Strangeness Asymmetry
(in the 21st Century)

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INT / Jlab workshop on
“Parton Distributions in the 21st Century”
18-20 Nov 2004
Based on:

■ “The parton structure of the Nucleon and Precision Measurement of the Weinberg Angle in Neutrino Scattering”
PRL 93, 041802 (2004); with F. Olness, D. Stump, J. Pumplin, M.H. Reno, and W.-K. Tung

■ “Neutrino Dimuon Production and the Strangeness Asymmetry of the Nucleon”
hep-ph/0312323; with CTEQ

■ “Target Mass Corrections to Electroweak Structure Functions and Perturbative Neutrino Cross Sections”
PRD 69, 034002 (2004); with M.H. Reno
To begin at the beginning:

Parton distributions in the 20\textsuperscript{th} Century
Nonperturbative sea quark distributions

\[ g \rightarrow q \bar{q} \]

- \( \bar{u}(x) = \bar{d}(x) = \bar{s}(x) = s(x) \) ?
- \( \alpha_s(k^2_\perp) \)
- \( \int d k^2_\perp \cdots = \int_{\mu^2} d k^2_\perp \cdots + \int_{\mu^2} d k^2_\perp \cdots \)

perturbative extrinsic
SU(3)\_flavour, CP

nonperturbative intrinsic
SU(3)\_flavour, CP
Nonperturbative sea quark distributions

\[ \bar{s}(x) < \frac{\bar{u}(x) + \bar{d}(x)}{2} \]

\[ \bar{u}(x) \neq \bar{d}(x) \]

Anything left?

Strangeness asymmetry \((s - \bar{s})(x)\)?
Strangeness Asymmetry

\[ \mathcal{O}_s^{\mu_1 \cdots \mu_n} = \bar{\psi}_s \gamma^{\mu_1} D^{\mu_2} \cdots D^{\mu_n} \psi_s \]

\[ \langle N | \mathcal{O}_s^{\mu_1 \cdots \mu_n} | N \rangle \propto \int_0^1 dx \ x^{n-1} \ [s(x) + (-1)^n \bar{s}(x)] \]

\[ \langle N | J_s^\mu | N \rangle \propto \int_0^1 dx \ [s(x) - \bar{s}(x)] = 0 \quad \text{(Sum rule)} \]

\[ \langle N | \mathcal{O}_s^{\mu_1 \cdots \mu_n} | N \rangle \neq 0 \quad n = 2, 3, 4, 5, \ldots \]

\[ (s - \bar{s})(x) \neq 0 \quad !!! \]
But there is more to it ...
NuTeV Anomaly

Atomic Parity Violation

SLAC E158: Parity Violation in Möller Scattering
The “NuTeV Anomaly”

The NuTeV $\sin^2 \theta_W$ measurement: 

It was inspired by, and is related (but not identical) to, the Paschos-Wolfenstein (1973) Ratio:

$$R^- = \frac{\sigma^\text{NC}_\nu - \sigma^\text{NC}_{\bar{\nu}}}{\sigma^\text{CC}_\nu - \sigma^\text{CC}_{\bar{\nu}}} \approx \frac{1}{2} - \sin^2 \Theta_{WW}$$  (isoscalar target, ...)

NuTeV  $\sin^2 \theta_W = 0.2277 \pm 0.0016$

LEP EWWG  $\sin^2 \theta_W = 0.2227 \pm 0.00037$

a 3.1 $\sigma$ discrepancy

Must be corrected for a target with a fractional neutron excess, $\delta N$, $S \neq \bar{S}$, ...

SM explanation(s) or Signal for New Physics?
NLO=$O(\alpha_s)$ corrections for light and charm quarks

Target mass corrections to EW structure functions along the OPE (Georgi & Politzer) approach:
$O(M_N^{2n}/Q^{2n})$

NLO DGLAP

Twist $\tau=2$

$O(m_c^{2n}/Q^{2n})$
Moment estimates

\[ \sigma_{\text{tot}} = \int dx dy \ (d\sigma/dx dy) \propto \int dx \ x q(x) = q(n = 2) \]

Approximations:

1. Neglect of evolution:
   Overestimates cross sections by about 15-20%

2. \( M_{W,Z} \rightarrow \infty \) limit of the boson propagator

3. Neglect of cuts

Good for:
Illustration or rough estimates of large effects (as in this talk).

