Gluon puzzle of gapless superconductivity

Igor A. Shovkovy

Institut für Theoretische Physik
J. W. Goethe Universität
Frankfurt am Main
Collaborator(s)

- Mei Huang  
  \textit{ITP, Goethe-University} \& \textit{Tsinghua University, Beijing}

References

- M. Huang and I. Shovkovy, in preparation
Matter at high density

We study this because we need to understand

(i) properties of dense matter that exists in the Universe

(densities in stars $\rho_c \gtrsim 5\rho_0$)

(ii) fundamental properties of QCD

($\mu_q \gtrsim \Lambda_{QCD}$: no lattice results)
[figure is taken from a talk by F. Weber]
Is there SC inside stars?

The answer is: **we do not know yet**

Arguments in favor:

- Relatively high densities in stars, $\rho_c \gtrsim 5\rho_0$, suggest that quarks may be deconfined
- An attractive diquark channel is likely to exist
- Temperatures are quite low, $T \lesssim 50$ keV, to allow pairing

Arguments against:

- Strongly coupled dynamics is not under control
- Matter may not necessarily be deconfined at existing densities
- Specific conditions inside stars (e.g., $\beta$-equilibrium) may not favor color superconductivity

The natural approach: **To give predictions and to test them**
Specific conditions inside stars

Matter in the bulk of a star is

(i) $\beta$-equilibrated: $\mu_d = \mu_u + \mu_e$

(ii) electrically and color neutral:
    $n_Q^{\text{el}} = 0, \quad n_Q^{\text{color}} = 0$

Otherwise, a star would not be stable!

• Coulomb energy (when $n_Q \neq 0$)

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_\odot c^2 \left( \frac{n_Q}{10^{-15} \text{e}/ \text{fm}^3} \right)^2 \left( \frac{R}{1 \text{ km}} \right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} \text{e}/ \text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{2\text{SC}} \gg M_\odot c^2$
Neutrality vs. color superconductivity

- The “best” 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in $\beta$-equilibrium) appears when $n_d \approx 2n_u$
- Electrons do not help (!):

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3}\mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

i.e.,

$$n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

The “best” Cooper pairing is distorted by the following mismatch parameter:

$$\delta \mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$
Mismatch vs. coupling strength

Mismatch parameter $\mu_e$ is **not** a free model parameter,

$$n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0 \implies \mu_e = \mu_e(\bar{\mu}_q, \Delta)$$

However, the diquark coupling strength ($\eta$) **is** a model parameter:

1. Weak coupling, $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing: **Normal phase is the ground state**

2. Strong coupling, $\eta \gtrsim 0.8$ — pairing wins over the mismatch between the Fermi surfaces: **2SC is the ground state**

3. Intermediate strength coupling, $0.7 \lesssim \eta \lesssim 0.8$ — the ground state is a new gapless color superconducting (g2SC) phase.
Quasiparticle spectrum in normal phase

Weak coupling (normal phase)

How does this spectrum change when Cooper pairs are formed?
Quasiparticle spectrum in 2SC phase

Strong coupling (2SC phase)

The energy gaps in the quasiparticle spectra are $\Delta - \delta \mu$ & $\Delta + \delta \mu$
Quasiparticle spectrum in g2SC phase

Intermediate coupling (gapless phase)

The energy gaps in the quasiparticle spectra are $0$ & $\Delta + \delta \mu$
Sarma phase in condensed matter

Type II superconductors in a constant magnetic field:

- Magnetic field originates from ferromagnetic order of impurities in $\text{La}_{1-x}\text{Gd}_x$ and $\text{Y}_{1-x}\text{Gd}_x\text{Os}_2$ [B. Matthias, H. Suhl & Corenzwit, Phys. Rev. Lett. 1 (1958) 92], [N. Phillips, B. Matthias, Phys. Rev. 121 (1961) 105]

- Pairing happens between spin-$\uparrow$ and spin-$\downarrow$ holes/electrons
- Fermi momenta of $\uparrow$- and $\downarrow$-quasiparticles are different
- The mismatch parameter $\delta \mu \sim H \sim n_{\text{impurity}}$

The gapless “Sarma” phase is unstable!

Igor Shovkovy

ITP, Goethe-University
Stability of g2SC phase


No Sarma instability in g2SC phase if $n_Q = 0$ is enforced locally!
**Finite temperature properties**

1. **Effective potential at** $T \neq 0$:

   ![Graph showing effective potential vs. temperature](image1)

   i.e., 2nd order phase transition


2. **Nonmonotonic** $\Delta(T)$ dependence:

   ![Graph showing nonmonotonic $\Delta(T)$](image2)

   Note: g2SC $\rightarrow$ 2SC $\rightarrow$ g2SC $\rightarrow$ NQ

3. **Extreme nonmonotonic** temperature dependence at weaker couplings:

   ![Graph showing extreme nonmonotonic behaviour](image3)

4. **$T_c/\Delta_0$ is not universal** (unlike in BCS), and it can be arbitrarily large!

