S_0 0^{-+}$  
2S: $^3S_1 1^{--}$; $^1S_0 0^{-+}$ ...

1P: $^3P_2 2^{++}$; $^3P_1 1^{++}$; $^3P_0 0^{++}$; $^1P_1 1^{+-}$  
2P ...

1D: $^3D_3 3^{--}$; $^3D_2 2^{--}$; $^3D_1 1^{--}$; $^1D_2 2^{+-}$  
2D ...

$J^{PC}$ forbidden to $qq$ are called “$J^{PC}$-exotic quantum numbers”:

$0^{--}$; $0^{+-}$; $1^{--}$; $2^{++}$; $3^{+-}$ ...

Plausible $J^{PC}$-exotic candidates = hybrids, glueballs (high mass), maybe multiquarks (fall-apart decays).
Charmonium \((cc)\)

A nice example of a QQ spectrum. Expt. states (blue) are shown with the usual L classification.

Above 3.73 GeV:
Open charm strong decays (DD, DD\(^*\) \(\ldots\)):
broader states except 1D\(_2\) 2\(^--\), 2\(^--\)

Below 3.73 GeV:
Annihilation and EM decays.
\((\rho\pi, KK^*, \gamma cc, \gamma\gamma, \ell^+\ell^- \ldots)\):
narrow states.
Fitted and predicted $cc$ spectrum

Coulomb (OGE) + linear scalar conf. potential model

blue = expt, red = theory.

\[
\begin{align*}
\alpha_s &= 0.5538 \\
b &= 0.1422 \text{ [GeV}^2\text{]} \\
m_c &= 1.4834 \text{ [GeV]} \\
\sigma &= 1.0222 \text{ [GeV]}
\end{align*}
\]
cc from LGT

What about LGT???
An e.g.: X.Liao and T.Manke,
hep-lat/0210030 (quenched – no decay loops)
Broadly consistent with the cc potential model spectrum. No radiative or strong decay predictions yet.

<- 1^-+ exotic cc-H at 4.4 GeV

Small L=2 hfs.

oops...

1^+ cc has been withdrawn.

FIG. 1. Quenched charmonium spectrum. The experimental values are shown as short horizontal lines. The long horizontal dashed lines mark the D\bar{D} and D^{*+}\bar{D} thresholds. The masses (please refer to appendix B 2 for one naming convention) plotted are \(x(1^{++})\), \(\rho(1^{--})\), \(b_1(1^{++})\), \(\rho \times \Lambda_c(1^{++})\), \(\rho \times \eta_c(1^{++})\), \(\rho \times \Omega_{c0}(1^{++})\), \(x \times D\bar{D}(2^{++})\), \(\rho \times D\bar{D}(2^{++})\), \(x \times D\bar{A}_{c0}(2^{--})\), \(x \times D\bar{A}_c(3^{--})\), \(x \times D\bar{A}_c(3^{**})\), \(x \times A_c(1^{**})\), \(x \times A_c(1^{**})\), \(x \times A_c(1^{**})\), \(x \times A_c(1^{**})\), \(x \times A_c(1^{**})\), \(x \times A_c(1^{**})\). Some of the low-lying mesons (\(x(1^{--})\), \(x(1^{--})\), \(b_1(1^{**})\)) are taken from Columbia group's previous work [14]. The lattice scale is set by the \(1^F_1 - 1S\) splitting. The numerical values are listed in table II.
2. How hadrons interact (2 opinions) and comparison with experiment (4 cases).

A. **Meson exchange.**
(traditional nuke)

(In terms of **hadron** d.o.f.s)
Easy to calculate
(Feynman diags) but
vertices (form factors) are
obscure.
MANY free params, fitted
to data.

B. **Constituent interchange**
(quark model).

(In terms of **q+g** d.o.f.s)
Much harder to calculate
(convolutions) but q-g
vertices are known.
Just attach ANY hadron
wfns.

e.g. for NN scat $\pi$, $\rho$, $\omega$, “O”, ...

Confusion theorem...
Hard to distinguish these:
Identical flavor flow.

