NJL-model study of color superconducting quark phases in compact stars

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Introduction: The QCD Phase Diagram

• schematic QCD phase diagram (2+1 flavors)

(Rajagopal, Wilczek, hep-ph/0011333)

- hadronic phase (H):
  \[ \langle \bar{\psi}\psi \rangle \neq 0, \langle \psi\psi \rangle = 0 \]

- quark-gluon plasma (QGP):
  \[ \langle \bar{\psi}\psi \rangle \approx 0, \langle \psi\psi \rangle = 0 \]

- two-flavor color superconductor (2SC):
  \[ \langle \bar{\psi}\psi \rangle \approx 0, \langle ud \rangle \neq 0 \]

- color-flavor locking (CFL):
  \[ \langle ud \rangle \approx \langle us \rangle = \langle ds \rangle \neq 0 \]
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• further possible phases:
  color superconducting crystals, CFL + kaon condensate, spin-1 condensates, gapless color superconductors . . .
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Topics to be discussed

- color-flavor unlocking transition:
  role of the strange quark mass

- neutral quark matter:
  homogeneous vs mixed-phase solutions

- application to compact stars:
  stability of quark matter cores
Color-flavor unlocking for realistic quark masses

- precondition for standard BCS pairing: \( |p_F^a - p_F^b| \lesssim \sqrt{2}\Delta_{ab} \)

- common chemical potential, \( M_u \approx M_d < M_s < \infty \)

- \( \mu \gg M_s \Rightarrow p_F^u = \sqrt{\mu^2 - M_u^2} \approx \sqrt{\mu^2 - M_s^2} = p_F^s \rightarrow \text{CFL} \)

- \( \mu \sim M_s \Rightarrow p_F^u \gg p_F^s \rightarrow 2\text{SC phase} \)
  (with or without unpaired s-quarks)

- favored state at intermediate densities?

- \( M_u, M_d, M_s \): effective ("constituent") quark masses
  - related to \( \langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle \)
  - \( T \) and \( \mu \) dependent
  - interdependence: masses \( \leftrightarrow \) diquark condensates
Approach

- microscopic treatment within QCD:
  - asymptotic densities $\rightarrow \alpha_s = \text{small} \rightarrow$ gluon exchange
  - optimistic estimate: $\mu > 1.5 \text{ GeV} \rightarrow \rho_B > 175 \rho_0$
  - Rajagopal and Shuster, PRD (2000): $\mu \gg 10^5 \text{ GeV} !!!$

- "model independent" studies:
  - expansions in $\Delta/\mu, M_s/\mu$
  - expansion parameters not necessarily small
  - misses $\mu$-dependence of $\Delta$ or $M_s$

- model calculations:
  - based on vacuum phenomenology
    $\rightarrow$ extrapolation of parameters into an unknown regime
  - relatively simple $\rightarrow$ allows for tackling more complex problems
  - NJL model: naturally suited for studying the competion of $\langle qq \rangle$ and $\langle \bar{q}q \rangle$ condensates
• NJL-type Lagrangian: \( \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq} \)

  • free part:
    \[
    \mathcal{L}_0 = \bar{\psi}(i\gamma^\mu - \hat{m})\psi, \quad \hat{m} = \text{diag}_f(m_u, m_d, m_s)
    \]

  • quark-antiquark interaction:
    \[
    \mathcal{L}_{\bar{q}q} = G \left\{ (\bar{\psi} \tau^a \psi)^2 + (\bar{\psi} i\gamma_5 \tau^a \psi)^2 \right\} - K \left\{ \det_f(\bar{\psi}(1 + \gamma_5)\psi) + \det_f(\bar{\psi}(1 - \gamma_5)\psi) \right\}
    \]

  • quark-quark interaction:
    \[
    \mathcal{L}_{qq} = H (\bar{\psi} i\gamma_5 \tau_A \lambda_{A'} C\bar{\psi}^T)(\psi^T C i\gamma_5 \tau_A \lambda_{A'} \psi)
    \]

• mean-field approximation:
  • six condensates: \( \langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle; \langle ud \rangle, \langle us \rangle, \langle ds \rangle \)
  \( \rightarrow \) six coupled gap equations for \( M_u, M_d, M_s; \Delta_{ud}, \Delta_{us}, \Delta_{ds} \)
Numerical results

