The QCD Phase Diagram: What About Isospin?

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Introduction

- **Experiments:** baryon AND isospin density
  - Neutron stars (high $\mu_B$, low $T$)
  - RHIC (low $\mu_B$, high $T$)

- Most studies: $\mu_I = 0$, except low $T$
  - Neutron stars
  - LOFF phase
  - What about higher $T$?

- QCD phase diagram with $\mu_I, \mu_B, T \neq 0$

- Random Matrix model
Outline

• Quick overview of QCD phase diagram
  ▶ $\mu_I = 0, \mu_B \neq 0, T \neq 0$
  ▶ $\mu_B = 0, \mu_I \neq 0, T \neq 0$

• Random Matrix Theory
  ▶ Description of the model
  ▶ Range of validity at $\mu_I, \mu_B, T = 0$

• Random Matrix model with $\mu_I, \mu_B, T \neq 0$

• QCD phase diagram ($N_f = 2, m_q \neq 0$)

• How to test these results?
QCD: $\mu_I = 0$, $\mu_B \neq 0$, $T \neq 0$

- Color Superconductivity, Critical endpoint
  - Random Matrix, NJL, Ladder QCD
  - Lattice $\rightarrow$ low $\mu_B$ only
QCD: $\mu_B = 0$, $\mu_I \neq 0$, $T \neq 0$

- Superfluidity, Tricritical point, Critical endpoint?
- Chiral Perturbation Theory, Lattice
- Phase diagram similar to $N_c = 2$, $\mu_B \neq 0$
Lattice: $N_c = 2, \mu_B \neq 0, T \neq 0$
Random Matrix Theory: $\mu_B, \mu_I, T = 0$

- QCD partition function

$$Z_{\text{QCD}} = \int [dA] \prod_f \det(iD + m_f) \, e^{-S_{\text{YM}}}$$

- Random Matrix Theory partition function

$$Z_{\text{RMT}} = \int [dW] \prod_f \det(iD + m_f) \, e^{-nG^2 \text{Tr}WW^\dagger}$$

$$\begin{vmatrix}
  m_1 & 0 & W & 0 \\
  0 & m_2 & 0 & W \\
  -W^\dagger & 0 & m_1 & 0 \\
  0 & -W^\dagger & 0 & m_2
\end{vmatrix}, \quad W = n \times n$$
Random Matrix Theory: $\mu_B, \mu_I, T = 0$

- Random Matrix Theory partition function
  - Same symmetry as QCD partition function
  - No dynamics

- Spectrum of Dirac operator
  
  Berbenni-Bitsch et al., PRL 80 (1998) 1146

  - Lattice, Chiral Perturbation Theory

  - Valid if $L \ll 1/m_\pi$
Random Matrix model: $\mu_B, \mu_I, T \neq 0$

- Partition function

$$
Z_{\text{RMT}} = \int [dW] \prod_f \det (iD + m_f + \mu_f \gamma_0) \; e^{-nG^2 \operatorname{Tr}WW^\dagger}
$$

$$
  \begin{pmatrix}
  m_1 & 0 & W + \Omega + \mu_1 & 0 \\
  0 & m_2 & 0 & W + \Omega + \mu_2 \\
  -W^\dagger - \Omega^\dagger + \mu_1 & 0 & m_1 & 0 \\
  0 & -W^\dagger - \Omega^\dagger + \mu_2 & 0 & m_2
  \end{pmatrix}
$$

$$
\Omega = \begin{pmatrix}
iT & 0 \\
0 & -iT
\end{pmatrix}
$$
Random Matrix model: $\mu_B, \mu_I, T \neq 0$

- Random Matrix model partition function

$$Z_{\text{RMT}} = \int [dW] \prod_f \det(iD + m_f + \mu_f \gamma_0) \ e^{-nG^2 \text{Tr}WW^\dagger}$$

1) Determinant as integral over fermions
2) Integration over $W$ (Gaussian)  
   $\Rightarrow$ Four fermion term
3) Hubbard-Stratonovich transformation  
   $\Rightarrow$ Mesons
4) Integration over fermions
   $\Rightarrow$ Effective action with mesons (exact map.)
Random Matrix model: $\mu_B, \mu_I, T \neq 0$

- Effective action identical to zero-momentum part of Chiral Perturbation Theory at $\mu_B, T = 0$

- Saddle point approximation of effective action
  ▶ Ansatz → order parameters

$$\begin{cases} 
\sigma_1 = \langle \bar{u}u \rangle \\
\sigma_2 = \langle \bar{d}d \rangle \\
\rho = \frac{1}{2}(\langle \bar{u}\gamma_5 d \rangle - \langle \bar{d}\gamma_5 u \rangle)
\end{cases}$$

▶ Study like a Landau-Ginzburg model
Random Matrix: $\mu_B \neq 0, \mu_I = 0$

- NO CSC, but Critical endpoint
- Lattice, Nambu–Jona-Lasinio, Ladder QCD
Random Matrix: $\mu_B = 0, \mu_I \neq 0$

- Superfluidity, but **NO** Tricritical Point
- Lattice, Chiral Perturbation Theory
Random Matrix: $\mu_I, \mu_B \neq 0, T = 0$

- Hadronic phase, Superfluid phase
- High $\mu_f \Rightarrow \langle \bar{q}_f q_f \rangle \ll 1$
Random Matrix: \( \mu_I, \mu_B, T \neq 0 \)
Random Matrix: $\mu_I, \mu_B, T \neq 0$

- **Doubling:** phase transition lines, critical endpoints
- **Critical endpoint at lower $\mu_B$ for fixed $\mu_I < m_\pi$**
  
  $\Rightarrow$ RHIC: 2 crossovers or first order phase trans.

$\Rightarrow$ Study of critical endpoint easier
How to test these results?

- NJL model if flavor-mix. four-fermion interaction \( \lesssim 10 - 15\% \) of non-flav. mixing term
- Ladder QCD
- Lattice
  - 3-color QCD: methods used for low \( \mu_B \)
  - 2-color QCD: large \( m_\pi \), fixed \( \mu_I < m_\pi \)
  
  \( \rightarrow \) Random Matrix model for 2-color QCD
Random Matrix model for 2-color QCD
Conclusions and Outlook

- Influence of small $\mu_I$ on QCD phase diagram
  - Doubling: phase trans., critical endpoints
  - Critical endpoint at lower $\mu_B$, same $T$

- Predictions can be tested on lattice
  - 3-color or 2-color QCD

- Consequences for RHIC:
  - Two crossovers or two first order phase transitions at low $\mu_B$
  - Critical end point at lower $\mu_B \Rightarrow$ more important