Syrme. N made of pions. “Es ist nicht einmal falsch.” -Pauli

\[ r_{\text{max.~ca.~} 1/m_{\text{meson}}} \]

This eliminates all but pions! \((1/m_{\pi} = 1.4 \text{ fm, but } 1/m_{\omega} = 0.25 \text{ fm.})\)

‘residual quark forces’ give a reasonable short-ranged NN interaction!
Strong evidence against meson exchange at small $r$:

NN core in meson exchange models is usually attributed to $\omega$ exchange.

NN implied core is ATTRACTION (G-parity transformation) - deeply bound state.

LEAR at CERN was built to study these states. They weren’t there.

However, $\pi$ exchange is certainly correct at large $r$: NN expt (high $L$ scat).

$p p \rightarrow \Lambda \Lambda$ clearly shows $K$ exchange.

$I=2 \quad \pi \pi \rightarrow \pi \pi$ correctly described by $\rho$ exchange OR constit. ex.

Perhaps $g_{NN\omega}$ is just overestimated? (3x expectations in mes. ex. model of $V_{NN}$.)
q-g interactions approach: Why these Born-order diagrams?

They are the lowest-order allowed q-g color-singlet scattering diagrams in QCD.

\[ I_{\text{color(capture_1)}} = \begin{align*}
\delta_{ii}/\sqrt{3} & \delta_{ij}/\sqrt{3} \\
\delta_{ij}/\sqrt{3} & \delta_{ij}/\sqrt{3}
\end{align*} \]

\[ = \frac{\delta_{ii} \delta_{jj}}{\sqrt{3} \sqrt{3}} \left( \frac{\lambda^a_{ij}}{2} \right) \delta_{ii} \left( \frac{-\lambda^a_{i'j'}}{2} \right) \delta_{jj} \delta_{ii} \delta_{jj} \]

\[ = -\frac{1}{36} \text{Tr}(\lambda^a \lambda^a) = -\frac{4}{9}. \]

Don’t higher-order q-g processes dominate?
Apparently not – it’s been checked variationally and in res. group formalism.
Born approx. is most of the answer (in channels w/o qg annihilation).
We are just lucky.
The simplest e.g. of had-had scat: \( I=2 \, \pi\pi \)
(A flavor-exotic 27 channel, no s-channel \( qq \) resonances, 
so no \( qq \) annihilation. Similar to the NN and BB' problems.)

The \( I=2 \, \pi\pi \) S-wave phase shift predicted by the constituent interchange 
diagrams is shown together with the expt. phase shift.

\textit{Not a fit} – standard quark model parameters. \( I=2 \, \square\square \) S-wave

\begin{figure}
\centering
\includegraphics[width=\textwidth]{diagram.png}
\caption{Phase shift diagram with constituent interchange diagrams A, B, C, D, T1, T2.}
\end{figure}

\textit{T.Barnes, N.Black and E.S.Swanson,}
How to calculate hadron-hadron forces in the quark model:
Start with the standard quark model qq interaction (as a qq scat T-matrix):

\[ T_{fi}^{OGE}(\vec{q}, \vec{p}_1, \vec{p}_2) = 4\pi \alpha_s \left[ \frac{1}{q^2} - \frac{1}{8m_1^2} - \frac{1}{8m_2^2} + \frac{i}{2q^2} \left( \frac{1}{m_1^2} \vec{s}_1 \cdot (\vec{q} \times \vec{p}_1) - \frac{1}{m_2^2} \vec{s}_2 \cdot (\vec{q} \times \vec{p}_2) \right) \right. \]

\[ \left. - \frac{2}{3m_1m_2} \vec{s}_1 \cdot \vec{s}_2 + \frac{1}{m_1 m_2 q^2} \left( \vec{s}_1 \cdot (\vec{q} \cdot \vec{p}_1) - \vec{s}_2 \cdot (\vec{q} \cdot \vec{p}_2) \right) - \frac{i}{m_1 m_2 q^2} \left( \vec{s}_1 \cdot (\vec{q} \times \vec{p}_2) - \vec{s}_2 \cdot (\vec{q} \times \vec{p}_1) \right) \right] . \]  

(3)

Linear scalar conf.

\[ T_{fi}^{\text{lin}}(\vec{q}, \vec{p}_1, \vec{p}_2) = \frac{6\pi b}{q^4} \left[ 1 - \frac{1}{2} \left( \frac{\vec{p}_1^2}{m_1^2} + \frac{\vec{p}_2^2}{m_2^2} \right) - \frac{i}{2} \left( \frac{1}{m_1^2} \vec{s}_1 \cdot (\vec{q} \times \vec{p}_1) - \frac{1}{m_2^2} \vec{s}_2 \cdot (\vec{q} \times \vec{p}_2) \right) \right] . \]
Convolve with quark model meson wavefunctions to generate a meson-meson scattering T-matrix.