Not good for:
Precision observables. (E.g. flips sign of NLO correction to \( R^V \))
Corrections to $R^-$ for illustration (this presentation):

• Let’s neglect cuts / exp. Issues: ideal Paschos-Wolfenstein relation [and look for big effects that are unaffected by $O(30\%)$ detector effect corrections]

• Let’s neglect evolution / scale dependence

• Let’s neglect NLO corrections

• Let’s assume the target material (mostly Fe) is isoscalar and that isospin is exact (→ T. Londergan’s talk)

• Let’s assume $m_c=0$

• Let me focus on the quark / antiquark asymmetry for strange sea quarks
Then:

\[ R^- \simeq \frac{1}{2} - \sin^2 \Theta_W - \left( \frac{1}{2} - \frac{7}{6} \sin^2 \Theta_W \right) \frac{[S^-]}{[Q^-]} \]

\[ [Q^-] \equiv \int_0^1 dx \ x \ \frac{u_V(x) + d_V(x)}{2} \]

\[ [S^-] \equiv \int_0^1 dx \ x \ (s - \bar{s}) (x) \]

[S^-] is not a local operator.

(Higher, odd moments are.)
(But there is more to it …)

... and that is not the end:
Theoretical expectations:
S. Brodsky & B.-Q. Ma (1996)
\[ p \Lambda K^+ \text{ fluctuation} \]

\[ s(x) = \int_x^1 \frac{dy}{y} f_{\Lambda/K^+}(y) q_{s/\Lambda} \left( \frac{x}{y} \right); \quad \bar{s}(x) = \int_x^1 \frac{dy}{y} f_{K^+/\Lambda}(y) q_{\bar{s}/K^+} \left( \frac{x}{y} \right) \]

More recent meson cloud results:

\[ \chi \text{ quark soliton model:} \]

And phenomenology from:

Figure 1: The momentum distributions for the strange quarks and antiquarks in the light-cone meson-baryon fluctuation model of intrinsic q\bar{q} pairs, with the fluctuation wavefunction of K^+\Lambda normalized to 1. The curves in (a) are the calculated results of \( s(x) \) (solid curves) and \( \bar{s}(x) \) (broken curves) with the Gaussian type (thick curves) and power-law type (thin curves) wavefunctions and the curves in (b) are the corresponding \( \delta_s(x) = s(x) - \bar{s}(x) \). The parameters are \( m_s = 330 \text{ MeV} \) for the light-flavor quark mass, \( m_s = 480 \text{ MeV} \) for the strange quark mass, \( m_D = 600 \text{ MeV} \) for the spectator mass, the universal momentum scale \( \alpha = 330 \text{ MeV} \), and the power constant \( p = 3.5 \), with realistic meson and baryon masses.
Perturbative effect?

Moch & Vermaseren & Vogt: \( u \rightarrow s \neq u \rightarrow \bar{s} \) @ 3-loop evolution

\[ [S^-]_{	ext{NNLO}} \approx -0.0005 \]
The CTEQ analysis
"CTEQ" Global Analysis

- Same ingredients as "CTEQ6" analysis
- Add CCFR-NuTeV dimuon data (next slide)
- Allow a non-symmetric strangeness sector:

**Parametrization** of the Strangeness sector (at some $Q = Q_0$)

$$s^+(x, Q_0) = A_0 x^{A_1} (1 - x)^{A_2} P_+(x; A_3, A_4, \ldots)$$
$$s^-(x, Q_0) = s^+ \tanh[a x^b (1 - x)^c P_-(x; x_0, d, e, \ldots)]$$

$$P_-(x) = (1 - \frac{x}{x_0} + dx^2 + ex^3 + \ldots)$$

Where $x_0$ is to be determined by the condition $[s^-] = 0$. **Sum rule and positivity are then satisfied.**

$$[s^-] \equiv \int_0^1 dx \ (s - \bar{s})(x) = 0$$
$$|(s - \bar{s})(x)| < (s + \bar{s})(x)$$
Data from CCFR/NuTeV (see P. Spentzouris' talk) determine the strange sea within a CTEQ global analysis.

NLO corrections are not yet included in current CTEQ analysis (to meet the LO acceptance correction model).

NLO corrections become sizable at high energy (HERA) but are well behaved for fixed target scattering (CCFR/NuTeV).
Portrait of the strange sea
(in an active iron target)
Strangeness Structure of the Nucleon: Dimuon Production in $\nu, \bar{\nu}$ Scattering

$$\frac{d\sigma_{\mu^+\mu^-}^{\pm\mp}}{dx\ dy} = \int d\Gamma d\Omega \frac{d\sigma_{\mu^+_c}^{\mp\mp}}{dx\ dy\ d\Gamma} \otimes D_c(\Gamma) \otimes \Delta_c(\Omega)|_{\mu^\pm > 5 \text{ GeV}}$$

M. Goncharov et al., NuTeV Collaboration PRD 64:110226 (2001)

Modeling needed for comparing theory with data.
Differential production cross section (NLO) and acceptance

D. Mason, F. Olness, SK: DISCO

Acceptance vs. rapidity

Acceptance vs. z

\[ d\sigma \]
CC charm: LO/NLO stability

PDF - hard scattering

NLO: T. Gottschalk
M. Glück, E. Reya, SK
ACOT
The perturbative NLO effects are small compared to the (data driven) uncertainty of extracting \([S^-]\) at LO.

