Neutrino Oscillations and the MaVaN Model

Kevin Weil
UW REU
Summer 2004
Outline

- Massive neutrinos and the Standard Model
- Neutrino oscillations
  - The standard picture
  - The MaVaN picture
- Outlook
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According to the Standard Model, all three neutrino masses are zero.

Nonzero masses can work, but require an extension of the model.

Difficult to measure masses because neutrinos rarely interact.

- A neutrino of moderate energy can penetrate many light years of lead!
- They’re passing through us right now.
Super-Kamiokande

- 50,000 ton underground water tank
- Photomultiplier tubes see results of electron neutrino interactions
  - But not the actual neutrinos
- Fewer results than expected
  - Resolution: neutrino oscillations, which can only happen with nonzero mass
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Neutrino Oscillations

- Consider only electron and muon neutrinos
  - “Flavor eigenstates” $|\nu_e\rangle, |\nu_\mu\rangle$
- Different from the eigenstates of the Hamiltonian
  - “Mass eigenstates” $|\nu_1\rangle, |\nu_2\rangle$
- Related by a (vacuum) mixing angle $\theta_0$
  - To get from one basis to the other, multiply by a unitary transformation $U(\theta_0)$:

$$
\begin{pmatrix}
|\nu_e\rangle \\
|\nu_\mu\rangle
\end{pmatrix} = 
\begin{pmatrix}
\cos(\theta_0) & \sin(\theta_0) \\
-\sin(\theta_0) & \cos(\theta_0)
\end{pmatrix}
\begin{pmatrix}
|\nu_1\rangle \\
|\nu_2\rangle
\end{pmatrix}
$$
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A Quick Derivation (1 of 4)

- In the mass eigenstate basis, the mass matrix is
  \[ M_{\text{mass}} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \]

- In the flavor eigenstate basis, it is then
  \[ M_{\text{flavor}} = U(\theta_0) M_{\text{mass}} U^\dagger(\theta_0) \]
  \[ = (m_2^2 - m_1^2) \begin{pmatrix} -\cos(2\theta_0) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) \end{pmatrix} + \frac{1}{2} (m_2^2 + m_1^2) \mathbb{1} \]
A Quick Derivation (2 of 4)

- In the non-relativistic limit, $\sqrt{p^2 + m^2} \approx p + \frac{m^2}{2p}$
- The kinetic energy is then
  \[
  T = p\mathbf{1} + \frac{1}{2p} M_{\text{flavor}}^2
  \]
  \[
  = \frac{1}{4p} \left( m_2^2 - m_1^2 \right) \begin{pmatrix} -\cos(2\theta_0) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) \end{pmatrix} + \frac{1}{4p} \left( 4p^2 + m_2^2 + m_1^2 \right) \mathbf{1}
  \]
- Wolfenstein (1978) derives potential term (MSW effect)
  - Matter almost entirely first-generation leptons and quarks
  - Weak charged current interactions single out the electron neutrino component

\[
V_{\text{MSW}} = \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}
\]
A Quick Derivation (3 of 4)

- Define $\delta m^2 = m_2^2 - m_1^2$
- Drop terms proportional to the identity
- Define $\Omega = \frac{2\sqrt{2} G_F n_e E}{\delta m^2}$

\[
H_{\text{eff}} = \frac{\delta m^2}{4} \begin{pmatrix}
- (\cos(2\theta_0) - \Omega) & \sin(2\theta_0) \\
\sin(2\theta_0) & \cos(2\theta_0) - \Omega
\end{pmatrix}
\]

- After some algebra and trigonometry:

\[
\sin^2(2\theta_m) = \frac{\sin^2(2\theta_0)}{\sin^2(2\theta_0) + (\cos(2\theta_0) - \Omega)^2}
\]

- Mixing angle changes in matter!
From QM,

\[ |\nu_j(t)\rangle = |\nu_j(0)\rangle \exp \left( -i \int_0^t E_j(t') \, dt' \right) \]

Use the adiabatic approximation (essentially assuming density varies slowly):

\[
P_{\nu_e \rightarrow \nu_e} = |\langle \nu_e(T) | \nu_e(0) \rangle|^2
= \cos^2(\theta_m) \cos^2(\theta_0) + \sin^2(\theta_m) \sin^2(\theta_0) + \frac{1}{2} \sin(2\theta_m) \sin(2\theta_0) \cos \left( \int_0^T (E_2(t') - E_1(t')) \, dt' \right)
\rightarrow \frac{1}{2} + \frac{1}{2} \cos(2\theta_m) \cos(2\theta_0)\]
No More Derivations!

The survival probability of electrons as functions of energy

Energy (MeV)
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Add a new scalar field, the “acceleron” $A$

Postulate a heavy sterile (dark) neutrino
- Mass dependent on expectation value $hA_i$, which is a function of $n_e$

Physical justification: measured dark energy density and neutrino energy density are similar
- MaVaNs can help explain this without fine tuning
- Many more cosmological justifications - see hep-ph/0309800

But do they agree with experimental results?
Simplify model by integrating out heavy sterile neutrino

- Assume \( M_{\text{sterile}} = K n_e^r \)

New Hamiltonian is

\[
H_{\text{MaVaN}} = \frac{1}{2E} \frac{m_D^4}{K^2} n_e^{-2r} \left( \frac{2\sqrt{2} K^2 G_F n_e^{2r+1} E}{m_D^4} + \sin^2(\theta) \begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix} \begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix} \right)
\]

- Radically different electron density dependence
Reproducing Measurements

- Theory of neutrino masses has a new basis
  - Must reproduce experimental results to be useful
- We can!
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- This is a very positive result -- no a priori reason that MaVaNs should reproduce experimental results
- Experiments like KamLAND provide further constraints to test
- Lots to explore!
Thanks

• To my two great advisors, Ann Nelson and Neal Weiner
• To my fellow REU students
• To UW Physics
• To the NSF