ROLE OF CORE POLARIZATION IN TWO-BODY INTERACTION

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Abstract: The correction to the interaction of two valence particles in $^{18}$O and $^{48}$Sc, due to perturbations of the closed shell wave functions, is calculated and found to be as large as 30% of the first-order interaction. The qualitative behaviour of this interaction is: attractive for $T = 1$, low $J$ states; repulsive for $T = 1$, high $J$ states; and small for $T = 0$ states.

1. Introduction

The interaction between two particles in a nucleus is modified by the presence of the other particles in several ways. Recent reaction-matrix calculations take account of the other particles with the Pauli operator and a modified nucleon propagator. This provides a static treatment of the closed shells, since they only participate by providing an effective potential well, and by their presence restricting the intermediate states in the two-particle wave function. It might be expected that polarization of the closed shells would be appreciable in the two-body effective interaction, as it is known to be important in one-body operators $^1-^3$).

We here calculate some dynamical effects of the closed-shell core on the particle-particle interaction $^4$. In lowest order, this comes about when one particle excites the core through a potential interaction, which is then de-excited by the other particle. For valence particles within a given shell, parity conservation requires the intermediate core particle wave functions to have the same parity as the hole, thus there has to be a jump of two shells for the intermediate state.

2. The Matrix Element

We do not want to consider the entire interaction between two particles through the core, because part of this will have already been included in a reaction-matrix calculation, and part gives a self-energy contribution to each particle which has no effect on the two-particle spectrum. So we consider only the graph shown in fig. 1 and its trivial variants made by antisymmetrizing the two interacting particles at an

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$^2$ This work was supported in part by the U.S. Atomic Energy Commission
$^3$ A somewhat different approach to the same problem is given in ref. $^4$.)
interaction line. The algebraic form of this interaction is the second-order perturbation theory expression

\[
V \frac{1}{T}
\]

We calculate \( V \) using the Kallio-Kolltveit potential and harmonic oscillator wavefunctions. It is more difficult to determine a value for the intermediate state energy \( T = E - E_0 \). A first guess would be the harmonic oscillator value,

\[
T = 2\hbar \omega_0.
\]

This cannot be very good, because the excited particle extends through the surface of the nucleus, where the infinite harmonic oscillator well description is poor, and also the highly excited particle sees more of the hard cores of the nucleons, invalidating the velocity-independent potential well picture. These two effects go in opposite directions.

[Diagram]

Fig. 1.

There are further corrections of higher order in perturbation theory which can be subsumed in a change in the intermediate state energy denominator. We can take the Tamm-Dancoff or random phase approximation description of the intermediate state, which will produce an energy shift of the state from its original position \( T_0 \). Then if the particle-hole matrix elements go as the corresponding core excitation matrix elements, which is reasonable, the energy denominator should be the eigenenergy of this intermediate state.

In sect. 3 formulae for the angular momentum structure of the interaction are given. Then simplifications are made which show qualitatively that the effective force is mostly pairing in nature; finally numerical computations are done for the interesting cases of \(^{18}\text{O}\) and \(^{42}\text{Sc}\).

3. Angular Momentum Algebra

We consider two particles of angular momentum \( j \) which scatter to states of angular momentum \( j' \), by exciting a particle in the h shell to the p shell and then de-exciting it.
A particle-particle force will be given with matrix elements
\[
\langle (j'h)^J | V | (jp)^J' \rangle \quad \langle (j'h)^J | V | (jp)^J' \rangle,
\]
which can by standard techniques be brought to a form more convenient for this calculation
\[
\langle j | V | (j'ph)^T \rangle = \sum_{J} \sqrt{\frac{2J+1}{2J+1}} U(j'ph;JL) \langle (jh)^J | V | (pj')^J \rangle.
\]
(2)

The overall matrix element is, before antisymmetrizing,
\[
\frac{1}{T} V = \sum_{L} \langle (j(1)j(2))^L | V(1, i) | [j'(1)(p(i)h)^T(j(2))^L] \rangle \frac{1}{T_{int}} \langle j(j'h^T) | V | (j'p)^J' \rangle.
\]
(3)

This is brought into a form involving only matrix elements of type (2) by recoupling the excitation in the second term, and interchanging the order of the particle and the excitation. The first operation requires a Racah coefficient and the second a phase factor. The result is finally
\[
\frac{1}{T} V = \sum_{L} (-1)^{L+j+j'} U(j'j;j) \langle j | V | (j'ph)^T \rangle \frac{1}{T_{int}} \langle j' | V | (jph)^T \rangle.
\]
(4)

With an L-S or isobaric spin formalism, coupling to S, T is included in a similar manner with the factors
\[
\sum_{T_e} (-1)^{T_e} U(\frac{1}{2}T_e; T_{tot}; \frac{1}{2}; \frac{1}{2}) V_{T_e} V'_{T_e},
\]
(5)
\[
\sum_{S_e} (-1)^{S_e} U(\frac{1}{2}S_e; S_{tot}; \frac{1}{2}; \frac{1}{2}) V_{S_e} V'_{S_e},
\]
(6)

where \(S_e\) is the spin of the particle-hole excitation of the core, and \(S_{tot}\) is the total spin of the system.

