Pion Phase Space Density and Entropy in RHIC Collisions

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The Lorentz scalar phase space density \( \langle f(m_T) \rangle \) is the dimensionless average number of pions per 6-dimensional phase space cell \( h^3 \). At midrapidity \( \langle f \rangle \) is given by the expression:

\[
\langle f(m_T) \rangle = \frac{1}{E_p} \frac{1}{\sqrt{?}} \left[ \frac{d^2N}{2p m_T dm_T dy} \right] \left[ \frac{?(\hbar c \sqrt{p})^3}{R_S R_O R_L} \right]
\]

- **Jacobian**
- **Purity**
- **Momentum Spectrum**
- **HBT “volume”**
We can do a global fit of the *uncorrected* pion spectrum vs. centrality by:

1. Assuming that the spectrum has the form of a Bose-Einstein distribution:
   \[ \frac{d^2N}{m_T dm_T dy} = \frac{A}{\exp(E/T) - 1} \]
   and

2. Assuming that $A$ and $T$ have a quadratic dependence on the number of participants $\nu$:
   \[
   A(p) = A_0 + A_1 \nu + A_2 \nu^2 \\
   T(p) = T_0 + T_1 \nu + T_2 \nu^2
   \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>31.1292</td>
</tr>
<tr>
<td>$A_1$</td>
<td>21.9724</td>
</tr>
<tr>
<td>$A_2$</td>
<td>-0.019353</td>
</tr>
<tr>
<td>$T_0$</td>
<td>0.199336</td>
</tr>
<tr>
<td>$T_1$</td>
<td>-9.23515E-06</td>
</tr>
<tr>
<td>$T_2$</td>
<td>2.10545E-07</td>
</tr>
</tbody>
</table>
**The Hanbury-Brown-Twiss Effect**

Coherent interference between incoherent sources!

For non-interacting identical bosons:

\[ A_{12} = \frac{1}{\sqrt{2}}[e^{i\mathbf{p}_1 \cdot (\mathbf{r}_1 - \mathbf{x})}e^{i\mathbf{p}_2 \cdot (\mathbf{r}_2 - \mathbf{y})} + e^{i\mathbf{p}_1 \cdot (\mathbf{r}_1 - \mathbf{y})}e^{i\mathbf{p}_2 \cdot (\mathbf{r}_2 - \mathbf{x})}] \]

so that

\[ \mathcal{P}_{12} = \int d^4x \, d^4y \, |A_{12}|^2 \rho(x) \rho(y) = 1 + |\bar{\rho}(q)|^2 \equiv C_2(q) \]

The “bump” results from the Bose-Einstein statistics of identical pions \( J^\pi = 0^- \).

Width of the bump in the \( i \)th momentum direction is proportional to \( 1/R_i \).
$R_{\text{out}}$ and $R_{\text{side}}$ are energy independent within error bars.

Smooth energy dependence in $R_{\text{long}}$

No immediate indication of very different physics

Fit $R_{\text{long}}$ to:

$$A = \frac{\tau_0}{\sqrt{m_T}}$$

AGS:  $A = 2.19 \pm .05$

SPS:   $A = 2.90 \pm .10$

RHIC:  $A = 3.32 \pm .03$

$A = \tau_0 T$ in 1$^{st}$ order $T/m_T$ calculation

$\tau_0 = \text{average freeze-out time}$

$T = \text{freezeout temperature}$
The momentum volume can be determined in two ways:

(1) Fit the correlation function with a 3D Gaussian and use the fit parameters to estimate the momentum volume $v_{mom}$,

$$v_{mom} = \left[ \frac{\mathcal{Q}(\hbar c \sqrt{p})^3}{R_S R_O R_L} \right]$$

(2) Direct summation of the 3D histogram channels.

$$v_{mom} = \int\int\int (C(q_o, q_s, q_l) - 1) dq_o dq_s dq_l$$

Method (1) is traditional, but Method (2) is less model-dependent and gives the best statistical accuracy.

