Infrared QCD and the renormalisation group

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• motivation
  QCD phase diagram

• renormalisation group
  momentum cutoff
  flow equation

• applications to QCD
  infrared analysis in Landau gauge
  signatures of confinement

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Phase diagram of QCD

- high temperature QCD
- high density QCD
- QCD at strong coupling
QCD at strong coupling

- **relevant for:**
  - confinement
  - chiral symmetry breaking
  - confinement-deconfinement phase transition
  - matter at realistic densities
  - hot QCD dynamics of soft excitations

- **lattice simulations**
  - ideal for
    - time independent observables
    - finite temperature
  - problematic for
    - dynamics, non-equilibrium
    - high baryonic density

- **renormalisation group methods**
  - analytic approach
  - complementary to lattice
  - Exact Renormalisation Group: very flexible
Exact Renormalisation Group

• **goal:** calculation of quantum effective action $\Gamma[\phi]$

• successive integrating-out of momentum modes via
  
  $S[\phi] \rightarrow S[\phi] + \Delta S_k[\phi]$, with
  
  $$\Delta S_k[\phi] = \int_q \phi(q) R(q) \phi(-q)$$

  leads to scale-dependent effective action $\Gamma_k$ with
  momentum modes $q^2 > k^2$ integrated out.

• infinitesimal integration $\Rightarrow$ flow equation:

  $$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + R \right)^{-1} \partial_t R$$

  initial condition: classical action $\Gamma_{k \to \Lambda} = S$

  endpoint: quantum effective action $\Gamma_{k \to 0} \equiv \Gamma$

  flow is infrared finite and ultraviolet finite
\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + R \right)^{-1} \partial_t R = \frac{1}{2} \]  

- **Infrared cutoff** \( R \)
  
  \[ \begin{align*}
  R(q^2 \to 0) &\approx k^2 \\
  R(k \to 0) &\to 0 \\
  R(q^2 \to \infty) &\to 0
  \end{align*} \]

- **Locality**
  
  Momentum integration peaked about
  
  \[ q^2 \approx k^2 \]
Example: scalar fields at criticality

- real scalar field theory: Ising universality class
- scaling behaviour
  flow equation for effective potential $u$ in $3d$:

\[
\partial_t u + 3u - \rho u' = \int_0^\infty dy \frac{-y^{5/2} r'(y)}{y(1 + r) + u' + 2\rho u''}
\]

scaling variables $y = \frac{q^2}{k^2}$, $\rho = \frac{\phi^2}{2k}$, $u(\rho) = \frac{U(\phi)}{k^3}$, $r = \frac{R}{q^2}$

- critical exponent $\nu$
  deduced from Wilson-Fisher fixed point $\partial_t u' = 0$

- optimisation for $\nu$
- global extrema exist

$\nu_{opt} = \min_R \nu(R)$.

- optimisation entails PMS
- physical value (N=1)

$\nu_{phys} \approx 0.63$. 
Gauge symmetry

• gauge symmetry $\Rightarrow$ link between Greens functions

Ward identity:

$$D_\mu \frac{\delta \Gamma}{\delta A_\mu} = \text{quantum corrections}$$

• flow equation:

momentum cutoff quadratic in the fields $\sim \int A R_k A$ is, a priori, incompatible with non-linear gauge symmetry.

modified Ward identity:

$$D_\mu \frac{\delta \Gamma}{\delta A_\mu} = \text{quantum corrections} + \text{cutoff terms}$$


cutoff terms $\sim \langle D_\mu \frac{\delta}{\delta A_\mu} \Delta S_k \rangle \neq 0$

• gauge invariance of physical Greens functions:

required in the infrared limit at $k = 0$.

not mandatory at $k \neq 0$. 
Confinement in Landau gauge QCD

• Kugo-Ojima confinement criterion
  gluonic mass gap and absence of Higgs mechanism ⇔ momentum behaviour of two-point functions:
  \[ \Gamma^{(2)}_A(p) = p^{2(1-2\kappa_C)}, \quad \Gamma^{(2)}_C(p) = p^{2(1+\kappa_C)} \]
  confinement: \( \kappa_C \geq 0 \)