(M.B and M. Oertel, NPA ’02)

- Parameters fixed to reproduce reasonable vacuum properties
- $T = 0$, equal chemical potentials:

$$M_u = M_d, \ M_s$$

$$\Delta_{ud}, \ \Delta_{us} = \Delta_{ds}$$

- Two distinct first-order phase transitions:
  
  normal $\rightarrow$ 2SC $\rightarrow$ CFL

- Strong interdependence masses $\leftrightarrow$ diquark condensates
first and second order phase transitions:
Quark matter in compact stars

• quark core of a neutron star:
  • quarks \((u, d, s)\) + leptons
  • after a few minutes: neutrinos untrapped

• additional constraints:
  • \(\beta\) equilibrium: \(d, s \leftrightarrow u + e^- + \bar{\nu}_e\) \(\Rightarrow\) \(\mu_d = \mu_s = \mu_u + \mu_e\)
  • electric charge neutrality: \(\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0\)
  • color singletness \(\Rightarrow\) color neutrality: \(n_r = n_g = n_b\)

• consequences:
  • unequal Fermi momenta for \(u\) and \(d\)
  • instability of the \(ud\) condensate (no 2SC phase) ??

(M. Alford K. Rajagopal, JHEP 0206 (2002) 031)
Limiting cases:

- **case 1:** $M_s$ small  
  (Alford, Rajagopal, JHEP ’02)
  
  - $M_s = 0$: $n_u = n_d = n_s$
  
  - Taylor expansion in $M_s$:
    \[
    p_F^d = p_F^u + \frac{M_s^2}{4\mu}, \quad p_F^s = p_F^u - \frac{M_s^2}{4\mu} \]
    equidistant Fermi momenta!
    
    ⇒ $us$ pairing as likely as $ud$ pairing
    
    ⇒ whenever $ud$ pairing is more favored than no pairing, 
      CFL is even more favored
    
    ⇒ no 2SC phase

- **case 2:** $M_s$ large  ⇒ no strange quarks
  
  - $n_d \simeq 2n_u$  ⇒  $p_F^d \simeq 2^{1/3} p_F^u \simeq \frac{5}{4} p_F^u$,
  
  - stability criterion for standard BCS pairing: $\Delta > \delta \mu/\sqrt{2}$
  
  - example: $p_F^u = 400$ MeV  ⇒  $p_F^d = 500$ MeV  ⇒  $\Delta > 70$ MeV
    
    ⇒ 2SC phase possible if interaction strong enough
Phase diagram in the $\mu$-$\mu_Q$ plane

(F. Neumann, M.B., M. Oertel, NPA ’03)

$$(\mu_3 = \mu_8 = 0)$$
Homogeneous neutral solutions

(Steiner, Reddy, and Prakash, PRD '02)

- pressure: $(CFL, 2SC, N)$

- CFL favored for large $\mu$
- 2SC favored for small $\mu$
- normal quark matter never favored
Homogeneous neutral matter: quark masses and gaps

\[ (M_u, M_d, M_s, \Delta_{ud}) \]
\[ (M_u = M_d, M_s, \Delta_{ud}, \Delta_{us} = \Delta_{ds}) \]

- masses and gaps in the 2SC phase:
  - \( \Delta_{ud} \sim 100 \text{ MeV} \)
  - \( M_s \sim \mu \gg M_u, M_d \quad \Rightarrow \quad \text{Taylor expansion in } M_s \text{ fails} \)
• CFL pairing energy: \( \sim 100 \) MeV per baryon (cf. Jes Madsen’s talk)

• no absolutely stable strange quark matter
Mixed quark phases

- **basic principle:** (Glendenning 1992)
  - 1st component positive
  - 2nd component negative
  - globally neutral

- **here:**
  - four chemical potentials: $\mu, \mu_Q, \mu_3, \mu_8$
  - three neutrality conditions:
    $$n_Q = n_3 = n_8 = 0$$

- **mixed phases:**
  - 2, 3, or 4 components
  - neutral along 1-dimensional lines
Mixed quark phases

