COOLING OF NEUTRON STARS WITH COLOR SUPERCONDUCTING QUARK CORES

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Collaboration:
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- EoS and QCD Phase Transition
- Stability of Hybrid Stars
- Cooling of Hadronic Stars
- Hybrid Star Cooling: 2SC + X Quark Matter
- Perspectives - a Conjecture

Picture taken from
http://www.astroscu.unam.mx/neutrones/NS-Picture/

David Blaschke
Seattle, June 2004
**INTRODUCTION**

**QCD Phase Diagram**

- $T_c \sim 170 \text{ MeV}$
- $T = 0, \mu = 0$
  - $\langle \bar{\psi} \psi \rangle \neq 0$ hadron phase
  - Chiral symmetry is broken.
- $T \to \infty, \mu \to \infty$
  - $\langle \bar{\psi} \psi \rangle = 0$ symmetry phase
  - All symmetries are restored.

**Moderate $T$ and $\mu$?**

- Heavy-ion collisions / compact stars interiors

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*S. Hands, The phase diagram of the QCD* physics/0105022 (2001)

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Motivation

Even at weak coupling, any attractive quark-quark interaction

\[ \downarrow \]

condensate of Cooper pairs

\[ \downarrow \]

Color superconductivity

To explore dense matter EoS \((T, \mu)\)

\[ \uparrow \]

To build a stable *realistic* neutron star

- with diquark condensates?
- with strangeness?

- with quark core - Quark matter EoS
- with hadron shell - Nuclear matter EoS
QUARK MATTER EoS MODEL

Mass gap $\phi_f$ and diquark gap $\Delta_f$ are order parameters for $\chi$SB and CSC $\beta$-equilibrium for the two flavor and three color case ($N_f = 2$, $u, d$, $N_c = 3$, $r, b, g$) requires different chemical potentials: $\mu_q = (\mu_u + \mu_d)/2$ and $\mu_I = (\mu_u - \mu_d)/2$.

$$\Omega(\phi, \Delta; \mu_q, \mu_I, T) = \frac{\phi^2}{4G_1} + \frac{\Delta^2}{4G_2} - \frac{2}{2\pi^2} \int_0^\infty dq q^2 (N_c - 2) \{2E_\phi

+ T \ln \left[ 1 + \exp \left( -\frac{E_\phi - \mu_q - \mu_I}{T} \right) \right] + T \ln \left[ 1 + \exp \left( -\frac{E_\phi - \mu_q + \mu_I}{T} \right) \right]

+ T \ln \left[ 1 + \exp \left( -\frac{E_\phi + \mu_q - \mu_I}{T} \right) \right] + T \ln \left[ 1 + \exp \left( -\frac{E_\phi + \mu_q + \mu_I}{T} \right) \right] \}$$

$$- \frac{4}{2\pi^2} \int_0^\infty dq q^2 \{ E_+ + E_- \}

+ T \ln \left[ 1 + \exp \left( -\frac{E^-_\phi - \mu_I}{T} \right) \right] + T \ln \left[ 1 + \exp \left( -\frac{E^-_\phi + \mu_I}{T} \right) \right]

+ T \ln \left[ 1 + \exp \left( -\frac{E^+_\phi - \mu_I}{T} \right) \right] + T \ln \left[ 1 + \exp \left( -\frac{E^+_\phi + \mu_I}{T} \right) \right] \} - \Omega_{vac}$$

The dispersion relations:

$$E_\phi^2 = q^2 + (m + g(q)\phi)^2$$

for quarks of unpaired color

$$E_\phi^{\pm 2} = (E_\phi \pm \mu)^2 + g(\bar{q})\Delta^2$$

for quarks of paired colors

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**Form Factors**

The following cases are considered: Gaussian ($g_G$), Lorentzian ($g_L$) and NJL cutoff ($g_{NJL}$).