• Schwinger-Dyson equation
  infrared coefficients \( \kappa_C \geq 0, \alpha_s \)
  inclusion of quarks
  problems: RG scaling and renormalisation

• Stochastic quantisation
  infrared coefficients \( \kappa_C \geq 0, \alpha_s \)
  resolution of Gribov problem
  problems: RG scaling and renormalisation

• Lattice
  infrared behaviour of gluon propagator
  problem: finite size scaling

• Flow equations
  heavy quark potential
  effective quark interactions
  problems: access to infrared regime
Flows in Landau gauge QCD

• infrared analysis

consider Greens functions for the regime

\[ k^2 \ll p^2 \ll \Lambda_{\text{QCD}}^2 \]

– deep infrared region
– physics already integrated-in
– trivial cutoff dependence

• fixed point behaviour

Greens functions in the deep infrared regime

\[ \Gamma_{k}^{(n)}(p_i \ll \Lambda_{\text{QCD}}) = z_n \cdot \hat{\Gamma}^{(n)}(p_i, p_i^2/k^2) \]

– \( z_n \) is \( k \)-independent up to RG scalings
– \( k \)-dependence of \( \hat{\Gamma}^{(n)} \) only via the ratio \( p^2/k^2 \)
Technical details

- truncation

General ghost and gluon two-point functions vertices $\Gamma^{(3)}$, $\Gamma^{(4)}$ from Slavnov-Taylor identities

Parametrisation \( x = p^2/k^2 \)

\[
\begin{align*}
\Gamma^{(2)}_{k,A}(p) &= z_A \cdot x^{\kappa_A} (1 + \delta Z_A(x)) \cdot p^2 \cdot \Pi(p) \\
\Gamma^{(2)}_{k,C}(p) &= z_C \cdot x^{\kappa_C} (1 + \delta Z_C(x)) \cdot p^2 \\
\Gamma^{(n)}_{k}(p) &= z_n \cdot S^{(n)}_{\text{cl}}
\end{align*}
\]

- RG scaling implies

\[
\kappa_A = -2\kappa_C, \quad \alpha_s = \frac{g^2}{4\pi} \frac{1}{z_A z_C^2}
\]

- integrated flow

Analyse \( \Gamma^{(2)}_k(p) - \Gamma^{(2)}_0(p) = \int_0^k \frac{dk'}{k'} \partial_{t'} \Gamma^{(2)}_{k'} \)

Trade scale- for momentum integration \( \frac{dk'}{k'} = -\frac{dx'}{2x'} \)

⇒ deduce infrared parameters
Infrared analysis of QCD

• fixed point equation
  evaluate the integrated flow in the deep infrared regime:

  \[ \delta Z(x) = \frac{\alpha_s}{\pi^2 N} \int_x^\infty \frac{dx'}{x'^{2+\kappa}} f(x'; x, \kappa, \delta Z) \]

  \[ \delta Z(x \to \infty) \to 0 \] physical regime
  \[ \delta Z(x \to 0) \to -1 + \ldots \] cutoff regime
  \[ \Rightarrow \text{deduce } \kappa, \alpha_s \text{ and } \delta Z \text{ from the limit } x \to 0. \]

• infrared coefficients
  zeroth order: \( \kappa_C = 0.59535 \quad \alpha_s = 2.9717 \)
  unique, identical to Dyson-Schwinger result

  \[ \kappa_{opt} = \text{extr}_R \kappa(R) \]
  maximum located at the non-iterated result
Infrared analysis (cont’d)

- solution for $\delta Z$
  - zeroth order: solution is $R$ independent
  - full iteration: optimal propagators near non-iterated solution

\[
r(x) = \frac{\gamma}{x(1 + x)}
\]
Conclusions and outlook

- **infrared regime of QCD**
  - fixed point behaviour
  - analytical access
  - infrared coefficients $\kappa_C$ and $\alpha_s$
  - support for Kugo-Ojima scenario
  - full vertex functions, fermions

- **finite temperature**
  - deconfinement phase transition
  - thermal pressure

- **finite density**
  - inclusion of quarks, chemical potential