*** Coming soon … Use $Q_{inv}^2(C(Q_{inv}) - 1)$ ***
### Momentum Volume from $R_0R_SR_L$ (upper) and $\Sigma H$ ist (lower)

<table>
<thead>
<tr>
<th>% $\sigma_{Tot}$</th>
<th>h</th>
<th>Low $p_T$</th>
<th>Mid $p_T$</th>
<th>High $p_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5%</td>
<td>345</td>
<td>9.14±0.10</td>
<td>14.38±0.17</td>
<td>22.32±0.31</td>
</tr>
<tr>
<td>± 2</td>
<td></td>
<td>12.44±0.01</td>
<td>17.19±0.01</td>
<td>22.42±0.07</td>
</tr>
<tr>
<td>5-10%</td>
<td>286</td>
<td>9.64±0.10</td>
<td>16.23±0.26</td>
<td>25.54±0.49</td>
</tr>
<tr>
<td>± 4</td>
<td></td>
<td>13.54±0.01</td>
<td>19.83±0.08</td>
<td>25.19±0.13</td>
</tr>
<tr>
<td>10-20%</td>
<td>221</td>
<td>11.83±0.14</td>
<td>18.35±0.31</td>
<td>30.65±0.70</td>
</tr>
<tr>
<td>± 6</td>
<td></td>
<td>15.26±0.01</td>
<td>22.31±0.13</td>
<td>32.43±0.25</td>
</tr>
<tr>
<td>20-30%</td>
<td>149</td>
<td>14.32±0.23</td>
<td>24.93±0.60</td>
<td>39.33±1.45</td>
</tr>
<tr>
<td>± 7</td>
<td></td>
<td>17.19±0.03</td>
<td>26.61±0.15</td>
<td>40.37±0.62</td>
</tr>
<tr>
<td>30-40%</td>
<td>97</td>
<td>20.63±0.39</td>
<td>31.58±0.87</td>
<td>56.47±2.78</td>
</tr>
<tr>
<td>± 8</td>
<td></td>
<td>22.79±0.11</td>
<td>36.64±0.33</td>
<td>55.56±1.69</td>
</tr>
<tr>
<td>40-50%</td>
<td>61</td>
<td>27.20±0.75</td>
<td>40.13±1.85</td>
<td>62.3±10.6</td>
</tr>
<tr>
<td>± 7</td>
<td></td>
<td>30.56±0.24</td>
<td>42.76±0.58</td>
<td>73.73±3.38</td>
</tr>
<tr>
<td>50-80%</td>
<td>18.4</td>
<td>47.4±2.2</td>
<td>56.9±16.7</td>
<td>107.9±225.9</td>
</tr>
<tr>
<td>± 9</td>
<td></td>
<td>50.55±1.25</td>
<td>76.74±1.51</td>
<td>132.1±159.2</td>
</tr>
</tbody>
</table>
Phase Space Density from Direct Histogram Sums

Central

NA49

Peripheral

STAR Preliminary
Entropy per Particle and Phase Space Density

An estimate of the average entropy per particle <S/N> can be obtained from a momentum integral over the local phase space density f(p)

\[
\langle S/N \rangle = \frac{\int_0^\infty \int d^3p [(f(p)+1)\ln(f(p)+1) - f(p)\ln(f(p))]}{\left\langle \int_0^\infty \int d^3p f(p) \right\rangle}
\]

Following Brown, Panitkin, and Bertsch, this can be approximated by a momentum integral over the average phase space density <f>

\[
\langle S/N \rangle \approx \frac{\int_0^\infty m_T d^2p_T [(<f>+1)\ln(<f>+1) - <f> \ln(<f>)]}{\int_0^\infty m_T d^2p_T <f>}
\]

Application of this relation requires a knowledge of the average phase space density <f> at all values of transverse momentum p_T. This cannot be obtained directly from measurements, but can be accomplished by fitting the <f> data with a model. We use the longitudinally averaged source function model of Tomasik and Heinz.
The longitudinal expansion has **reduced** the phase space density and broken the rule that the PSD goes to a Bose-Einstein distribution when $\eta_t=\rho_t=0$ (no flow).

The reduction in the PSD leads to a need for a non-zero chemical potential $\mu_0$ to reach high enough PSD values to match RHIC/STAR observations.
Because the longitudinal expansion reduces the phase space density, a non-zero chemical potential $\mu_0$ is required to reproduce the most central data.

