Giant resonances in hot nuclei

Dipole oscillations in which all the protons in a nucleus move relative to all the neutrons are now seen not only in nuclear absorption processes, but also in the emission spectra of highly excited nuclei.

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Nuclei interact with the external environment through a number of different fields—electromagnetic, weak and hadronic. The collective excitations induced by these interactions are known as giant resonances. The best-known example is the giant dipole resonance, which is stimulated when the electric field of an incident gamma ray exerts a force on the positively charged protons in a nucleus, moving them relative to the uncharged neutrons (see figures 1 and 2). Other giant resonances that have been studied are the monopole, quadrupole and spin–isospin modes of oscillation. The spin–isospin mode involves charge-changing processes, in particular beta decay. The quadrupole and monopole giant resonances are most easily seen with fields that act equally on neutrons and protons, because in these modes the neutrons and protons oscillate in the same mode.

The giant resonances are collective oscillations, and the various modes of oscillation depend on specific aspects of the nuclear force to sustain them. In the monopole mode, the motion is radial and the frequency depends on the compressibility of the nucleus. In the dipole and the spin–isospin resonances, the protons and neutrons are excited out of phase and the proton–neutron interaction provides the restoring force.

From a general theoretical point of view, these modes are interesting because they allow us to study collective phenomena in a small system of particles. We can understand the physics either from the viewpoint of quantum mechanics or in classical macroscopic terms. It is remarkable that collective motion is sustained in a small quantum system and that it can be interpreted in the language of classical mechanics.

To observe the giant dipole resonance, which was discovered nearly 30 years ago, one irradiates a substance with a broadband beam of gamma rays. The nuclei absorb mainly photons whose frequencies are near the resonant frequency. To excite the other modes one bombards the nuclei with protons, neutrons or alpha particles, measuring the energy and momentum transferred.

Until recently, one could study giant resonances only as excitations of nuclear ground states. Thanks to the availability of heavy-ion accelerators it is now possible to study the giant dipole resonance in highly excited, or “hot,” nuclei and in nuclei spinning with high angular momenta.

In this article we describe the new experiments on giant dipole resonances in hot nuclei and discuss the theory that attempts to explain what they show. The new studies are significant because many kinds of nuclear transformations, such as radioactivity and fission, involve cascades of gamma rays from excited nuclear states. To describe the de-excitation process requires knowledge of the photon interactions of the excited states. The common assumption, known as the Brink–Axel hypothesis, has been that the frequency and other properties of the giant dipole resonance are unaffected by any excitation of the nucleus. This longstanding assumption, put forward by David Brink and Peter Axel in 1956, could not be tested directly until recently.

Making hot nuclei

The new experimental results come principally from studying heavy-ion collisions. Nuclear collisions with heavy ions put energy into a nucleus in such a way that it is shared by many degrees of freedom. This is to be contrasted with excitation by protons, electrons or other elementary probes: These projectile nuclei interact with only a small part of the target when they transfer a lot of energy, so the energy goes off in a few knocked-out nucleons instead of getting distributed over the whole nucleus.

The energy is transferred more evenly in reactions induced by heavy ions because initially it is already shared among the nucleons of the projectile. The excited nuclei produced in heavy-ion collisions may keep their energy long enough to come to an internal
Apparatus at the University of Washington, Seattle, for experiments on giant dipole resonances in excited nuclei. Heavy ions from an accelerator enter through the vacuum line at the top of the photograph. The nuclei of these ions react with target nuclei in the hemispherical target chamber, visible in the photograph, forming excited, or "hot," nuclei in which giant dipole oscillations are induced. Ions that do not interact with the target travel out the exit pipe at the lower left. The sodium iodide detector to the right of the target chamber analyzes the giant dipole oscillations by detecting the gamma radiation that the resonating nuclei emit as the oscillations die out. Inspecting the apparatus are, from left to right, graduate student Jerry Feldman; research assistant professor Cindy Gossett; professor Kurt Snover; Marta Kicinska-Habior, visiting scientist from Warsaw; and graduate student John Behr.