(This expectation is confirmed by preliminary NLO results.)
\[
\xi s'(\xi, Q^2)_{\text{eff}} \equiv \frac{1}{2} \frac{\pi (1 + Q^2/M^2_W)^2}{G_F^2 M_N E_{\nu}} |V_{cs}|^{-2} \frac{d^2 \sigma_{\nu N \to cX}}{dx dy} \\
= (1 - \frac{m_c^2}{2M_N E_{\nu} \xi}) \xi s'(\xi, Q^2) + \mathcal{O}(\alpha_s)
\]

\[
\xi \equiv x \left(1 + \frac{m_c^2}{Q^2}\right) \quad s' \equiv s + \frac{|V_{cs}|^2}{|V_{cd}|^2} d
\]
Quality of fit to the neutrino dimuon data

\[ Q^2/\text{GeV}^2 \quad 2. \quad 50. \]

(Data points are color-coded according to \(x\); for each \(x\), they are ordered in \(y\).)
Typical fit results
Vs. Bjorken x

positive $[S^-]$
Lagrangian multiplier results for \([S^-]\):

**Rule of thumb:**
The 3 \(\sigma\) anomaly corresponds to \([S^-] \approx 100' +0.5\)

- **NuTeV/CCFR data**
- **Other (less) sensitive data**
Further uncertainties:

- LO $\to$ NLO PDF analysis
- charm mass
- charm fragmentation and decay

Are considered in our estimate for the range of $[S^-]$ (! conclusions).

The general features of the LM parabola stay the same, the minimum shifts by an amount that is consistent with the width of the $\chi^2$ vs. $[S^-]$ parabola.
Results on the strangeness asymmetry

- The central value is \([S^-] = 0.0017\) or \([S^-] / [S^+] = O(10\%)\).
- We estimate that \(-0.001 < [S^-] < 0.004\).

Implication on the NuTeV anomaly

Based on a NLO calculation of the Paschos & Wolfenstein ratio:

A value of \([S^-] = 0.0017\) (central value) can reduce the NuTeV anomaly from a \(\sim 3\ \sigma\) effect to \(\sim 1.5\ \sigma\); a value of \([S^-] \sim 0.003 - 0.004\) would then reduce it to within \(1\ \sigma\). There are further effects (not discussed here): Isospin violation, higher twist, …

*The current combined analysis of perturbative and non-perturbative corrections to the Paschos & Wolfenstein ratio suggests that the Weinberg angle measurement and global data sets used in QCD parton structure analysis can all be consistent with the SM.*
Models as seen from a non-model-builder
A democratic (yet incomplete) "model slide". Curves from: Brodsky & Ma, Cao & Signal, Wakamatsu.
Model building

Theoretical basis:
Chiral symmetry (breaking)
Light Cone Quantization / LC Wave functions

Asymmetry

Qualitative features:
# of crossings with x-axis?
Sign of second moment?
Perturbative effect?

Moch & Vermaseren & Vogt: \( u \to s \neq u \to \bar{s} \) @ 3-loop evolution

Catani & de Florian & Rodrigo & Vogelsang; PRL (2004):

\[ [S^-]_{\text{NNLO}} \approx -0.0005 \]
***** backup slides *****
The CCFR ansatz:

\[
s(x, Q^2) = \kappa \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1 - x)^{\alpha_q}
\]

\[
\bar{s}(x, Q^2) = \kappa \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1 - x)^{\alpha_q}
\]

violates Strangeness # SR and QCD evolution.