<table>
<thead>
<tr>
<th>Form Factor</th>
<th>Expression</th>
<th>$\Lambda$ [GeV]</th>
<th>$G_1 \Lambda^2$</th>
<th>$m$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>$g_G^2(q) = \exp\left(-q^2/\Lambda_G^2\right)$</td>
<td></td>
<td>$g_G$</td>
<td>1.025</td>
</tr>
<tr>
<td>Lorentzian</td>
<td>$g_L^2(q) = \frac{1}{\Lambda^2 + (q/\Lambda_L)^4}$</td>
<td></td>
<td>$g_L$</td>
<td>0.894</td>
</tr>
<tr>
<td>NJL</td>
<td>$g_{NJL}^2(q) = \Theta(1 - q/\Lambda_{NJL})$</td>
<td></td>
<td>$g_{NJL}$</td>
<td>0.900</td>
</tr>
</tbody>
</table>

The parameters $\Lambda$, $G_1$, $G_2$ and $m$ are fixed at $T = \mu = 0$ by $m_{\pi} = 140$ MeV, $f_{\pi} = 93$ MeV, $m_q(0) = 330$ MeV and by choosing $G_1/G_2 = 4/3$.


**Quark Matter in Compact Star Cores**

The quark matter core of compact stars consists of $u$, $d$, $e$, $\nu_e$, $\bar{\nu}_e$ under the conditions:

- **$\beta$-equilibrium**
  
  \[
  d \longleftrightarrow u + e^- + \bar{\nu}_e \quad \mu_d = \mu_u + \mu_e + \mu_{\bar{\nu}_e} \]
  
  \[
  \mu_e + \mu_{\bar{\nu}_e} = -2\mu_I
  \]

- **Charge neutrality**
  
  \[
  \frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0
  \]
  
  \[
  n_q + 3n_I - 6n_e = 0
  \]
**Thermodynamical Potential, Gap Equations**

The extrema of $\Omega$ correspond to **Gap equations** for the order parameters $\phi_0, \Delta_0$:

$$\left. \frac{\partial \Omega}{\partial \phi} \right|_{\phi=\phi_0; \Delta=\Delta_0} = \left. \frac{\partial \Omega}{\partial \Delta} \right|_{\phi=\phi_0; \Delta=\Delta_0} = 0$$

As $T$ or $\mu$ changes, $\Omega$ can have several local minima in the $\phi, \Delta$ plane. The **lowest minimum** describes the lowest free energy state and is preferred. As an example, $\Omega(T, \mu)$ at $T = 0$, $\mu = 0.292$ GeV is plotted. Two degenerate minima can coexist at the values: $\phi = 0.4$ GeV, $\Delta = 0$ and $\phi = 0$, $\Delta = 0.072$ GeV. This corresponds to a **first order transition** at which two phases with equal pressure can coexist.

$$\Omega(\phi_0, \Delta_0; \mu_q, \mu_I, T) = \epsilon - TS - \mu_q n_q - \mu_I n_I = -P, \quad n_i = \left. \frac{\partial \Omega}{\partial \mu_i} \right|_{T=\phi=\phi_0; \Delta=\Delta_0}$$
The diquark condensate does not feel the difference in partial or complete restoration of the chiral symmetry.

At low temperatures color superconductivity at \( \mu \) immediately above the first order phase transition. Color superconductivity persists up to \( T_c^\Delta = 45 \text{ MeV} \).

The onset of the diquark condensation is found at higher \( \mu_q \) if the temperature is increased.

BCS theory predicts that \( T_c^\Delta = 0.57 F(\mu)^2 \Delta_0(T = 0) \).

\[ m_q = 0 \text{ (left) and } m_q = 2.41 \text{ MeV, (right)} \]
The maximum of the diquark condensate ($\approx 150$ MeV) roughly form factor independent.

The critical values of the phase transition temperature and chemical potential depend on smoothness of form factors.

At $T = 0$ the critical chemical potentials are: $\mu_{qc}^G = 320$ MeV, $\mu_{qc}^L = 345$ MeV and $\mu_{qc}^{N\,JL} = 360$ MeV.

**EOS for Quark Star Matter**

**Temperature and Antineutrino Effects**

\[ P + \epsilon = sT + \mu_q n_q + \mu_I n_I, \quad n_q = n_u + n_d, \]
\[ n_I = n_u - n_d \]

**Temperature Effect**

- \( T = 0 \)
- \( T = 40 \text{ MeV} \)
- \( T = 50 \text{ MeV} \)
- \( T = 20 \text{ MeV} \)

**Antineutrino Effect**

- \( \mu_{\bar{\nu}_e} = 0 \)
- \( \mu_{\bar{\nu}_e} = 100 \text{ MeV} \)
- \( \mu_{\bar{\nu}_e} = 200 \text{ MeV} \)

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The integral parameters $M$ and $R$ feel the existence of the diquark condensation: the EoS becomes softer. At higher temperature both $M_{\text{max}}$ and $R_{\text{max}}$ decrease approximately 10% compared to $T = 0$ and $\Delta = 0$ cases.