Figure 1
Nuclear vibratory motion known as the giant dipole resonance. Neutrons and protons, shown in red and green in this schematic diagram, acquire velocities in opposite directions under the influence of an external electric field, as the top right diagram indicates. Under this motion, the two kinds of nucleons partly separate from each other (bottom right). The separation requires energy, giving rise to a restoring force (bottom left). The amplitude of the motion is greatly exaggerated in this figure; the actual displacement is on the order of one percent of the nuclear radius when a single quantum of vibration is present.

Figure 2

Statistical equilibrium. The primary time scale in such a reaction is the nucleon traversal time, which is of the order of $10^{-22}$ seconds because the dimensions of a nucleus are a few fermis and the nucleon velocity at the Fermi surface is 30% of the velocity of light. This time is of the same order of magnitude as the period of the giant dipole vibration. After the nucleons collide, equilibration takes place in a few traversal times. The time it takes the equilibrated system to decay by particle emission is much larger than the equilibration time—in the range $10^{-21}-10^{-18}$ sec, depending on the excitation energy. During this time the dipole motion is present as a thermal excitation and it may be observed by the emission of photons. The quantitative interpretation of the photon rates is based on the hypothesis of statistical equilibrium.

The key assumptions in the statistical description of nuclear decay are that all states at a given excitation energy are equally populated and that from the calculated inverse reaction rate, one can infer the rate at which the nucleus decays by emitting a given particle. This is the essence of Niels Bohr's compound-nucleus model. It has long been used to describe nuclei with excitation energies on the order of tens of MeV, and heavy-ion reactions have extended the domain of the model to nuclei produced with excitations of hundreds of MeV.

The hypothetical energy-level scheme of a nucleus shown in figure 3 helps illustrate the workings of the statistical description of photon decay using the Brink–Axel hypothesis. The black lines represent the ordinary energy levels of the nucleus, starting from the ground state and extending to a very high density of levels at high excitation. The giant dipole resonances of these levels, according to the Brink–Axel hypothesis, are an identical set of levels displaced upward by the giant-dipole-resonance frequency. These are shown in red. If the nucleus is in statistical equilibrium at some high excitation, there is a nonzero probability that it is in one of the red states, where it can decay to the base state by emitting a dipole photon. The probability depends essentially on how fast the level density increases between the energy of the base state and the energy of the initial excited nucleus. The rate at which the level density increases is parameterized by temperature in statistical mechanics, with the ratio of level densities given by $e^{-\Delta E/kT}$. Thus, assuming statistical equilibrium and knowing the nuclear temperature, we can infer the properties of the giant dipole resonance in emission as well as absorption processes.

To make an analogy with optical physics, the color of an object can be determined in two ways. The easy way is to illuminate it with white light and see what wavelengths it absorbs. But another way, in principle, would be to heat the object and see the same wavelengths emitted.

Experiments

Even before the heavy-ion measurements, reactions with protons showed some preference for photon emission at the dipole frequency for states at higher excitation. However, it was necessary to use heavy-ion beams to achieve statistical equilibrium, and a group at Berkeley did the first experiments to show the thermal emission of giant-dipole photons. The probability of photon emission is quite small because the nuclear temperatures achieved are low compared with the dipole frequency. In their first experiments, the Berkeley group bombarded heavy targets with a beam of 170-MeV argon ions. This produces a fused nucleus with a maximum temperature of about 3 MeV. Because the dipole energy is about 15 MeV, the probability of forming a dipole state is only about $e^{-5}$. Furthermore, any highly excited state is much more likely to decay by particle emission than by photon emission. The observed photon spectrum is a steeply falling curve, close to exponential. The giant dipole resonance is seen as a change in the slope of the curve in the range of the resonance frequency.
Nuclear energy levels in schematic representation. The levels depicted in black are the ordinary nuclear states. The levels shown in red are giant-dipole excitations. These states are built from the ordinary ones by the addition of a quantum of vibrational excitation, which raises the energy of the state by $\hbar\omega_{\text{dipole}}$. The probability that an excited nucleus will decay by emitting a giant-dipole-oscillation photon is proportional to the relative density of the red states at that energy. This picture is idealized in that the states are depicted as distinct, sharp levels. In reality they are broad and strongly mixed. However, this broadening and mixing is not relevant for the statistical arguments. Figure 3.