BPZ: large-x peak of \( s^{-}(x) \) not natural; 2 oscillations not supported by theoretical expectations.
SM: Corrections to the P-W Relation Due to Strangeness Asymmetry, Isospin Violation, ... etc.

\[ R^- \approx \Delta^2_u + \Delta^2_d \]
\[ - \delta N \left( \frac{U_v - D_v}{V_p} \right) (3\Delta^2_u + \Delta^2_d) \]
\[ + \frac{\delta U_v - \delta D_v}{2V_p} (3\Delta^2_u + \Delta^2_d) \]
\[ + \frac{\delta S}{V_p} (2\Delta^2_d - 3(\Delta^2_d + \Delta^2_u)\epsilon_c), \]

where
\[ \delta N \equiv (A - 2Z)/A \]
\[ V_p = U_p - \overline{U}_p + D_p - \overline{D}_p \]
\[ \delta D_v \equiv D_p - \overline{D}_p - U_n + \overline{U}_n \]
\[ \delta U_v \equiv U_p - \overline{U}_p - D_n + \overline{D}_n \]
\[ \delta D \equiv \overline{D}_p - \overline{U}_n \]
\[ \delta U \equiv \overline{U}_p - \overline{D}_n \]
\[ \delta S \equiv \langle S \rangle - \langle \overline{S} \rangle \]
\[ \epsilon_c : \text{charm-mass kinematic correction factor} \]

CCFR-NuTeV: PR D65, 111103 (2002)

Strangeness asymmetry
CC charm: LO/NLO stability

PDF - hard scattering - fragmentation

NLO: M. Glück, E. Reya, SK
What do we know about \((s - \bar{s})(x)\)?

Before any data, two things:

- \([s^-] \equiv \int_0^1 dx \ (s - \bar{s})(x) = 0\) (exact sum rule) \(s^-(x_0) = 0\)
- \(|(s - \bar{s})(x)| < (s + \bar{s})(x)\) (positivity)

For more information, we have to ask data:

- **dimuon production**: \(\nu_\mu s \rightarrow \mu^- c \rightarrow \mu^\pm x\) (CCFR, NuTeV, ...)
- further (insignificant) constraints:
  - \(F_3^\nu \sim \sum_q (q - \bar{q})(x)\)
  - **s g ! c W**- background to sign-selected W production ("W charge asymmetry") at the Tevatron
Representative PDF sets obtained with this global QCD analysis:

5 representative fits obtained using LM method

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<th>B+</th>
<th>A</th>
<th>B</th>
<th>C</th>
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<td>0.97</td>
<td>1.00 (141)</td>
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<td>Inclusive II</td>
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<td>1.00 (2349)</td>
<td>1.00</td>
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Normalized $\chi^2$ values

Processes having some sensitivity to $s^-$

Processes having no sensitivity to $s^-$ (majority)
The Lagrange Multiplier Method in Global Analysis

2-dim \((i,j)\) rendering of \(d\)-dim PDF parameter space

Contour plots of \(\chi^2_{\text{global}}\)

Constrained fits using modified \(\chi^2\) function:

\[
\Psi(\lambda, a) = \chi^2_{\text{global}}(a) + \lambda X(a)
\]

and vary \(\lambda\) over an appropriate range.
Future prospects for $[S^-]$?

- $W$ and associated charm (jet) production: conceivable @ Tevatron, RHIC, LHC
- Do statistics (efficiency driven) and high scale permit to access a small nonsinglet asymmetry?
- $CC$ charm @ HERA: ditto

- Lattice:
  The moment $[S^-]$ itself does not correspond to a local operator.
  Higher, uneven moments ($n=3,5,...$)

\[
\int_0^1 dx \ x^{n-1} \ (s-\bar{s})(x) \sim \langle N | \ \bar{\Psi} \gamma^{\mu_1} D^{\mu_2} ... D^{\mu_n} \Psi | N \rangle
\]

can be related to local operators and could presumably clarify the sign of the $x!1$ behaviour, though not the magnitude of $[S^-]$.

- Semi-Inclusive DIS?

Baur, Halzen, Keller, Mangano, Riesselmann
Digested results on “standard” QCD corrections:

- NLO, TMC, (and $m_\mu$) corrections
  - shift $R_{QCD}^{\nu,\bar{\nu}}$ by an amount that is of the order of the experimental accuracy for $R_{e,exp}^{\nu,\bar{\nu}}$ (<1%).
  - shift $R_{QCD}^{-}$ by an amount that is small compared to the experimental error attributed to $\sin^2 \theta_W$ (<1%).
  - [Seeming discrepancies with analytic NLO estimates by Bogdan Dobrescu and Keith Ellis (PRD 04) have been understood.]