This is just “Born order”, but resonating group studies confirm that it is usually a good approximation.

Only 4 diagrams in meson-meson scattering!

\[ T_{\mu}^{(C1)}(AB \rightarrow CD) = \int \int d^3q \, d^3p \, \Phi_C^*(2\vec{p} + \vec{q} - (1 + \lambda)\vec{C}) \, \Phi_D^*(2\vec{p} - \vec{q} - 2\vec{A} - (1 - \lambda)\vec{C}) \]
\[ T_{\mu}(\vec{q}, \vec{p}, -\vec{p} + \vec{C}) \, \Phi_A(2\vec{p} - \vec{q} - (1 + \lambda)\vec{A}) \, \Phi_B(2\vec{p} - \vec{q} - (1 - \lambda)\vec{A} - 2\vec{C}) , \]

\[ T_{\mu}^{(C2)}(AB \rightarrow CD) = \int \int d^3q \, d^3p \, \Phi_C^*(-2\vec{p} + \vec{q} + 2\vec{A} - (1 + \lambda)\vec{C}) \, \Phi_D^*(-2\vec{p} - \vec{q} - (1 - \lambda)\vec{C}) \]
\[ T_{\mu}(\vec{q}, \vec{p}, -\vec{p} - \vec{C}) \, \Phi_A(-2\vec{p} + \vec{q} + (1 - \lambda)\vec{A}) \, \Phi_B(-2\vec{p} + \vec{q} + (1 + \lambda)\vec{A} - 2\vec{C}) , \]

\[ T_{\mu}^{(C1)}(AB \rightarrow CD) = \int \int d^3q \, d^3p \, \Phi_C^*(2\vec{p} + \vec{q} - (1 + \lambda)\vec{C}) \, \Phi_D^*(2\vec{p} - \vec{q} - 2\vec{A} - (1 - \lambda)\vec{C}) \]
\[ T_{\mu}(\vec{q}, \vec{p}, -\vec{p} - \vec{A} - \vec{C}) \, \Phi_A(2\vec{p} - \vec{q} - (1 + \lambda)\vec{A}) \, \Phi_B(2\vec{p} + \vec{q} - (1 - \lambda)\vec{A} - 2\vec{C}) , \]

\[ T_{\mu}^{(C2)}(AB \rightarrow CD) = \int \int d^3q \, d^3p \, \Phi_C^*(-2\vec{p} + \vec{q} + 2\vec{A} - (1 + \lambda)\vec{C}) \, \Phi_D^*(-2\vec{p} - \vec{q} - (1 - \lambda)\vec{C}) \]
\[ T_{\mu}(\vec{q}, \vec{p}, -\vec{p} - \vec{A} - \vec{C}) \, \Phi_A(-2\vec{p} + \vec{q} + (1 - \lambda)\vec{A}) \, \Phi_B(-2\vec{p} + \vec{q} + (1 + \lambda)\vec{A} - 2\vec{C}) . \]
Do some very messy overlap integrals, combine with color and spin matrix elements. This gives $T_{fi} (AB \rightarrow CD)$.

Then FT to get a $V(r)$ or project with $Y_{LM}$ to get scat. phase shifts.

\[
T_{fi}^{(C1)} (AB \rightarrow CD) = \frac{1}{\pi^3 \beta^6} \exp \left\{ - \frac{1}{3 \beta^2} \left[ (1 + \lambda)^2 \mathcal{A}^2 - 2 \lambda (\mathcal{A}^2 + \mathcal{A} \cdot \mathcal{C}) \right] \right\} \\
\iint d^3q d^3p \exp \left\{ - \frac{2}{\beta^2} (\mathcal{p} - \mathcal{p}_0)^2 \right\} \exp \left\{ - \frac{3}{8 \beta^2} (\mathcal{q} - \mathcal{q}_0)^2 \right\} T_i(q, \mathcal{p}, -\mathcal{p} + \mathcal{C}),
\]