More recently, groups have done experiments at a number of laboratories, including the Max Planck Institute in Heidelberg, the University of Washington, Brookhaven National Laboratory and the Niels Bohr Institute in Copenhagen. The effect is confirmed, and are we even beginning to get detailed information about the properties of the giant dipole in the excited system. In one of the experiments, Kurt Snover's group at the University of Washington bombarded Sm$^{154}$ with a 60-MeV beam of carbon ions to make the deformed nucleus Eu$^{166}$. Figure 4 shows the raw spectrum of gamma rays. The gamma emission rate falls over six orders of magnitude as the gamma energies go from 5 to 25 MeV. The spectrum shows an undulation—the primary evidence of the giant dipole resonance. The solid line drawn through the data points is the prediction of the statistical theory. As we mentioned above, this theory depends only on the density of levels and the assumed photon-absorption cross sections. For the curve in the figure, the absorption cross section was assumed to be the same as in the ground state.

The ground-state dipole in Eu$^{166}$ is different from dipoles in spherical nuclei because of the deformation. Motion along the longer axis has a lower frequency than motion along either short axis, which implies that the dipole is split into two components. The double-peaked structure persists in the excited states reached through heavy-ion collisions. This is clearer in figure 5, which is a plot of the same data rescaled by an exponential factor to roughly divide out the effects of the density of levels on the probability of formation of the dipole state. The flat-topped peak between 10 and 15 MeV is the giant dipole resonance. For a spherical nucleus the peak has a nearly Lorentzian shape. Thus this experiment shows that the deformation found in nuclear ground states persists to excitation energies on the order of 50 MeV. This is not surprising: Nuclear theory predicts that the shell effects giving rise to deformations remain important to temperatures on the order of 2 MeV. In this experiment the average temperature is somewhat less than that.

Experiments at higher energies can, in principle, tell us about much more than changes in nuclear shape. In particular, the frequency of the dipole is intimately related to the size of the nucleus and the forces between nucleons. However, there is no consensus yet on the behavior of the dipole at higher energies. Some groups find that the dipole frequency decreases at high excitation energy; others find no frequency shift. In all cases the resonance becomes broader, as figure 6 indicates. The collective motion is more quickly damped in an environment of thermal excitations than in a cold Fermi liquid.

Theory

The giant dipole resonance is excited by the electric field of the photon, which exerts a force on the positively charged protons, moving them away from the neutrons. The neutrons themselves are electrically neutral, so the field has no direct influence on them. However, because the center of mass of the nucleus remains at rest, the neutrons move in the opposite direction, as shown in figure 2. Due to the strong attraction between protons and neutrons, separating them in this way requires a substantial amount of energy, which is the origin of the restoring force of the vibrational motion.

One can formulate the theory of the vibrational motion at various levels of sophistication. At the most detailed level, one can describe the nuclear ground states with quantum-mechanical mean-field theory using the Hartree–Fock equations and interactions that are usually somewhat phenomenological. One can treat the vibrations with the time-dependent extension of this theory. The method works remarkably well in describing many nuclear properties, but is rather complicated.

In fact, the structure of the giant dipole resonance is rather simple and its major features can be understood.
with simpler theory. Empirically, three quantities characterize the giant dipole resonance in spherical nuclei:

- the absorption cross section integrated over frequency
- the center frequency of the resonance
- the resonance width.