Tolman-Oppenheimer-Volkoff equations

$$\frac{dP}{dr} = -\frac{[\epsilon(r) + P(r)][m(r) + 4\pi r^3 P(r)]}{r[r - 2m(r)]}$$

$$m = 4\pi \int_0^r r^2 \epsilon(r) dr$$

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During the cooling evolution the star must lose an amount of energy: the energy release will lead to an equivalent mass defect. We estimated it to about $0.1 \, M_\odot$.

- It accidentally corresponds to the simple estimate (Hong et al., PLB 516 (2001) 362) $(\Delta/\mu)^2 \, M_\odot \sim 10^{52} \, \text{erg}$
**PNS Evolution with Neutrino Trapping**

- **Mass defect** $\Delta M \approx 0.05 \div 0.4 \, M_\odot$
  $\Rightarrow 10^{53} \div 10^{54} \, \text{erg}$
- Effect due to Un-Trapping of (Anti-)Neutrinos
- Explosion possible: Phase transition $1^{st}$ Order
- GRB with time delay $\Rightarrow$ Cannon Ball Model

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**Phase Transition: Nuclear Matter → Quark Matter**

- Walecka model \((p,n,e)+(\sigma,\omega)\) → Bag model \((u,d,e)\)
- Maxwell construction: \(P_{\text{Had}} = P_{\text{Quark}}\)
  \[\mu_{\text{B}}^{\text{Had}} = \mu_{\text{B}}^{\text{Quark}}\]

EoS at \(T = 0\) \(B^{1/4}\) [MeV]

\[
P[\text{MeV/fm}^3] \quad \varepsilon[\text{MeV/fm}^3]
\]

- \(B^{1/4} = 220\)
- \(B^{1/4} = 200\)
- \(B^{1/4} = 190\)
- \(B^{1/4} = 180\)

\[
\mu = 1472.5 \text{ MeV}
\]

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STABILITY OF HADRONIC AND HYBRID STARS

**Gaussian model**
- RMF
- RMF - SM_{G}^{(S)}
- RMF - SM_{G}^{(N)}
- RX J185635-3754
- EXO 0748-676
- Quark core - SM_{G}^{(S)}
- Quark core - SM_{G}^{(N)}

**Lorentzian model**
- RMF
- RMF - SM_{L}^{(S)}
- RMF - SM_{L}^{(N)}
- RX J185635-3754
- EXO 0748-676
- Quark core - SM_{L}^{(S)}
- Quark core - SM_{L}^{(N)}

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**Stability of Rotating Hybrid Stars**

- **Mass - Radius relation**
  HHJ - Gaussian formfactor model

- **Phase diagram for rotating stars**
  Springer LNP 578 “Phys. of NS Int.”

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**Figure 1:**
- Mass-Radius relation
  - HHJ-Gaussian formfactor model

**Figure 2:**
- Phase diagram for rotating stars
  - Configurations: Hadronic, Quark Core, Black Holes
  - Lines: RSt line (normal), RSt line (2SC), phTC line (normal), phTC line (2SC), BH line (normal), BH line (2SC)

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**Citation:**
S. DAVID BLASCHKE
SEATTLE, JUNE 2004
Evolution of the surface temperature $T_s$ of a hadronic star

Data:
Yakovlev et al.,

Calculation:
D.B., Grigorian, Voskresensky,
astro-ph/0403170
**The nuclear URCA Process**

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

Neutrinos leave the star \((\lambda_\nu \gg R)\): \(\mu_\nu = 0\)

\(-\)Equilibrium: \(\mu_n = \mu_p + \mu_e\)

Momentum conservation: \(\vec{p}_{F,n} = \vec{p}_{F,p} + \vec{p}_{F,e} \iff |\vec{p}_{F,n}| \leq |\vec{p}_{F,p}| + |\vec{p}_{F,e}|\)