   ![Graph showing $T_c/\Delta_0$](image4)

*Igor Shovkovy*

*ITP, Goethe-University*
**Higgs/Meissner effect in g2SC**

- Higgs effect, i.e., $SU(3)_c \to SU(2)_c$ without NG bosons
  - there exists unitary gauge in which NG boson fields are “eaten” by 5 gluons

- Is there Meissner effect?
  - low energy spectrum looks like in normal quark matter

- Improved HDL approximation plus (NG) collective modes:
$$\Pi^{AB,\mu\nu}(q) = \begin{cases} 
\delta^{AB} \Pi^\mu_1^\nu, & \text{for } A, B = 1, 2, 3, \\
\delta^{AB} \Pi^\mu_{4+}, & \text{for } A, B = (4 + 5i), (6 + 7i), \\
\delta^{AB} \Pi^\mu_{4-}, & \text{for } A, B = (4 - 5i), (6 - 7i), \\
\left( \begin{array}{cc} \Pi^\mu_{88} & \Pi^\mu_{8\gamma} \\
\Pi^\mu_{\gamma 8} & \Pi^\mu_{\gamma \gamma} \end{array} \right), & \text{for } A, B = 8, \gamma,
\end{cases}$$

where
$$\Pi^\mu_\alpha(q) = \left( g^{\mu\nu} - u^\mu u^\nu + \frac{\vec{q}^\mu \vec{q}^\nu}{q^2} \right) H_a(q) + u^\mu u^\nu K_a(q)$$
$$- \frac{\vec{q}^\mu \vec{q}^\nu}{q^2} L_a(q) + \frac{u^\mu \vec{q}^\nu + \vec{q}^\mu u^\nu}{|\vec{q}|} M_a(q)$$

Screening masses:
$$m^2_{M,a} \equiv -H_a(0) \quad \text{and} \quad m^2_{D,a} \equiv -K_a(0)$$
$\Pi^{AB,\mu\nu}$ with $A, B = 1, 2, 3$

Meissner and Debye screening masses:

\[
m^2_{M,1} \equiv -H_1(0) \simeq 0, \quad \text{(no Meissner effect)}
\]

\[
m^2_{D,1} \equiv -K_1(0) = \frac{2\alpha_s}{\pi} \left( \frac{(\mu^-)^2 \delta\mu}{\sqrt{(\delta\mu)^2 - \Delta^2}} + \frac{(\mu^+)^2 \delta\mu}{\sqrt{(\delta\mu)^2 - \Delta^2}} \right) \theta(\delta\mu - \Delta)
\]

\[
\simeq \frac{4\alpha_s \bar{\mu}^2 \delta\mu}{\pi \sqrt{(\delta\mu)^2 - \Delta^2}} \theta(\delta\mu - \Delta),
\]

where

\[
\bar{\mu} \equiv \frac{\mu_{g+d} + \mu_{r+u}}{2} \quad \text{(average Fermi momentum)}
\]

\[
\delta\mu \equiv \frac{\mu_{g+d} - \mu_{r+u}}{2} \quad \text{(mismatch between Fermi momenta)}
\]

\[
\mu^\pm \equiv \bar{\mu} \pm \sqrt{\delta\mu^2 - \Delta^2} \quad \text{(boundaries of “blocking” region)}
\]
\( \Pi^{AB,\mu\nu} \) with \( A, B = 8, \gamma \) (Debye screening)

\[
K_{88} \simeq \frac{4\alpha_s \bar{\mu}^2}{\pi},
\]
\[
K_{\gamma\gamma} \simeq \frac{8\alpha \bar{\mu}^2}{3\pi} \left( 1 + \frac{3\delta \mu \theta(\delta \mu - \Delta)}{2\sqrt{(\delta \mu)^2 - \Delta^2}} \right),
\]
\[
K_{8\gamma} = K_{\gamma 8} \simeq 0
\]

There is no mixing (static, long-range Debye screening).
However, a mixing will appear in the “natural basis”,
\[
\tilde{A}_\mu^8 = A_\mu^8 \cos \varphi + A_\mu^\gamma \sin \varphi,
\]
\[
\tilde{A}_\mu^\gamma = A_\mu^\gamma \cos \varphi - A_\mu^8 \sin \varphi,
\]