The \(T\)-sum is explicitly
\[
VV'(T_{tot} = 1) = VV'(T_e = 0) + \frac{1}{2} VV'(T_e = 1),
\]
(7)
\[
VV'(T_{tot} = 0) = VV'(T_e = 0) - VV'(T_e = 1).
\]
(8)

For the oxygen calculation, it was convenient not to use isobaric spin but to treat neutrons and protons separately. The sum over intermediate spin configurations is decomposed into a sum over particle-particle interactions. This requires the extra Racah coefficient of eq. (2), and is
\[
V^2(S = 1) = \frac{1}{2} V_s V_s + \frac{3}{4} V_s V_t + \frac{1}{2} V_t V_s,
\]
(9)
\[
V^2(S = 0) = -\frac{1}{2} V_s V_s + \frac{3}{4} V_s V_t + \frac{1}{2} V_t V_s,
\]
(10)

where \(V_s\) and \(V_t\) denote matrix elements of \(V\) for particles in a singlet and a triplet state, respectively.

The above equations hold for the interaction of neutrons through the proton core. For interactions through the neutron core, the identity of the particles allows more
graphs; for a spatially symmetric force, the result will be to double the singlet \( V \) and cancel the triplet \( V \) component.

For the case of the diagonal interaction of two particles in an \( L \)-shell, formula (3) reduces to

\[
\langle (ll')|V \frac{1}{T} V|ll' \rangle = \sum (-1)^{l_1} U(lLJl; ll') \frac{1}{T} \langle l|V||l(|ph)z| \rangle^2.
\]  

(11)

When the valence particles are coupled to \( J = 0 \), the \( U \) coefficients are unity; for other \( J \), and reasonably larger \( l \), they are small and of variable sign. Thus on the average, for \( J \neq 0 \), the contributions of the different multipolarities of the core excitation are small and tend to cancel. We think of this in the following way: when the two particles are coupled to \( J = 0 \), their wave function is so restricted that when one particle excites the core, the other is necessarily in a position to de-excite it. For other \( J \) there is more freedom to the wave function and therefore less of a chance that the intermediate core state be coupled properly to the valence particles.

For \( J = 0 \), the matrix element is

\[
\frac{1}{T} \left( \sum_{\text{even } L} \langle l|V|l(|ph)L\rangle^2 - \sum_{\text{odd } L} \langle l|V|l(|ph)L\rangle^2 \right).
\]  

(12)

It can be shown that odd \( L \) are much less important than even \( L \), so in effect the multipolarities are coherent for \( J = 0 \), and provide an attractive force. The argument proceeds by expanding a local interaction

\[
V(12) = \sum_K a_K(r_1r_2)[Y^K(1)Y^K(2)]^0
\]

for the direct part of a matrix element in \( L-S \) coupling:

\[
\langle l(1)|V||l(1)(|ph)L\rangle = \delta_{KK}\langle l(1)|Y^K||l(1)\rangle\langle |Y^K|(|ph)L\rangle \times \text{radial integral}.
\]

(13)

The factor

\[
\langle l|Y^K|l \rangle = (2l+1)^{K+1} \begin{pmatrix} 2K+1 \\ 4\pi \end{pmatrix} \begin{pmatrix} l & l & K \\ 0 & 0 & 0 \end{pmatrix}
\]

is non-zero only for \( K \) even, and therefore \( L \) must be even in the matrix element.

Thus we find that the effective force in \( L-S \) coupling simulates a pairing force. This is different in character from the short-range force, which is strong in the state of highest \( L \) as well as \( L = 0 \).

