Non Fermi liquid effects in dense matter

Kai Schwenzer, Thomas Schäfer
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Introduction

Possible phases at high density ...

... all involve condensed excitations and are no Fermi liquids

Rajagopal, Wilzcek, hep-ph/0011333
Alford, Kouvaris, Rajagopal, hep-ph/0311286
Kryjevski, Kaplan, Schaefer, hep-ph/0404290
Motivation

☐ (Normal) quark matter phase at high density is basically ruled out

However:

☒ Important to check the stability of superfluid phases
☒ Many considered phases include un-gapped quark excitations (g)2SC, gCFL, ...
☒ Simpler environment to study gluonic effects than in superfluid phases
High density effective theory

- Effective degrees of freedom at high density: excitations around the Fermi surface

- "Integrate out" high energy excitations

  - No anti-particles

- N-point functions in the effective theory from matching procedure

- Counterterm to ensure gauge invariance

Effective Lagrangian

\[ L_{\text{HDET}} = \sum \bar{\psi}_v \gamma^i (iv \cdot D) \psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots \]

Involves an explicit cutoff \( \Lambda \)

- Fermi surface covered with patches of size \( \Lambda \)
- Involves only relative momenta and energies

Consistent power counting scheme

Large \( \mu \) plays similar role to large \( N_f \)
Unscreened gluons

Gluonic excitations in the medium

Static electric gluons are screened by Debye mass

\[ V(r) \propto \frac{\exp(-m_D r)}{r} \quad m_D^2 = \frac{N_f g^2 \mu^2}{2\pi^2} = 2m^2 \]

Magnetic gluons are only dynamically screened

\[ \Pi_t(k_0, k) = m^2 \frac{k_0}{k} \left( 1 - \left( \frac{k_0}{k} \right)^2 \right) \frac{1}{2} \log \left( \frac{k_0 + k}{k_0 - k} \right) + \frac{k_0}{k} \]

Effective gluon propagator

\[ D_{ij}(k_0, k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta |k_0|/|\vec{k}|} \]
1-loop self energy

- Inclusion of dynamically screened gluons

\[-i \Sigma(p) = - \int \frac{d^4 k}{(2\pi)^4} \Gamma^a_\mu S(p + k) \Gamma^b_\nu D^{ab}_{\mu\nu}(k)\]

- Approximate analytic result

\[\Sigma(\omega, l) = \gamma \omega \log \left(\frac{\Lambda}{\omega}\right)\]

\[\gamma = \frac{g^2 C_F}{12\pi^2}\]

- Strong infrared enhancement

- Independent of \(\vec{p}\) to leading order

\[\text{Breakdown of perturbation theory } \omega < \Lambda \exp \left(-\frac{9\pi^2}{g^2}\right) ?\]
Scales in dense matter

\[ \omega_{bcs} = \mu \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \]

\[ \omega_{nfl} = \mu \exp\left(-\frac{9\pi^2}{g^2}\right) \]

• **HDET cutoff** \( \Lambda \) chosen at the damping scale \( m \)

• **Pairing instability**  

• **Non Fermi liquid effects** at very low scales

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Fermi sea

\( \omega_{nfl} \)

\( \Lambda \approx m \)

\( \mu \)

screening & damping
Dyson-Schwinger equation

Selfconsistent analysis of the self energy

\[-i \Sigma(p_0) = g^2 C_F \int \frac{d^4k}{(2\pi)^4} \frac{1-(\vec{v} \cdot \hat{k})^2}{p_0+k_0-l_{p+k}+\Sigma(p_0+k_0)} \frac{1}{k_0^2-k^2+i\eta|k_0|/|k|}\]

\[k_l \ll k_t \rightarrow \text{integrations decouple}\]

Gluon propagator dominated for \[k \approx (\eta k_4)^{\frac{1}{3}}\]

