Elliptic Flow from Recombination and Fragmentation in Heavy Ion Collisions

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    Centrality dependence of hadron spectra
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- Summary
Experimental Data (1)

**Au+Au at sqrt(s_{NN})=200GeV**

V2 

\[ \left| \eta \right| = 3 \sim 4 \]

\( v_2 \) min. bias r.p. 

- Positives: \( h^+, \pi^+, K^+, p \)
- Saturation in \( v_2 \) of baryon occurs at higher \( p_T \) than one of meson.

\[ \frac{dN}{d\varphi} \approx v_0 (1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi)) \]

\( dN \) vs. \( p_T \) for different particles:

- \( K_S^0 \)
- \( \Lambda + \bar{\Lambda} \)
- \( h^+ + h^- \)

**Hydro calculations**

- \( \pi \)
- \( K \)
- \( p \)
- \( \Lambda \)

**At Low\( p_T \)**

Hydro works well

\( p_T \) (\( \text{GeV/c} \))

\( \text{Parameter } v_2 \)

\text{Transverse Momentum } p_T \ (\text{GeV/c})

- **PHENIX Preliminary**

- **PHENIX** : nucl-ex/0210012

- **STAR Preliminary (Au+Au; 200 GeV; |y|<1.0)**
Proton Puzzle at RHIC

\[ \frac{p}{\pi} \text{ ratio } \sim 1 \ (P_T > 2 \text{ GeV}) \text{ at central collisions} \]

Why is it interesting?
Proton Puzzle

Hadronization from Fragmentation At high $P_T$

Fast parton in the vacuum with a color string: $a \rightarrow h + X$

$$E \frac{dN_h}{d^3P} = \int_0^1 \frac{dz}{z^2} \frac{E}{z} \frac{dN_a}{d^3(P/z)} D_{a \rightarrow h}(z)$$

- $p/\pi << 1$
- Energy loss should affect pions and protons in the same way.

RHIC Data

ratio of KKP fragmentation functions for $p$ and $\pi$ from $u$ quarks
• Suppression in $R_{AA}$ of baryon occurs at higher $P_T$ than one of meson.

• There are some correlations between $R_{AA}$ and $v_2$.

There is the hadronization mechanism which describes these experimental data.

Recombination + fragmentation
Model $P_T > 2$ GeV
Recombination + Fragmentation Model

- **Recombination at low $p_T$**
  - Recombination occurs at an instant
  - The parton spectrum is shifted to higher $p_T$ in the hadron spectrum.

  \[ q\bar{q} \rightarrow M \quad qqq \rightarrow B \]

- **Fragmentation at high $p_T$**
  - The parton spectrum has thermal part (quarks) and a power law tail (quarks and gluons) from pQCD.
  - The parton spectrum is shifted to lower $p_T$ in the hadron spectrum.
Recombination – a non-relativistic model (1)

Quarks and antiquarks in volume V

\[
\langle x \mid q, p_1 p_2 \rangle = V^{-1} e^{i(p_1x_1 + p_2x_2)}
\]

\[
\langle x \mid M, \mathbf{P} \rangle = V^{-1} e^{i(PR)} \Phi_M (y)
\]

\[
R = \left( x_1 + x_2 \right)/2, y = (x_1 - x_2)
\]

\[
\langle q, p_1 p_2 \mid M, \mathbf{P} \rangle = \frac{(2\pi)^3}{V^{3/2}} \delta^3(\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2) \Phi_\pi (r)
\]

\[
q = (p_1 + p_2)/2, r = (p_1 - p_2)
\]

Total number of mesons

\[
N_\pi = \sum_{a,b} V^3 \int \frac{d^3q}{(2\pi)^3} \frac{d^3r}{(2\pi)^3} \frac{d^3P}{(2\pi)^3} w_a (p_1) w_b (p_2) \langle q, p_1 p_2 \mid M, \mathbf{P} \rangle^2
\]

Degeneracy factor : \(C_M\)