\[
T_{fi}^{(C2)} (AB \rightarrow CD) = \frac{1}{\pi^3 \beta^6} \exp \left\{ - \frac{1}{3 \beta^2} \left[ (1 + \lambda)^2 \mathcal{A}^2 - 2 \lambda (\mathcal{A}^2 + \mathcal{A} \cdot \mathcal{C}) \right] \right\} \\
\iint d^3q d^3p \exp \left\{ - \frac{2}{\beta^2} (\mathcal{p} - \mathcal{p}_0)^2 \right\} \exp \left\{ - \frac{3}{8 \beta^2} (\mathcal{q} - \mathcal{q}_0)^2 \right\} T_f(q, \mathcal{p}, -\mathcal{p} - \mathcal{C}),
\]

\[
T_{fi}^{(T1)} (AB \rightarrow CD) = \frac{1}{\pi^3 \beta^6} \exp \left\{ - \frac{1}{4 \beta^2} \left[ (1 - \lambda)^2 (\mathcal{A}^2 + \mathcal{A} \cdot \mathcal{C}) \right] \right\} \\
\iint d^3q d^3p \exp \left\{ - \frac{2}{\beta^2} (\mathcal{p} - \mathcal{p}_0)^2 \right\} \exp \left\{ - \frac{1}{2 \beta^2} (\mathcal{q} - \mathcal{q}_0)^2 \right\} T_f(q, \mathcal{p}, \mathcal{p} - \mathcal{A} - \mathcal{C}),
\]

\[
T_{fi}^{(T2)} (AB \rightarrow CD) = \frac{1}{\pi^3 \beta^6} \exp \left\{ - \frac{1}{4 \beta^2} \left[ (1 + \lambda)^2 (\mathcal{A}^2 - \mathcal{A} \cdot \mathcal{C}) \right] \right\} \\
\iint d^3q d^3p \exp \left\{ - \frac{2}{\beta^2} (\mathcal{p} - \mathcal{p}_0)^2 \right\} \exp \left\{ - \frac{1}{2 \beta^2} (\mathcal{q} - \mathcal{q}_0)^2 \right\} T_f(q, \mathcal{p}, \mathcal{p} - \mathcal{A} + \mathcal{C}).
\]
Note the dominance of the spin-spin OGE force. The color factors alone sum to zero for the scattering of color-singlets; thus the spin-indep terms destructively interfere between diagrams. $S*S$ has compensating phases.

$I=2$ \[\square\square\] S-wave

Largest term is biggest! KN and NN calcs will neglect others.
The “BB” meson-meson test case. Actually has a well-defined $V(r)$.

\[ B = b \ g \]
\[ m_b = \text{infty} \]

\[ b \ r \ b \]

Comparison of our Born-order “BB” meson-meson $V(r)$ with LGT results from UKQCD. (I,S) case 1 of 2.

Fig. 3. The $V_{BB}^{(I,S)}$ potential, showing individual contributions.

Fig. 4. Comparison of the $V_{BB}^{(I,S_{\text{sat}}=2)}$ quark model potential (solid is calculated, dashed is smeared by $a = 0.18 \text{ fm}$) with the $V_{BB}^{(I,S_{\text{sat}}=1)}$ LGT potential of Ref. [6].
Comparison of our Born-order “BB” meson-meson $V(r)$ with LGT results from UKQCD.

$(I,S)$ case 2 of 2.

The full $I=1$ $BB$ potential is given by

$$V_{BB}^{(I=1)}(r) = -\frac{2\alpha_s}{9r} \left\{ 1 + (2/\pi)^{1/2} \beta r - 4 \operatorname{Erf}(\beta r/2) \right\} e^{-\beta^2 r^2/2} + \frac{2^{1/2}}{9\pi^{1/2}} \frac{\alpha_s \beta^3}{m^2} e^{-\beta^2 r^2/2}$$

$$+ \left[ \frac{\beta r e^{-\beta^2 r^2/2}}{2^{3/2} \pi^{1/2}} - \left( \beta r + \frac{2}{\beta r} \right) \operatorname{Erf}(\beta r/2) e^{-\beta^2 r^2/2} - \frac{2}{\pi^{1/2}} e^{-3\beta^2 r^2/4} \right].$$

(47)

Fig. 5. Comparison of the $V_{BB}^{(I=0,S_{tot}=2)}$ quark model potential (solid is calculated, dashed is smeared by $a = 0.18$ fm) with the $V_{BB}^{(I_{light}=0,S_{light}=1)}$ LGT potential of Ref. [6].
A flavor-exotic 10 channel. It's a very weak P-wave phase shift. These are short-ranged forces.