- These results do not permit reliable conclusions on the significance of the anomaly: *None of the above R’s is actually measured. Only an experimental reanalysis can provide a definitive answer.*
- The results do indicate that *the corrections can be relevant within the accuracy of the data.*
**SM:** Corrections to the P-W Relation Due to Strangeness Asymmetry, Isospin Violation, ... etc.

\[ R^- \approx \Delta^2_u + \Delta^2_d - \delta N \left( \frac{U_v - D_v}{V_p} \right) (3\Delta^2_u + \Delta^2_d) \]

\[ + \frac{\delta U_v - \delta D_v}{2V_p} (3\Delta^2_u + \Delta^2_d) \]

\[ + \frac{\delta S}{V_p} (2\Delta^2_d - 3(\Delta^2_d + \Delta^2_u)\epsilon_c), \]

where

\[ \delta N \equiv (A - 2Z)/A \]

\[ V_p = U_p - \bar{U}_p + D_p - \bar{D}_p \]

\[ \delta D_v \equiv D_p - \bar{D}_p - U_n + \bar{U}_n \]

\[ \delta U_v \equiv U_p - \bar{U}_p - D_n + \bar{D}_n \]

\[ \delta \bar{D} \equiv \bar{D}_p - \bar{U}_n \]

\[ \delta \bar{U} \equiv \bar{U}_p - \bar{D}_n \]

\[ \delta S \equiv \langle S \rangle - \langle \bar{S} \rangle \]

\[ \epsilon_c : \text{charm-mass kinematic correction factor} \]

\[ \Delta^2_{u,d} = (\epsilon_{L}^{u,d})^2 - (\epsilon_{R}^{u,d})^2 \quad \text{(Weinberg Angle)} \]

CCFR-NuTeV: PR D65, 111103 (2002)

**Emphasis from now on**
Status (theory) of the NuTeV anomaly

The cross section ratios $R_{\nu,\bar{\nu},-}$ that are closely related to the NuTeV extraction of the Weinberg angle have been thoroughly reevaluated by many authors under several SM corrections:

- pQCD corrections (NLO & masses)
- electroweak corrections
- nucleon parton structure effects & uncertainties

The exact impact of the corrections on the extraction of $\sin^2 \theta_W$ cannot be quantified in theory because of the involved Monte Carlo / detector effects in modeling “long” and “short” events in the NuTeV experiment. The closely related theoretical observables receive corrections that are of the order of the assigned experimental errors or bigger. Partonic uncertainties (strangeness asymmetry, ...) do even survive in the ideal Paschos-Wolfenstein ratio.

These effects have, so far, not been assigned a systematic error in the NuTeV value of $\sin^2 \theta_W$. Claims that they cancel in the NuTeV (LO Monte Carlo) analysis procedure are a posteriori and have not been substantiated by an analysis that takes them into account ab initio. Before a careful re-assessment of all theoretical uncertainties (pQCD & non-pQCD & electroweak) the $3 \sigma$ discrepancy with the SM cannot be taken at face value.
MRST analysis of isospin violations:

\[ u^v_n(x) = d^v_p(x) + \kappa f(x) \]

with \( s_0 \int dx f(x) = 0 \) finds \( \kappa f(x) < 3-10\% \) (8 \( x \))

Best fit value of \( \kappa \) removes half of the discrepancy with the SM.

Within a permissible variation of \( \chi^2_{MRST} \)
\[-0.007 < \delta \sin \Theta_W < 0.007\]
the full 3 \( \sigma \) are covered.
Summary and conclusions

• Our measured asymmetry is at most consistent with zero
  » both LO and NLO
  » with different parameterizations
• $s$ tends toward negative values at the lowest $x$-bin data points
• number sum-rule forces positive turn
• below lowest $x$-data-point

Collaboration with phenomenologists (Amundson, Kretzer, Olness, Tung) provided us with necessary tools

And Dave Mason at Moriond

Panagiotis Spentzouris <spentz@fnal.gov>  

DIS 2004
\[ F_i^c(x, z, Q^2) = s'(\xi, \mu_F^2) \ D_c(z) + \frac{\alpha_s(\mu_F^2)}{2\pi} \int_{\xi}^{1} \frac{d\xi'}{\xi'} \int_{\max(z, \xi_{\text{min}})}^{1} \frac{d\zeta}{\zeta} \left[ H_i^q(\xi', \zeta, \mu_F^2, \lambda) \ s'(\frac{\xi}{\xi'}, \mu_F^2) \right. \\
\left. + \ H_i^g(\xi', \zeta, \mu_F^2, \lambda) \ g'(\frac{\xi}{\xi'}, \mu_F^2) \right] D_c(\frac{\zeta}{\zeta}) \]

\[ H_i^{q,g}(\xi', \zeta, \mu_F^2, \lambda) \to \tilde{H}_i^{q,g}(\xi', \zeta, \mu_F^2, \lambda) = H_i^{q,g}(\xi', \zeta, \mu_F^2, \lambda) \times \Theta_{\Xi}(\xi', \zeta) \]