Quark distribution

Non-relativistic wave function

\[
\Phi_M (r) = N_M e^{-r^2/2\Lambda_M^2}
\]
Recombination-a nonrelativistic model (2)

- Use thermal quark spectrum: \( w(p) = \exp(-p/T) \)
- for a Gaussian meson wave function with momentum width \( \Lambda_M \)
- Meson spectrum

\[
\frac{dN_M}{d^3P} = C_M \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} w\left(\frac{1}{2}P - q\right) w\left(\frac{1}{2}P + q\right) |\hat{\phi}_M(q)|^2
\]

\[
= C_M \frac{V}{(2\pi)^3} \left[ w\left(\frac{1}{2}P\right) \right]^2 \left( 1 - \frac{2\Lambda_M^2}{TP} + \ldots \right)
\]

- Baryon spectrum

\[
\frac{dN_B}{d^3P} = C_B \frac{V}{(2\pi)^3} \left[ w\left(\frac{1}{3}P\right) \right]^3 \left( 1 - O(\Lambda_B^2 / TP) \right)
\]
Input for Quantitative Calculation

Recombination

\[ E \frac{dN_{\text{rec}}^{B.M}}{P_T dP_T} = C_{B,M} \frac{\tau P_T A_T}{(2\pi)^2} \int dy d\eta \cosh(\eta - y) w_a^n(\sigma, P/n) \]

Spectrum of parton

\[ w_a(\sigma, P) = \gamma_a e^{-P\cdot v(\sigma)/T} e^{-\eta^2/2\Delta^2} f(\rho, \phi) \]

\( \gamma_a \): fugacity factor, \( \Delta \): rapidity width

\( f(\rho, \phi) \): transverse distribution

T=175 MeV, \( \tau=5 \) fm, \( v_T=0.55 \)

Fragmentation

\[ E \frac{dN_h}{d^3P} = \sum_a \int_0^1 \frac{dz}{z^2} D_{a\rightarrow h}(z) E_a \frac{dN_a}{d^3P_a} \]

Spectrum of parton

\[ \frac{dN_a^{\text{pert}}}{d^2P_T dy_{y=0}} = K \frac{C}{(1 + P_T / B)^\beta} \]

• \( K=1.5 \) : roughly account for higher order corrections

• \( C, B, \beta \) are taken from a leading order pQCD calculation

Energy Loss:

\[ \Delta P_T(P_T) = \varepsilon(0) \sqrt{P_T} \quad \varepsilon(0) = 0.82 \text{ GeV}^{1/2} \]
Comparison with Experimental Data I

Hadron spectra,
Hadron ratio,
Central dependence of hadron spectra
Hadron Spectra I

\[ \frac{1}{2\pi P_T} dN/dP_T \text{ (GeV)}^2 \]

- **π^0** PHENIX 0-10% cent.
  - Recombination (R)
  - Fragmentation (F)
  - Reco+Frags (R+F)

- **π^+** PHENIX 0-5% cent.
- **π^-** PHENIX 0-5% central

- **K^0_s** STAR 0-5% cent.
- **K^+** PHENIX 0-5% cent.
- **K^-** PHENIX 0-5% cent.
  - K STAR ??? cent.

\[ P_T \text{ (GeV)} \]

2 4 6 8 10 2 4 6 8 12 2 4 6 8 10 12
Hadron Spectra II

\[ \frac{1}{2\pi P_T} \frac{dN}{dP_T} \text{ (GeV}^2) \]

- p PHENIX 0-5% cent.
- pbar PHENIX 0-5% cent.
- \( \Phi \) Reco

- \( \Lambda+\bar{\Lambda} \) STAR 0-5% cent.
- \( \Xi+\bar{\Xi} \) STAR 0-5% cent. (a.u.)
- \( \Omega+\bar{\Omega} \) STAR min. bias
  - \( \Omega+\bar{\Omega} \) R b=0 fm
  - \( \Omega+\bar{\Omega} \) R b=10 fm

\[ P_T \text{ (GeV)} \]