The $I=2$ $K\pi$ P-wave phase shift predicted by the same 4 constituent interchange again not a fit – standard quark model parameters. (2 sets)

$I=\frac{3}{2}$ $K\bar{\pi}$ P-wave

$\delta_{P}^{(3)}$ (degrees)
Another application “elsewhere”
Had-had scat and RHIC

Has the QGP been discovered? One cannot be sure w/o a QGP signature.

One popular suggestion (Matsui and Satz):
dramatic reduction of the $J/\psi$ production cross section, since the QGP will screen the long range confining potential.

Competing effect:

$c\bar{c}$ states may also be destroyed in secondary collisions with some of the $10^4$ light hadrons also produced in the collision. “comover dissociation”.

Are they? What are the $c\bar{c}$ – light hadron dissociation cross sections for slowly moving hadrons? (Project suggested to us by B. Mueller during a DOE review.)
Meson-meson scattering in the quark model.

\[ H_I = \text{full quark model interaction.} \]

Just attach wfn's and calc m.e.s

most recent publication:
T. Barnes, E.S. Swanson, C.Y. Wong and X.M. Xu,
KN Scattering... a completely new set of diagrams, combinatorics, color factors, spin factors, flavor factors, and a new set of terrible overlap integrals. If this works...

\[
\begin{align*}
I_{\text{space}}(D_1) &= + \frac{8\pi\alpha_s}{3m_q^2} \frac{1}{(2\pi)^3} \int \int \int \int d\bar{a} d\bar{b}_1 d\bar{b}_2 \phi_A(2a - \frac{2\rho A}{1+\rho}) \phi_*(2a + \frac{2C}{1+\rho} - 2A) \\
&\quad \cdot \phi_B(b_1, b_2, -A - b_1 - b_2) \phi_*(b_1 + A - C, b_2, -A - b_1 - b_2), \\
I_{\text{space}}(D_2) &= + \frac{8\pi\alpha_s}{3m_q^2} \frac{1}{(2\pi)^3} \int \int \int \int d\bar{b}_1 d\bar{c} d\tilde{a} \phi_A(2c - \frac{2A}{1+\rho} - 2C) \phi_*(2c - \frac{2\rho C}{1+\rho}) \\
&\quad \cdot \phi_B(b_1, c, -A - b_1 - c) \phi_*(d_1, A - C + b_1 + c - d_1, -A - b_1 - c), \\
I_{\text{space}}(D_3) &= + \frac{8\pi\alpha_s}{3m_q^2} \rho \frac{1}{(2\pi)^3} \int \int \int \int d\bar{a} d\bar{b}_2 d\bar{c} \phi_A(2a - \frac{2\rho A}{1+\rho}) \phi_*(2c - \frac{2\rho C}{1+\rho}) \\
&\quad \cdot \phi_B(a - A + C, b_2, -a - b_2 - C) \phi_*(a, b_2, -a - b_2 - C), \\
I_{\text{space}}(D_4) &= + \frac{8\pi\alpha_s}{3m_q^2} \rho \frac{1}{(2\pi)^3} \int \int \int \int d\bar{a} d\bar{b}_1 d\bar{c} \phi_A(2a - \frac{2\rho A}{1+\rho}) \phi_*(2c - \frac{2\rho C}{1+\rho}) \\
&\quad \cdot \phi_B(b_1, c, -A - b_1 - c) \phi_*(A - C - a + b_1 + c, a, -A - b_1 - c).
\end{align*}
\]
4. Stellar QCD: motivation

Models of supernova remnants require a realistic nuclear / strange matter EOS. This can be calculated from pairwise BB’ forces \( (B=p, n, \Lambda, \Sigma,...) \).

These BB’ interactions were traditionally calculated from meson exchange models in the NN system. However this is unphysical at short distances (baryon wfns overlap), and these forces presumably arise from quark-gluon interactions. These (OGE) work well for the short-ranged NN repulsion.