Charge neutrality: \(n_p - n_e = 0 \iff \frac{p_{F,p}^3}{3\pi^2} = \frac{p_{F,e}^3}{3\pi^2}\)

Triangle inequality: \(p_{F,n} \leq 2p_{F,p} \Rightarrow n_n \leq 8 n_p \Rightarrow \frac{n_p}{n_p + n_n} = x_p \geq \frac{1}{9}\)

Luminosity: \(L_\nu = (2\pi)^4 \int \frac{d^3p_n}{(2\pi)^3 2E_n} \cdots \int \frac{d^3p_{\nu}}{(2\pi)^3 2E_{\nu}} \delta^3(\vec{p}_i) \delta(E_i) \ |M_{fi}|^2 f_n(1 - f_p)(1 - f_e)\)

Emissivity: \(\epsilon_\nu = \frac{L_\nu}{V} \sim 10^{27} \left(\frac{m_n^*}{m_N^*}\right) \left(\frac{n_e}{n_0}\right)^{1/3} \left(\frac{T}{10^9 K}\right)^6 \frac{\text{erg}}{\text{cm}^3 \text{s}}\)

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**HADRON MATTER: DU CRITICAL DENSITIES AND CRITICAL MASSES OF STARS**

**DU critical densities**

\[ n_c = 2.7 \, n_0 \] non linear Walecka (NLW)

\[ n_c = 5.0 \, n_0 \] AV 18 + \( \delta v \) (HHJ)

**DU critical masses**

\[ M_c = 1.25 \, M_\odot \] - NLW

\[ M_c = 1.84 \, M_\odot \] - HHJ

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D A V I D   B L A S C H K E

S E A T T L E , J U N E  2 0 0 4
The energy flux per unit time $l(r)$ through a spherical slice at distance $r$ from the center is:

$$l(r) = -4\pi r^2 k(r) \frac{\partial (Te^\Phi)}{\partial r} e^{-\Phi} \sqrt{1 - \frac{2M}{r}}.$$  

The factor $e^{-\Phi} \sqrt{1 - \frac{2M}{r}}$ corresponds to relativistic corrections of time and distance scales.

The equations for energy balance and thermal energy transport are:

$$\frac{\partial}{\partial N_B}(le^{2\Phi}) = -\frac{1}{n}(\epsilon_\nu e^{2\Phi} + c_V \frac{\partial}{\partial t}(Te^\Phi))$$

$$\frac{\partial}{\partial N_B}(Te^{2\Phi}) = -\frac{1}{k} \frac{le^\Phi}{16\pi^2 r^4 n}$$

where $n = n(r)$ is the baryon number density, $N_B = N_B(r)$ is the total baryon number in the sphere with radius $r$ and

$$\frac{\partial N_B}{\partial r} = 4\pi r^2 n(1 - \frac{2M}{r})^{-1/2}$$

COOLING CURVES: HADRONS (NORMAL) STARS

Our crust ($T_s - T_{in}$)

$\pi$ -condensate and MMU

D.B., Grigorian, Voskresensky, astro-ph/0403170

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SEATTLE, JUNE 2004
Neutron and Proton pairing gaps

Crust model

Yakovlev, Gnedin, Kaminker, Levenfish, Potekhin, astro-ph/0306143;
Takatsuka, Tamagaki, nucl-th/0402011.

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**Cooling curves: Hadronic (superconducting) stars**

- with AV18 gaps, $\pi$-condensate, MMU ($3P_2$-suppressed)
- with Gaps from Yakovlev at al. 2003

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Cooling curves: Hadronic (superconducting - anomalous) stars

- with AV18 gaps, $\pi$ -condensate, MMU ($3P_2$ - is NOT suppressed)
- with Gaps from Yakovlev at al. 2003

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Bag Model Fit and Hybrid Star Configuration

- Mass - Radius relation
  HHJ - Gaussian formfactor model

- Bag model Fit for Gaussian SM EoS
  H. Grigorian et al., PRC 69 (2004)

![Graph showing mass-radius relation for HHJ model with and without 2SC](image1)

![Graph showing bag model fit for Gaussian SM EoS](image2)
**Neutrino processes in quark matter: Emissivities**