2 4 6 8 10

2 4 6 8 10 12

2 4 6 8 10 12
Hadron Ratios vs. $p_t$
Centrality dependence

### Fragmentation

\[ dN^\text{pert}_a(b) = \frac{T_{\text{AuAu}}(b)}{T_{\text{AuAu}}(0)} dN^\text{pert}_a = \frac{N_{\text{coll}}(b)}{N_{\text{coll}}(0)} dN^\text{pert}_a \]

The values of \( N_{\text{coll}}(b) \) are given by PHENIX collaboration.

### Energy loss

\[ \Delta p_T(b, p_T) = \varepsilon(b) \sqrt{p_T} \frac{\langle L \rangle}{R_A} \]

\( \langle L \rangle \): average length, \( \varepsilon(b) = \frac{1 - e^{-(2R_A-b)/R_A}}{1 - e^{-2}} \)

### Recombination

Transverse area of the overlap zone

\[ A_T(b) = \frac{l(b)w(b)\pi}{R_A^2} A_T(0) \]
Centrality dependence

- R+F model reproduces the centrality dependence of $\pi$ and $p/\pi$ ratio.
High-\(p_t\) Suppression

- \(R_{AA}\) as a function of \(P_T\) is determined by contribution of recombination and Fragmentation.
- We can reproduce \(R_{CP}\) of proton and pion as a function of \(P_T\).
Comparison with Experimental data II

Elliptic Flow
Elliptic Flow (1)

- Elliptic flow is sensitive to initial geometry
  - At low $P_T$
    - Pressure gradient
    - Collision plane > Perpendicular plane
  - At high $P_T$
    - Energy loss
    - Perpendicular plane > Collision plane

- Total elliptic flow
  \[
  v_2(p_t) = r(p_t)v_2^{\text{recomb}}(p_t) + \left(1 - r(p_t)\right)v_2^{\text{frag}}(p_t)
  \]
  
  $r(p_t)$: relative weight of the fragmentation contribution in spectra
Elliptic Flow (2) : partons at low $p_t$

azimuthal anisotropy of parton spectra is determined by elliptic flow:

$$\frac{d^2N}{p_t \, dp_t \, d\phi_p} = \frac{1}{2\pi} \left[ \frac{dN}{p_t \, dp_t} \right] \left( 1 + 2v_2 \cos(2\phi_p) \right) \quad (\phi_p: \text{azimuthal angle in } p\text{-space})$$

with Blastwave parametrization for parton spectra:

$$v_2(p_t) = \langle \cos(2\phi_p) \rangle = \frac{1}{\int_0^{2\pi} d\phi_s \cos(2\phi_s) I_2 \left( \frac{p_t \sinh(\rho(\phi_s))}{T} \right) K_1 \left( \frac{m_t \cosh(\rho(\phi_s))}{T} \right)}$$

azimuthal anisotropy is parameterized in coordinate space and is damped as a function of $p_t$:

$$\rho(\phi_s) = \frac{1}{2} \ln \left( \frac{1 + \beta_t}{1 - \beta_t} \right) \left( 1 + \alpha_p(p_t) \cos(2\phi_s) \right) \quad \text{and} \quad \alpha_p(p_t) = -\alpha_0 \frac{1}{1 + (p_t / p_0)^2}$$
Elliptic Flow (3) : partons at high $p_t$

Azimuthal anisotropy is driven by parton energy/momentum loss $\Delta p_t$

$$\Delta p_t = \eta \left(1 - \exp\left[-\frac{2R_A - b}{R_A}\right]\right) \sqrt{p_t} \bar{L} \left(1 + \alpha (p_t) \cos(2\phi_s)\right)$$

- $L$: average thickness of the medium
- $\alpha(p_t)$: damping function for low $p_t$

