1. The r-process takes place in a plasma with $T_9 = 3$ and neutron density $\rho_n = 10^{22}/\text{cm}^3$. What is the neutron separation energy of the nuclei taking part in the r-process?

2. In class we modeled neutron stars assuming a pure neutron gas, $T=0$, and an ideal equation of state (Fermi gas). The point of this problem is to calculate the admixture of protons that must exist. a) Use as your variables the neutron density $n_n$; express the proton density as $n_B - n_n$, where $n_B$ is the baryon density that we will hold fixed. Write down an expression for the total energy density $\rho_n + \rho_p + \rho_e$, treating the neutrons and protons as nonrelativistic and the electrons as fully relativistic, as we did in class. By nonrelativistic I mean that you should retain the mass contribution to the energy density only.

b) The condition for proton-neutron chemical equilibrium is that

$$\frac{d\rho}{dn_n} = 0$$

Evaluate this to show that the proton density is independent of the neutron density and proportional to $(m_n - m_p)^3$.

c) The answer in b) should bother you because the nonrelativistic limit involves neglecting $k_f$ relative to $m_n$. Yet our answer doesn’t depend on $m_n$ but rather $m_n - m_p$, which is much smaller. Thus repeat the calculation without the relativistic nucleon assumption by using (justify)

$$\rho_n + \rho_p = \frac{1}{\pi^2 \hbar^3} \left( \int_0^{(3\pi^2 \hbar^3 n_n)^{1/3}} (k^2 + m_n^2)^{1/2} k^2 dk + \int_0^{(3\pi^2 \hbar^3 (n_B - n_n))^{1/3}} (k^2 + m_p^2)^{1/2} k^2 dk \right)$$
It is still ok to assume fully relativistic electrons. Show that

\[ \frac{n_p}{n_n} = \frac{1}{8} \frac{\left(1 + \frac{Q^2}{C^2 n_n^{2/3}}\right)^3}{\left(1 + \frac{m_n^2}{C^2 n_n^{2/3}}\right)^{3/2}} \]

where

\[ Q^2 = m_n^2 - m_p^2 \quad \text{and} \quad C = (3\pi^2 \hbar^3)^{1/3} \]

Now rewrite this result by eliminating C in favor of the critical density defined in class

\[ \rho_c = \frac{m_n^4}{3\pi^2 \hbar^3} \]

d) Compare the answers in b) and c) for \( \frac{n_p}{n_n} \) at \( m_n n_n = .001, .1, \) and 1 \( \rho_c \). Find the limit where the answer in c) agrees with the answer in b), and therefore show why b) is such a miserable approximation. What is the limit of the answer in c) when \( n_n \to \infty \)?