Of the three quantities, the integrated cross section is most fundamental to nuclear theory. One can calculate this integral from a sum rule equivalent to the Thomas-Reiche-Kuhn sum rule, also known as the \( f \)-sum rule in atomic physics. The sum rule describes the response of the system to an impulsive electric field. Immediately after the field acts, the particles have a net momentum but have not yet moved to a different position. Thus the impulsive response is independent of dynamics and is determined only by the kinds of charged constituents in the system and their masses. If we consider nuclei to be made of protons and neutrons only, the integrated cross section in a Lorentzian fit to the dipole agrees with the sum rule to within 10%. The presence of pions and other subnuclear constituents modifies the sum rule, but only by adding very-high-frequency contributions. In any case dealing with the temperatures and excitation energies that are accessible in these experiments, the nucleons are the basic degrees of freedom and the sum rule is very reliable.

The frequency of the resonance is a more interesting property to study, because it is sensitive to the size of the nucleus. This dependence is easiest to see with the aid of another integral, the polarizability, that is, the ratio of an induced dipole moment to an applied static electric field. It is equal to an integral over the photon-absorption cross section divided by the square of the frequency. Thus if the polarizability is known one can use it and the dipole sum rule to determine the frequency of the resonance. The nuclear polarizability is not directly measurable, but its behavior can be inferred from the short-range nature of the nuclear force. As shown in the box on the opposite page, the polarizability is proportional to the number of nucleons and to the square of the linear dimension of the nucleus. From the Thomas-Reiche-Kuhn sum rule and the polarizability equation, one finds that the dipole resonance frequency scales inversely with the linear dimension of the nucleus along the electric field.
Dipole frequency and nuclear size

One can derive the relationship between the frequency of the giant dipole oscillation and the size and shape of the nucleus. We do this here using sum rules, following an argument of Arkady Migdal of the Moscow Engineering and Physics Institute. The first sum rule, known as the Thomas-Reiche-Kuhn sum rule, is independent of nuclear interactions and is given by

$$\int \sigma \omega \, d\omega = (2 \pi^2 e^2 / mc) (NZ/A)$$

Here \( \sigma \) is the photon-absorption cross section; \( N, Z \) and \( A \) are the neutron, proton and nucleon numbers; \( m \) is the nucleon mass; and \( e \) is the proton charge. We also need the sum rule for the electric polarizability \( \alpha \):

$$\int (\alpha / c^2) \, d\omega = 2 \pi^2 \alpha / c$$

To see the relationship between the polarizability and the forces and dimensions of the nucleus, we examine the distortion in nuclear charge density caused by a polarizing field. The energy in the presence of an external field is the sum of two terms. The first is the field energy, which we write in terms of the electric field, taken to be in the \( x \) direction, and the deviation \( \delta \rho \) of the proton density from equilibrium:

$$E_1 = -e \int x \delta \rho \, d^3 r$$

The second contribution to the energy is the internal energy of the nucleus. It must vary quadratically with the deviation \( \delta \rho \) of the charge density from equilibrium, because the initial state is stationary. The short-range character of the interaction implies that the energy depends only on the charge-density deviations \( \delta \rho(r) \) and \( \delta \rho(r') \) at nearby points. Hence we can write the total internal energy as an integral over a local energy density:

$$E_2 = \frac{1}{2} \int b(\rho) (\delta \rho(\mathbf{r}))^2 \, d^3 r + \text{constant}$$

The energy-density coefficient \( b \) is related to the nuclear symmetry energy. We next minimize the total energy, the sum of \( E_1 \) and \( E_2 \), with respect to possible choices of the deviation \( \delta \rho \) of the charge density from equilibrium. The energy is a minumum for the choice

$$\delta \rho = -e x / b$$

The polarizability \( \alpha \) is defined as the ratio of the induced dipole moment to the electric field:

$$\alpha = \left( e \int x \delta \rho \, d^3 r / \delta \rho \right) / e = \alpha \left( x^2 / b \right) d^3 r$$

We may then determine the frequency of the dipole by using sum rules in a ratio:

$$\omega^2 = 1 / (1 / \omega^2) \nu = \int \sigma \, d\omega / \left( \alpha / c^2 \right) \, d\omega = (N\alpha / mA) \left( x^2 / b \right) d^3 r$$

From the final expression we see that the frequency measures directly the linear dimension along the electric field, provided the nucleus is large enough that the energy-density coefficient \( b(\rho) \) may be considered constant.