The unquenched parton $p_t$ distribution is defined according to:

$$\frac{d^2N}{p_t \, dp_t \, d\phi} = \frac{1}{2\pi^2 R^2} \int_0^R d^2r \frac{dN (p_t - \Delta p_t (\phi))}{p_t \, dp_t}$$

$v_2$ is then calculated via:  
$$v_2 \left( p_t \right) = \langle \cos(2\phi_p) \rangle$$
Parton Number Scaling of Elliptic Flow

From v2 of Parton

- At low $P_T$ (recombination)
  - meson and baryon $v_2$ can be obtained from the parton $v_2$

\[
v_2^M (p_t) = \frac{2v_2^p \left( \frac{p_t}{2} \right)}{1 + 2 \left( v_2^p \left( \frac{p_t}{2} \right) \right)^2} \quad \text{and} \quad v_2^B (p_t) = \frac{3v_2^p \left( \frac{p_t}{3} \right) + 3 \left( v_2^p \left( \frac{p_t}{3} \right) \right)^3}{1 + 6 \left( v_2^p \left( \frac{p_t}{3} \right) \right)^2}
\]

- neglecting quadratic and cubic terms, one finds a simple scaling law:

\[
v_2^M (p_t) = 2v_2^p \left( \frac{p_t}{2} \right) \quad \text{and} \quad v_2^B (p_t) = 3v_2^p \left( \frac{p_t}{3} \right)
\]

v2 of Baryon and meson
Elliptic Flow (4)

Input: Parton → Output: Hadron

\[ v_2(p_t) = r(p_t) v_{2\text{recomb}}(p_t) + \left(1 - r(p_t)\right) v_{2\text{frag}}(p_t) \]

- Gray region shows the uncertainty of limiting of R and F.

Parton elliptic flow

Relative weight of recombination

- Gray region shows the uncertainty of limiting of R and F.
Recombination only

- Recombination describes the flavor dependence.
- This behavior is consistent with experimental data.
• Pt~6 GeV, There are deviation between p and pion.
• At high Pt, all hadron merges, because energy loss does not have the flavor dependence.
• At high Pt, parton number scaling breaks down.
• We need the experimental data at high Pt.
Summary

- We can reproduce the experimental data.
  - Hadron spectra
  - Hadron ratio
  - Centrality dependence
  - High $P_T$ suppression of $R_{AA}$ and $R_{CP}$
  - Elliptic Flow

Recombination + Fragmentation Model provides the solution of understanding RHIC data!

Hydro, thermal model

Future plan
  - $d+Au$ Collisions
Back up
Recombination – a relativistic model

- Hadronization occurs on hypersurface $\Sigma$
- Use local light cone coordinates (hadron defining the + axis)
- $w_a(r,p)$: single particle Wigner function for quarks at hadronization
- $\Phi_M$ & $\Phi_B$: light-cone wave-functions for the meson & baryon respectively
- $x$, $x'$ & $(1-x)$: momentum fractions carried by the quarks
- Integrating out transverse degrees of freedom yields:

$$E \frac{dN_M}{d^3P} = \sum \int d\sigma \frac{P \cdot u}{(2\pi)^3} \sum_{\alpha,\beta} \int dx w_\alpha(R, xP^+) \bar{w}_\beta(R, (1-x)P^+) \left| \bar{\phi}_M(x) \right|^2$$

$$E \frac{dN_B}{d^3p} = \sum \int d\sigma \frac{P \cdot u}{(2\pi)^3} \sum_{\alpha,\beta,\gamma} \int dx dx' w_\alpha(R, xP^+) w_\beta(R, x' P^+) w_\gamma(R, (1-x-x')P^+) \left| \bar{\phi}_B(x, x') \right|^2$$
Parton Number Scaling of $v_2$

- In leading order of $v_2$, recombination predicts:

$$v_2^M (p_t) = 2v_2^p \left( \frac{p_t}{2} \right)$$

$$v_2^B (p_t) = 3v_2^p \left( \frac{p_t}{3} \right)$$

P. Soerensen, UCLA & STAR @ SQM2003