(F. Neumann, M.B., M. Oertel, NPA ’03)

• composition: $N \ 2SC \ 2SC_{us} \ SC_{us+ds} \ CFL$

• 9 different mixed phases
• 2-, 3-, and 4-component systems
• “exotic” components: $SC_{us+ds}, 2SC_{us}$
Stability of the mixed phases

- bulk energy gain:

- not stable if surface tension \( \gtrsim 10 \text{ MeV/fm}^2 \)

- BUT: surface tension can be small (Reddy and Rupak, nucl-th/0405054)
Application to compact stars

(Baldo, M.B, Burgio, Neumann, Oertel, Schulze, PLB '03, M.B, Neumann, Oertel, Shovkovy, PLB '04)

- homogeneous neutral NJL quark matter
- various hadronic EOS:
  - BHF (nucleons and leptons only) (Baldo et al.)
  - BHF (nucleons, hyperons, and leptons) (Baldo et al.)
  - relativistic mean field w/ hyperons (Glendenning)
  - chiral SU(3) model (Hanauske et al.)

- construct sharp phase transition
- solve Toman-Oppenheimer-Volkoff equation
Example: chiral SU(3) hadronic EOS

- hadron-quark phase transition (H, N, 2SC, CFL)

- $\mu_{\text{crit}}(H \rightarrow \text{CFL}) < \mu_{\text{crit}}(H \rightarrow N)$
- 2SC solution irrelevant

- solutions of the TOV equation:
  no stable configuration with pure quark matter core!
Other hadronic EOS

- BHF without hyperons: practically the same result

- BHF with hyperons: 
  
  \((H, N, 2SC, CFL)\)

  no phase transition at all!

- relativistic mean field:
  - \(H \not\rightarrow N\)
  - \(H \rightarrow CFL\)
  - phase transition renders star unstable
Summarizing the results up to this point:

NJL quark matter can compete with hadronic matter only if there is a non-negligible fraction of strange quarks.

→ strong increase of the energy density at the phase transition
→ star gets unstable
→ no pure quark matter core in compact stars

stable hybrid stars in the bag model:

strange quark masses and bag constant typically smaller than in NJL

BUT: recent example for stable hybrid stars in two-flavor NJL

(Shovkovy et al., PRD (2003))

still possible if strange quarks are included?
parameter dependence?
Different NJL-model parameters

- literature fits to pseudoscalar spectrum in vacuum
- so far: \( M_u^{\text{vac}} = 368 \text{ MeV}, \ M_s^{\text{vac}} = 550 \text{ MeV} \)  
  (Rehberg, Klevansky, H"ufner, PRC ’96)
- alternative: \( M_u^{\text{vac}} = 335 \text{ MeV}, \ M_s^{\text{vac}} = 527 \text{ MeV} \)  
  (Hatsuda & Kunihiro, Phys. Rep. ’94)
- impact on the pressure:

Rehberg, Klevansky, H"ufner:  
(N, 2SC, CFL, \( H = \chi_{\text{SU}(3)} \))
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  Hatsuda & Kunihiro:
  \( (N, 2SC, CFL, \ H = \chi SU(3)) \)
**Results**

- most results qualitatively unchanged
- only exception: $\chi_{SU(3)} \rightarrow 2SC \rightarrow CFL$
- in this case:
  - modest increase of the energy density at $H \rightarrow 2SC$, strong increase at $2SC \rightarrow CFL$
- TOV: stable $2SC$ core, unstable $CFL$ core
Conclusions

- NJL-model study of quark-matter cores in compact stars:
  - three \(\langle qq \rangle\) and three \(\langle \bar{q}q \rangle\) condensates under the constraints imposed by electric and color neutrality
  - 2 quark \(\times\) 4 hadronic EOS w/ and w/o diquark condensation:
    - only one case with stable pure quark matter core
  - stable case: 2SC phase with very few strange quarks
  - no stable CFL-matter core
- These results can at best be strong hints because the model parameters fixed in vacuum may be completely off at high densities.
- However, they
  - provide a counter example to the “model independent” prediction of absence of the 2SC phase in compact stars.
  - demonstrate the possible importance of \(\mu\)-dependent constituent masses and gaps and their interplay.