- **Quark direct Urca (QDU)** the most efficient processes
  \[ d \rightarrow u + e + \bar{\nu} \] and \[ u + e \rightarrow d + \nu \]
  \[ \epsilon_{\nu}^{\text{QDU}} \simeq 9.4 \times 10^{26} \alpha_s Y_e^{1/3} \zeta_{\text{QDU}} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1}, \]
  Compression \( u = n/n_0 \simeq 2 \), strong coupling \( \alpha_s \simeq 1 \)

- **Quark Modified Urca (QMU) and Quark Bremsstrahlung (QB)**
  \[ d + q \rightarrow u + q + e + \bar{\nu} \] and \[ q_1 + q_2 \rightarrow q_1 + q_2 + \nu + \bar{\nu} \]
  \[ \epsilon_{\nu}^{\text{QMU}} \sim \epsilon_{\nu}^{\text{QB}} \simeq 9.0 \times 10^{19} \zeta_{\text{QMU}} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}. \]

- **Suppression due to the pairing**
  \[ \text{QDU}: \zeta_{\text{QDU}} \sim \exp\left(-\Delta_q/T\right) \]
  \[ \text{QMU and QB}: \zeta_{\text{QMU}} \sim \exp\left(-2\Delta_q/T\right) \text{ for } T < T_{\text{crit},q} \simeq 0.4 \Delta_q \]

- **\( e+ e \rightarrow e+ e+ \nu+ \bar{\nu} \)**
  \[ \epsilon_{\nu}^{ee} = 2.8 \times 10^{12} Y_e^{1/3} u^{1/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}, \]
  becomes important for \( \Delta_q/T >> 1 \)


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EVOLUTION OF THE TEMPERATURE PROFILE

Temperature profiles in MeV

Δ = 50 MeV

Δ = 0.1 MeV

Δ = 0

Radius in km

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COOLING CURVES: HYBRID STARS WITH 2SC QUARK MATTER

- 2SC phase - 1 color is unpaired (mixed superconductivity)
- 2SC + X, $\Delta X = 1$ MeV

D.B., Grigorian, Voskresensky, astro-ph/0403171

D. BLASCHKE
SEATTLE, JUNE 2004
Cooling Curves: Hybrid Stars with 2SC Quark Matter

- $2SC + X$ phase, $\Delta X = 100$ keV
- $2SC + X$ phase, $\Delta X = 30$ keV

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**Conjecture about Magnetized Quark Stars: Neutrino Beaming and GRB**

- Magnetic vortex structure
  Characteristic length scales: $\xi, \lambda, \lambda_D$

  \[
  N_{\text{vort}} \Phi'_q = B_{\text{in},s} \pi R^2, \quad \Phi'_q \simeq 6 \Phi_0,
  \]

- Channeling of neutrino propagation


\[
\xi < \lambda < \lambda_D
\]
Conjecture about Magnetized Quark Stars: Neutrino Beaming and GRB

- Beaming angle as function of $T$ and $B$

$$\theta_\nu \sim \arcsin \frac{\tilde{\lambda}_\nu}{R}, \quad \tilde{\lambda}_\nu = \lambda_\nu \left(\frac{V}{N_{V_0} V_{V_0}}\right)^\alpha$$

- Evolution of luminosity, temperature, $\theta_\nu$

THEORY OF EoS WITH CONSTRAINTS

- Nuclear Matter - 2SC Quark Matter Phase transition depends on details of the modeling, e.g. Coupling Constants, Form Factors \( \Rightarrow \) Improve by using DSE Models
- 2SC Quark Matter Core is possible for NLW - Gaussian separable model
- Rôle of CFL phase at higher densities?

EXPERIMENTAL OBSERVABLES

- Late time Cooling:
  \( \Rightarrow \) Onset of Enhanced Cooling depends on Size of Quark Core, i.e. Mass/Spin of the Star
  \( \Rightarrow \) Coupling of Rotational & Cooling evolution (in progress)

- Early Evolution (PNS):
  \( \Rightarrow \) Explosive release of Binding energy is possible in antineutrino untrapping
  \( \Rightarrow \) Second pulse of \( \bar{\nu} \) from supernovae
  \( \Rightarrow \) Anisotropic Emission (magnetic vortices) may explain GRB - beaming