Pick up the fermion pole in the \[l_k\] integration

Self energy is approximately 1-loop exact
RG analysis

Previous result in: Boyanovsky, de Vega, Phys. Rev. D 63 (2001)

\[ S^{-1}(\omega, l) = \omega \left( \frac{\omega}{\Lambda} \right)^\gamma - l \]

In contradiction with DS-result

Broken Lorentz invariance due to Fermi see

\[ \mathcal{L} = \psi_v^\dagger (\omega - v_F l) \psi_v + g v_F \psi_v^\dagger \hat{\nabla} \psi_v + \ldots \]

Self energy and coupling

\[ \Sigma(\omega, l) = \frac{g^2 v_F}{9\pi^2} \omega \log\left( \frac{\Lambda}{\omega} \right) \]

\[ \alpha = \frac{g^2 v_F}{4\pi} \]

Bare fields and couplings

\[ \psi_{0,v} = Z^{1/2} \psi_v \]

\[ g_0 = \frac{Z_F}{Z^2} g \]

\[ v_{0,F} = Z_F v_F \]

\[ G_0^{(n)}(\omega_i, v_{0,F} l_i, \alpha_0) = Z^{n/2} G^{(n)}(\omega_i, v_F l_i, \alpha) \]
RG solution

Callan Symanzik equation

\[
\left\{ \Lambda \frac{\partial}{\partial \Lambda} + \beta(\alpha) \frac{\partial}{\partial \alpha} - \gamma_F(\alpha) l_i \frac{\partial}{\partial l_i} + \frac{n}{2} \gamma(\alpha) \right\} G^{(n)}(\omega_i, l_i, \alpha) = 0
\]

One loop results

\[
\beta(\alpha) = -\gamma_F(\alpha) \alpha \quad \gamma(\alpha) = -\gamma_F(\alpha) = \frac{4\alpha}{9\pi}
\]

RG equation for the two point function

\[
\left\{ \Lambda \frac{\partial}{\partial \Lambda} + \gamma \left[ \alpha \frac{\partial}{\partial \alpha} + l \frac{\partial}{\partial l} + 1 \right] \right\} S(w, l, \alpha) = 0
\]

\[\checkmark \text{ same result: } S^{-1}(\omega, l) = \omega \left( 1 + \gamma \log \left( \frac{\Lambda}{\omega} \right) \right) - v_F l\]

Neglecting the running of the coupling \( \beta(\alpha) = 0 \)
yields scaling with an anomalous dimension

Higher order logarithmic terms are absent

\[
S^{-1}(\omega, l) = \sum_k a_k \alpha^k \omega \left[ \log \left( \frac{\Lambda}{\omega} \right) \right]^k - v_F l \quad a_k = 0, \ k > 1
\]
Fermionic self energy

solid: full numeric
dashed: leading log
dotted: anom. dim.
Asymptotic behavior

Ratio between resummed and one loop result

Strong coupling @ $\mu \rightarrow$ nonperturbative for small $\mu$
Fermionic excitations

- Quark propagator is not given by a simple pole but contains a cut.

- Spectral density has not fully Breit-Wigner form:
  \[ \rho(\omega) = \frac{\gamma \omega}{[\omega(1 + \gamma \log(\Lambda/\omega)) - l]^2 + \pi^2 \gamma^2 \omega^2} \]

- Vanishing wavefunction renormalization and Fermi velocity at the Fermi surface.

- “Jump” at the Fermi surface vanishes.

- Strongly IR-modified dispersion relation.
Conclusion and Outlook

- Dense matter is not a Fermi liquid even in the normal phase
- Effects result from kinematics at the Fermi surface
  \[ C_v = N_f (N_c^2 - 1) \frac{g^2 \mu^2}{72 \pi^2} T \log \left( \frac{\Lambda}{T} \right) \]
- Modified neutrino emissivity
- Higher order n-point functions nonperturbative?
- Impact on thermal conductivity, ...?