How about gauge symmetry? — No problem.
\[ \Pi^{AB,\mu\nu} \text{ with } A, B = 8, \gamma \text{ (Meissner screening)} \]

\[ H_{88} \simeq \frac{4\alpha_s \bar{\mu}^2}{9\pi} \left( 1 - \frac{\delta \mu \theta(\delta \mu - \Delta)}{\sqrt{(\delta \mu)^2 - \Delta^2}} \right), \]

\[ H_{\gamma\gamma} \simeq \frac{4\alpha \bar{\mu}^2}{27\pi} \left( 1 - \frac{\delta \mu \theta(\delta \mu - \Delta)}{\sqrt{(\delta \mu)^2 - \Delta^2}} \right), \]

\[ H_{8\gamma} = H_{\gamma 8} \simeq \frac{4\sqrt{\alpha \alpha_s \bar{\mu}^2}}{9\sqrt{3\pi}} \left( 1 - \frac{\delta \mu \theta(\delta \mu - \Delta)}{\sqrt{(\delta \mu)^2 - \Delta^2}} \right) \]

This becomes diagonal in the new basis:

\[ \tilde{A}^8_\mu = A^8_\mu \cos \varphi + A^\gamma_\mu \sin \varphi, \]

\[ \tilde{A}^\gamma_\mu = A^\gamma_\mu \cos \varphi - A^8_\mu \sin \varphi, \]
where the mixing angle is determined by

\[
\sin \varphi = \sqrt{\frac{\alpha}{3\alpha_s + \alpha}},
\]
\[
\cos \varphi = \sqrt{\frac{3\alpha_s}{3\alpha_s + \alpha}}.
\]

Then, the Meissner screening masses are

\[
m^2_{M,\tilde{8}} \equiv \frac{4(3\alpha_s + \alpha)\bar{\mu}^2}{27\pi} \left(1 - \frac{\delta \mu \ \theta(\delta \mu - \Delta)}{\sqrt{(\delta \mu)^2 - \Delta^2}}\right),
\]

\[
m^2_{M,\tilde{\gamma}} \equiv 0 \quad \text{i.e., no Meissner effect connected with } \tilde{U}(1)_{\text{em}}.
\]

Note that \(m^2_{M,\tilde{8}} < 0\) in the g2SC phase.

This means that there is a \textbf{plasma} (magnetic) type instability.

Note that spin-1 condensates around \(\mu^\pm\) remove the instability.
\( \Pi^{AB, \mu \nu} \) with \( A, B = (4\pm) \)

Meissner and Debye screening masses:

\[
m_{M,4\pm}^2 \equiv -H_{4\pm}(0) \\
\simeq \frac{4\alpha_s \bar{\mu}^2}{3\pi} \left[ \frac{\Delta^2 - 2\delta \mu^2}{2\Delta^2} + \theta(\delta \mu - \Delta) \frac{\delta \mu \sqrt{\delta \mu^2 - \Delta^2}}{\Delta^2} \right] ,
\]

\[
m_{D,4\pm}^2 \equiv -K_{4\pm}(0) \\
\simeq \frac{4\alpha_s \bar{\mu}^2}{\pi} \left[ \frac{\Delta^2 + 2\delta \mu^2}{2\Delta^2} - \theta(\delta \mu - \Delta) \frac{\delta \mu \sqrt{\delta \mu^2 - \Delta^2}}{\Delta^2} \right] ,
\]

Note that \( m_{M,4\pm}^2 < 0 \) when

\[
0 < \Delta < \sqrt{2}\delta \mu \quad \text{(i.e., in g2SC and 2SC phases)}
\]

Thus, there is a \textbf{plasma} (magnetic) type instability...
Overview of the screening properties

Without spin-1 condensates

\[ A = 1, 2, 3 \quad \text{— red solid line} \]
\[ A = 4, 5, 6, 7 \quad \text{— green long-dash line} \]
\[ A = \tilde{8} \quad \text{— blue short-dash line} \]
Overview of the screening properties

With spin-1 condensates

\[ A = 1, 2, 3 \quad \text{— red solid line} \]
\[ A = 4, 5, 6, 7 \quad \text{— green long-dash line} \]
\[ A = \tilde{8} \quad \text{— blue short-dash line} \]
Magnetic instability $m_{M,4\pm}^2 < 0$

Could gluons “feel” the local maximum of the effective potential? This is possible, but this is not everything ...

What is the origin of instability in 2SC phase when $\delta \mu < \Delta < \sqrt{2}\delta \mu$?
So, the ground state is ...

something like this

Spontaneously induced currents

Spontaneous symmetry

Broken rotational chromo-magnetization

Nonlinear gluon dynamics

Gluon BEC
**Summary**

- The g2SC phase is a new state of matter that may exist in cores of compact stars (gCFL $\rightarrow$ [Rajagopal, Kouvaris, Alford, hep-ph/0311286])
- There is no Sarma instability in g2SC phase if the neutrality is enforced locally
- The spectrum of low-energy excitations in the g2SC phase has extra gapless modes (these should affect transport properties)
- Finite temperature properties of the g2SC phase are rather unusual [$\Delta(T)$ is nonmonotonic; $T_c/\Delta_0$ is nonuniversal]
- There is a new type plasma instability for a range of parameters in g2SC phase and even in 2SC phase!
- Part of plasma instability is removed by spin-1 condensates
- There are additional (BEC) gluon condensates in 2SC phase