Damping. One important theoretical question that remains is understanding the damping of the giant dipole resonance and how it is affected by excitation energy in the system. The damping is due to coupling of the collective motion to other degrees of freedom, particularly single-particle motion and collective surface vibrations. Damping from the single-particle coupling is well known in plasma physics as "Landau damping," but it does not appear to be the major factor in nuclear resonances. Coupling to surface vibrations, however, is quite important and accounts for most of the observed widths in cold nuclei. The availability of these degrees of freedom increases as the excitation energy grows. This is a trivial observation for classical systems—viscosity increases with temperature—but for quantum systems the dependence can be more subtle. For example, in the Fermi liquid He the quantum sound vibrations become strongly damped as the temperature is increased.

In the case of the giant dipole resonance in nuclei, the temperatures achieved are low compared with the dipole frequency, so the temperature-dependent effects should be small. Francesco Bortignon of the University of Padua and his collaborators tried recently to calculate the damping using finite-temperature perturbation theory. According to their calculations, the increase in damping would not be seen at the temperatures so far achieved.

Finite-temperature perturbation theory is based on a mean-field description of the ensemble of nuclear states, which ignores the fluctuations in nuclear shape from one state to the next. In fact, these fluctuations may be the single most important source of the width at high excitation. The observed broadening could just be a superposition of splittings due to the various deformations. To calculate the broadening...
Rescaled data showing giant-dipole-resonance peak. The measured spectrum from the gamma decay of the $^9$Be nucleus, shown in the previous figure, is here displayed with an exponential factor divided out. The giant dipole resonance now appears as a flat-topped peak. The solid curve shows a theoretical fit to the data from a computer simulation of the statistical decay process. The ingredients of the calculation are the nuclear-level-density formula, the neutron-absorption probability and the characteristics of the giant dipole resonance. The last are determined from the ground-state resonance, which is shown schematically in the figure below. The calculated dipole resonance is the sum of two Lorentzian functions because the nucleus is deformed, with two principal axes of different length. Figure 5

Rotational perturbations

One can also use heavy-ion reactions to study the dependence of the giant dipole resonance on rotational angular momentum. Because of their large masses, heavy ions can bring larger angular momenta to nuclear excitations than can other particles. To measure the angular momentum, however, one needs a special detector such as a “crystal ball”—a large array of scintillation detectors that completely surrounds the target. This array measures the total number of gamma rays that come out of the excited nucleus. The nucleus gets rid of most of its angular momentum by emitting low-energy gamma rays in the final stage of the de-excitation process. These photons are demonstrably quadrupolar, carrying off two units of angular momentum apiece. One determines the initial angular momentum by counting the number of photons emitted. If one measures another quantity at the same time, one can determine its dependence on angular momentum by plotting it as a function of photon multiplicity.

Using this technique, a collaboration of physicists at Heidelberg and Darmstadt found a surprising result: The frequency of the dipole oscillation decreases with increasing angular momentum, dropping by 20% when the angular momentum in the nucleus reaches $50h$. If future studies confirm this finding, the theory of nuclear rotation will have to be reevaluated. At present, it is based on mean-field theory in a rotating coordinate system. As in the case of the heated
nuclei, the predicted additional forces from the rotational motion are insufficient to affect materially the average density of the nucleus. The nucleus becomes more deformed when rotating at high velocity, but this causes opposite shifts of the dipole frequency along the different axes, and these shifts cancel on average.