Pion phase space density depends on $\mu_0$ and $T$ in essentially the same way, changing the PSD strength but not its shape. However, the spectrum slope has very different dependences on $\mu_0$ and $T$, breaking this ambiguity.

Therefore, fitting PSD and spectra together constrains the parameters. However, the lowest curves would prefer a negative $\mu_0$-value to reproduce the spectrum slope while fitting the PSD.
T&H Fits to STAR Phase Space Density

Phase space density $\sim 1$

Multiparticle and laser-like stimulated emission effects?

Central

NA49

Peripheral

STAR Preliminary
Aside: Incoherent Fraction $\varepsilon$ from 3-Particle HBT

Preliminary

Mid-Central

Central

Quadratic Fit

Quartic Fit

WA98

NA44

STAR

$\pi^+$

$\pi^-$
Entropy per Particle S/N from $\pi^-$ Phase Space Density

\[
S/N = \frac{\int_0^\infty m_T p_T dp_T [(< f > + 1) Ln(< f > + 1) - < f > Ln(< f >)]}{\int_0^\infty m_T p_T dp_T < f >}
\]
The thermal estimate of the $\pi$ entropy per particle can be obtained by integrating a Bose-Einstein distribution over 3D momentum:

$$
S/N = \frac{\int_{0}^{\infty} m_{T} p_{T} dp_{T} [(f_{BE} + 1) \ln(f_{BE} + 1) - f_{BE} \ln(f_{BE})]}{\int_{0}^{\infty} m_{T} p_{T} dp_{BE}} \int_{0}^{\infty} m_{T} p_{T} dp_{BE}
$$

where $f_{BE} = \frac{1}{\exp[(m_{T} - \mu_{\pi})/T] - 1}$

<table>
<thead>
<tr>
<th>$T/m_{\pi}$</th>
<th>$\mu_{\pi}/m_{\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.3 0.6 0.9</td>
</tr>
<tr>
<td>0.2</td>
<td>7.37481 5.86225 4.30277 2.43181</td>
</tr>
<tr>
<td>0.4</td>
<td>5.13504 4.33169 3.45065 2.25166</td>
</tr>
<tr>
<td>0.6</td>
<td>4.46843 3.89106 3.23476 2.28837</td>
</tr>
<tr>
<td>0.8</td>
<td>4.16727 3.70431 3.16747 2.36967</td>
</tr>
<tr>
<td>1.0</td>
<td>4.00256 3.61107 3.15191 2.45851</td>
</tr>
<tr>
<td>1.2</td>
<td>3.90175 3.56032 3.15728 2.54375</td>
</tr>
<tr>
<td>1.4</td>
<td>3.83522 3.53137 3.17146 2.62195</td>
</tr>
<tr>
<td>1.6</td>
<td>3.78887 3.51456 3.18916 2.69244</td>
</tr>
<tr>
<td>1.8</td>
<td>3.75521 3.50489 3.20786 2.75553</td>
</tr>
<tr>
<td>2.0</td>
<td>3.72997 3.49958 3.22638 2.8119</td>
</tr>
</tbody>
</table>
STAR Preliminary

Bose Gas: $m = m_p, T = 120$ MeV,
$m = m_p, T = 140$ MeV,
$m = m_p, T = 160$ MeV,

Landau Limit: $m = 0, T$ anything

Entropy per Particle S/N with Thermal Estimates

Central

Peripheral

T=120 MeV, $\mu_\pi=20.1$ MeV or
T=167 MeV, $\mu_\pi=0$
Concluding Caveats

1. This is still a work in progress. STAR HBT data at $S_{nn}^{1/2}=130$ GeV is being re-analyzed with several technical improvements and with 6 $K_T$ bins. Results presented here could change.

2. The $R_O R_S R_L$ phase space density estimates depend on the Gaussian-ness of the source. There are some indications of non-Gaussian sources.

3. If one seeks to avoid Item 2 above with more model-independent analysis techniques, the smoothness and normalization high-q region of the HBT histogram becomes quite important.

4. Systematic errors can be large and are not easy to estimate.