Deconfinement phase transition with various gauge groups

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- deconfinement in Yang-Mills theory
- phase transition & center symmetry
- universal behavior & Svetitsky-Yaffe conjecture
- $SU(N)$ gauge theory
- $Sp(N)$ gauge theory
- conjecture when transition universal

Michele Pepe (Bern) and Uwe-Jens Wiese (Bern & MIT)
Deconfinement & center symmetry

Finite temperature $\beta = 1/T$

Physical quantities periodic in time
$A_{\mu}(x)$ periodic up to gauge trans. $\Omega(x)$

Gauge group $G$ twist $\Omega(x) \rightarrow z \Omega(x)$
$z \in H$ (center of $G$) Global symmetry

Polyakov loop $\Phi = \text{Tr} \mathcal{P} \exp(i \int dx A_4(x))$ $\Phi \rightarrow z \Phi$

$\langle \Phi \rangle = \exp(-\beta F)$ free energy $F$ of static quark

$\langle \Phi \rangle = 0 \Rightarrow F \rightarrow \infty$ confined center symmetry intact

$\langle \Phi \rangle \neq 0 \Rightarrow F$ finite deconfined center symmetry broken

**Svetitsky-Yaffe conjecture:** if $(d+1)$-dim. gauge theory has 2nd order deconfinement phase transition $\Rightarrow$ belongs to universality class of $d$-dim. scalar field theory with symmetry $H$

2nd order $\xi \propto (T - T_{\text{crit}})^{-\nu} \rightarrow \infty$ $\nu$ critical exponent

$\xi \gg \beta$ dimensional reduction

conjecture does **NOT** say phase transition **MUST** be 2nd order
SU(N) Yang-Mills in 4-dim. center of SU(N) is Z(N)

N = 2  2nd order phase transition
critical exponents same as 3-dim. Z(2) symmetric scalar field theory
i.e. Ising universality class

N = 3  weak 1st order phase transition  not universal

N = 4, 6, 8  1st order phase transition  stronger as N increases

For N ≥ 5  ∃ 3-dim. Z(N) universality class  U(1) XY model

Pisarski-Tytgat conjecture: SU(N = ∞) has 2nd order
deconfinement phase transition, SU(3) with 1st order transition is a
small deviation from N = ∞ — seems not to be true

SU(N) Yang-Mills in 3-dim.

N = 2, 3, 4(?)  2nd order phase transition
belong to universality classes of 2-dim. Z(2), Z(3), Z(4) symmetric
scalar field theory

All cases support Svetitsky-Yaffe conjecture

4-dim. 2nd order phase transitions rare

Other possibilities?
Symplectic groups \( Sp(N) \subset SU(2N) \quad U \in Sp(N) \)

\[
U^* = JUJ^\dagger \quad J = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \quad \Rightarrow \quad U = \begin{pmatrix} W & X \\ -X^* & W^* \end{pmatrix}
\]

\( Sp(N) \) has \( (2N + 1)N \) generators and rank \( N \)

Special cases \( Sp(1) = SU(2) \quad Sp(2) \simeq SO(5) \)

\( Sp(N) \) Yang-Mills

\( Sp(2) \) in 4-dim.

no sign of any bulk transition separating strong and weak coupling

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\( 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \)

\( \vee \quad \wedge \quad \triangledown \quad \triangleright \quad \triangleright \quad \uparrow \)

\( Sp(N) \) gauge theory
Looks like 1st order deconfinement phase transition

susceptibility $\chi = V(\langle \Phi^2 \rangle - \langle \Phi \rangle^2) \propto V$ at $T_{\text{crit}}$

Specific heat/volume extrapolation agrees with non-zero latent heat

phase transition is 1st order
Conjecture for 4-dim.

$Sp(3)$ in 4-dim. also has 1st order deconfinement phase transition

What other possibilities?

$SO(N)$ groups

Exceptional groups

$SO(3) \simeq SU(2)$  \hspace{1cm} $SO(4) \simeq SU(2) \times SU(2)$

$SO(5) \simeq Sp(2)$  \hspace{1cm} $SO(6) \simeq SU(4)$

$G(2), E(8), F(4)$ have trivial center — expect no phase transition

$E(6), E(7)$ have center $Z(3), Z(2)$ respectively

**Conjecture:** only $Sp(1) = SU(2) \simeq SO(3)$ has universal 2nd order deconfinement phase transition in 4-dim.

size of group (number of “gluons”) makes transition 1st order

$Sp(2)$: 10 generators  \hspace{1cm} $Sp(3)$: 21 generators

center of group does not predict order of phase transition
1st order transition is physical

\[ T_C / \sqrt{\sigma} = 0.688(2) \text{ for 4-dim. } Sp(2) \]
\[ = 0.709(3) \text{ for 4-dim. } SU(2) = Sp(1) \]
\[ = 0.646(3) \text{ for 4-dim. } SU(3) \]

\( Sp(2) \) seems to be closer to \( SU(2) = Sp(1) \) than \( SU(3) \) is
$Sp(N)$ in 3-dim.

$Sp(2)$ has 2nd order transition

$\langle |\Phi| \rangle L^{\beta/\nu}$ universal behavior with 2-dim. Ising critical exponents $\beta, \nu$

**Supports** Svetitsky-Yaffe conjecture

$Sp(3)$ has weak 1st order transition

Similar to 4-dim.

“large” groups have 1st order transition
Critical exponents of 2-d Ising universality class
\[\nu = 1 \quad \beta = 1/8 \quad \gamma = 7/4\]
2nd order deconfinement phase transitions with universal behavior rare

conjecture that only $Sp(1) = SU(2) \simeq SO(3)$ have universal
deconfinement transitions in 4-dim.

$Sp(1) = SU(2), Sp(2), SU(3)$ and maybe $SU(4)$ universal
deconfinement in 3-dim.

phase transition driven by size of group, not by center

interesting to study more gauge groups systematically