The only effects predicted in rotating nuclei are additional splittings of the giant dipole resonance and, if the nuclei are deformed, an angular anisotropy of the emitted photons. We can understand these effects classically. Imagine the long axis of the nucleus to be a dipole antenna that is rotating so as to present a constantly changing aspect to an observer. The electromagnetic radiation will be amplitude modulated at the rotational frequency, implying that the dipole oscillation frequency will be split by that amount.

The same effect occurs in the quantum calculations of the giant dipole resonance in Hartree–Fock theory. However, the maximum rotational rates achieved are only about 2 MeV/s. The resulting rotational splitting of 2 MeV is much smaller than the damping width of the giant dipole resonance, which is 4–7 MeV. Hence it does not appear possible to observe this effect directly.

On the other hand, it is possible to see the predicted anisotropy in the angular distribution of photons. The effect arises because the rotation of a prolate nucleus tends to be about an axis perpendicular to the long axis. The lower-frequency component of the dipole will tend to be emitted perpendicular to the long axis as well, because that axis is acting as the antenna. This, together with the fact that the rotational angular momentum in collisions with heavy ions is perpendicular to the beam direction, leads us to expect that the lower-frequency giant-dipole-resonance photons will be emitted preferentially perpendicular to the beam direction. The Heidelberg-Darmstadt group found this to be the case in their experiment.

**Level density.** The recent experiments have called into question earlier findings on several properties of highly excited nuclei, but they have confirmed at least one aspect of the previous findings: the nuclear level density. As we mentioned above, high-energy phenomena are very sensitive to the rate of change of level density. This is conventionally parameterized with a formula for the level density that Hans Bethe derived from the Fermi gas model. The main parameter in the formula is the single-particle level density. If the sum rule is assumed to be satisfied, then the
yield of dipole photons may be used to infer that parameter. This determines the parameter far more accurately than was possible from other measurements. The level density in a normal Fermi system is proportional to the effective mass of the particles at the Fermi surface. The value extracted implies that nucleons are a little bit heavier inside nuclei than in free space, but further discussion of this finding would take us too far afield.

**Prospects**

Several laboratories are pursuing the interesting question, In how highly excited a nucleus can one measure the giant dipole resonance? There are many issues here: First, is a compound nucleus formed in collisions at high excitation energy? Studies of other particles emitted in heavy-ion reactions indicate that interacting projectile and target nucleons may produce a very short-lived equilibrated zone of very high temperature. However, this zone loses its energy by direct particle emission without transferring it to the remainder of the target and projectile. Thus the global equilibration necessary for analyzing the giant-dipole photon yields may not be attained. Even if the equilibration does occur, the giant dipole resonance may no longer exist. The resonance phenomenon requires there to be a restoring force in the system, and the system must last long enough, say a few times $10^{-22}$ sec, to undergo a cycle or so of the oscillation. If the nucleus does not last that long, the peak in the frequency distribution will disappear. The presence of the dipole with the predicted strength would confirm the continued existence of compound nuclei at higher temperature. Mean-field theory predicts that nuclei can exist to temperatures of at least 5 MeV, but whether this can be seen experimentally is unknown.

Besides the problem of forming the equilibrated system, another issue arises at higher energies, concerning the photon emission mechanism. Heavy-ion collisions can produce photons with energies much higher than that of the giant dipole resonance. For example, the observed photon spectrum from energetic heavy ions falls much less steeply than the spectrum from less energetic heavy ions, and extends up to 100 MeV. It would be difficult to attribute these energetic photons to the decay of an equilibrated compound system when the target is large, because the temperature would be too low. The photons may be bremsstrahlung from collisions between individual protons and neutrons in the first stages of the nucleus–nucleus interaction. If this is so, the complete description of the photon yields will be much more complicated. Even if a statistical description is applied to these photons, it is likely to be for a small zone rather than the entire system. The origin of these higher-energy photons is an interesting problem and is a focus of current study by both theorists and experimenters.

**References**