Remarkably, stellar models are still (only) using the meson exchange picture to describe forces between strange baryons.

GOAL (no doubt to be redefined in future):

What are the quark model predictions for strange baryon forces? What is the resulting EOS?
Quark model calculation of BB' forces

(Only such group active in USA.
Also 1 in Japan, 1 in Brazil, 1-3 in Germany.)

Convolve the quark model $H_I$ with baryon $|qqq>$ wavefunctions. Result is a BB -> BB T-matrix.

This has been done by ca.10 groups since about 1980.
(Usually resonating group – our formalism is the Born-order version of this. Easier and gives very similar results.)

General concl: dominant m.e. is the $S*S$ contact hyperfine part of OGE, this (not exchange) gives the NN short-ranged core.
With Gaussian wfns can derive analytic NN phase shifts and $V_{NN}(r)$s.

ca. $10^4$ terms per $(l,S)$ channel, 36-dim overlap integrals (12 after $\delta$-fns)
(3 indep. calcs to check: also, overall fermion $A$ only appears after sum)

1 of 8 topologically independent diagrams.
Four independent 12-dim overlap integrals, evaluated analytically for Gaussian nucleon wavefunctions. New combinatoric factors, flavor, color and spin factors for each diagram.

\[
I_1 = \kappa_{ss} \int \int da_1 da_2 \Phi_A(a_1, a_2, A - a_1 - a_2) \Phi_C^*(a_1, a_2, C - a_1 - a_2)
\]

\[
\cdot \int \int db_2 db_3 \Phi_B(-A - b_2 - b_3, b_2, b_3) \Phi_D^*(-C - b_2 - b_3, b_2, b_3);
\]

\[
I_2 = \kappa_{ss} \int \int \int \int da_1 da_3 db_2 dc_3 \Phi_A(a_1, A - a_1 - a_3, a_3) \Phi_C^*(a_1, C - a_1 - c_3, c_3)
\]

\[
\Phi_B(C - A + a_3, b_2, -C - a_3 - b_2) \Phi_D^*(a_3, b_2, -C - a_3 - b_2);
\]

\[
I_3 = \kappa_{ss} \int \int \int \int \int da_1 da_2 da_3 dd_1 \Phi_A(a_1, a_2, A - a_1 - a_2) \Phi_C^*(a_1, a_2, C - a_1 - a_2)
\]

\[
\Phi_B(C - a_1 - a_2, -A - C + a_1 + a_2 - b_3, b_3) \Phi_D^*(d_1, -C - b_3 - d_1, b_3);
\]

\[
I_4 = \kappa_{ss} \int \int \int \int da_1 da_3 db_1 db_3 \Phi_A(a_1, A - a_1 - a_3, a_3) \Phi_C^*(a_1, C - a_1 - b_1, b_1)
\]

\[
\Phi_B(b_1, -A - b_1 - b_3, b_3) \Phi_D^*(a_3, -C - a_3 - b_3, b_3).
\]

Our \(V_{NN}(r)\) results from OGE S*S term:
Quark model $V_{NN}(r)$:
“standard” NN hard core repulsion.
No t-channel meson exchange.

Other channels depend on spin and angular matrix elements.
No simple “general rule” for attraction or repulsion.
Some flavor 10x10 channels have attractive cores, as does one 8x8 channel. ($\Sigma\Sigma$, $l=0$, $S=0$)

Note no interm. range attr. at Born order.
J. Stone:
$V_{NN}$ -> energy density in pure neutron matter from variants of the Skyrme model and other phenom. models, versus our quark model result.

Skyrme models have ca. 10-15 phenom. parameters fitted to nuclei and nucl matter at saturation density.

Skyrme blue used in prev. neutron star models.

Quark model (red) has 2 params; OGE strength and baryon wfn. length scale. These are standard values.
Something new for the meeting.
Quark model $V_{\Lambda\Lambda}(r)$:
$\Lambda\Lambda$ ($S=0$) hard core repulsion.
No $H$ dibaryon.
Future:

MR future:

“Core” forces between all flavor 8 and 10 baryon pairs in the quark model.
Resulting EOS.

LR future:

QCD origins of intermediate range attraction.
(Meson exchange models as well as quark forces.)
KN and NN spin-orbit force
(L*S force = “Holy Grail of NN quark model calculations.” - N. Isgur)