4. Numerical Computations

4.1. THE \(^{18}\text{O} \) NUCLEUS

To find the correction to the interaction of the two sd neutrons in \(^{18}\text{O} \), we apply formulae (4), (9) and (10), allowing both 0s- and 0p-shell excitations. There is reason to think that \( T_{\text{int}} \) should be different in these cases; however, the overall uncertainty in this quantity is so large that this small error can be safely ignored. In the
calculation we use \( T_{\text{int}} = 25 \text{ MeV} \) (ref. 5)). The Kallio-Kolltveit force was calculated \(^\dagger\) with wave functions of harmonic oscillator constant \( \hbar \omega = 13.5 \text{ MeV} \). Table 1 gives the strength of the force in relative \( ns \) states of this harmonic oscillator.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative coordinate matrix elements of the Kallio-Kolltveit force.</td>
</tr>
<tr>
<td>( \langle 0s</td>
</tr>
<tr>
<td>( V_s )</td>
</tr>
<tr>
<td>( V_t )</td>
</tr>
</tbody>
</table>

The reasonableness of this off-diagonal force may be checked by calculating the radius of the nucleus with this force and comparing with the experimental. With this criterion the force is too large, unless the \( G \)-matrix corrections are taken into account (ref. 7)). However, these corrections may be small for the valence particles in \( ^{18}\text{O} \).

The contribution to \( V^2 \) for each multipolarity of core excitation is given in table 2. The exclusion principle is violated in some of the intermediate states.

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution of various core states to ( V^2 ) in ( ^{18}\text{O} ).</td>
</tr>
<tr>
<td>Valence state ( L_q )</td>
</tr>
<tr>
<td>( dd \rightarrow dd )</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>( ss \rightarrow ss )</td>
</tr>
<tr>
<td>( dd \rightarrow ss )</td>
</tr>
</tbody>
</table>

Table 3 gives the final value of the matrix elements, with the first-order interaction of the Kallio-Kolltveit force for comparison.

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second-order energy perturbation from core polarization in ( ^{18}\text{O} ).</td>
</tr>
<tr>
<td>Configuration</td>
</tr>
<tr>
<td>( (d_4d_4)^L=0 )</td>
</tr>
<tr>
<td>( (d_4d_4)^L=2 )</td>
</tr>
<tr>
<td>( (d_4d_4)^L=4 )</td>
</tr>
<tr>
<td>( (s_4s_4)^{a=0} )</td>
</tr>
<tr>
<td>( (d_4d_4)^{a=1} \rightarrow (s_4s_4)^{a=1} )</td>
</tr>
</tbody>
</table>

4.2. THE \( ^{48}\text{Sc} \) NUCLEUS

Here the interest is in the corrections to the \( f_2^*f_2^* \) interaction due to core polarization. Because there are so many intermediate states of core excitation, it was convenient to

\(^\dagger\) These matrix elements and the ones of Ca were kindly supplied by H. Mavromatis and L. Zamick.
use a computer for the preliminary angular momentum algebra, converting the particle-particle matrix elements to particle-core excitation matrix elements. Table 4 gives the strengths through the various multipoles of $L$ and $T$ in the core. Again, to avoid the complications of fractional parentage calculation, the exclusion principle is ignored for some of the states.

<table>
<thead>
<tr>
<th>$L$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V^*(T_0 = 0)$</td>
<td>3.65</td>
<td>2.47</td>
<td>101.62</td>
<td>10.55</td>
<td>49.00</td>
<td>7.50</td>
<td>8.86</td>
<td>3.62</td>
</tr>
<tr>
<td>$V^*(T_0 = 1)$</td>
<td>2.58</td>
<td>11.53</td>
<td>67.92</td>
<td>38.38</td>
<td>35.52</td>
<td>36.03</td>
<td>7.30</td>
<td>36.40</td>
</tr>
</tbody>
</table>

Finally, the energy corrections to the two-particle spectrum, with $T_{int} = 19$ MeV, are given in table 5.

<table>
<thead>
<tr>
<th>$L$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; (f^2 f^2)^J</td>
<td>V \frac{1}{T} V (f^2 f^2)^J &gt;$</td>
<td>-1.02</td>
<td>-0.25</td>
<td>-0.27</td>
<td>+0.11</td>
<td>+0.27</td>
<td>+0.15</td>
<td>+0.55</td>
</tr>
</tbody>
</table>

5. Conclusion

Mottelson has argued that, when the harmonic oscillator picture is valid, there will be a polarization of the core with the same quadrupole moment as the polarizing particle 8), and analogously the interaction of two particles through the quadrupole excitations of the core will be the same as the first-order $P_2$ interaction of the particles. We find that the quadrupole component of the core polarization is the most important, but that the other even multipoles contribute enough to give the overall force a pairing character. This is opposite of what is found from the $G$-matrix corrections to the two-body interaction; these corrections cut down interactions in high relative $s$-states more than in low relative $s$-states, which means the force is cut down more for particles coupled to low angular momentum than for high 9).

Because calculations with pairing force have been so successful in heavy nuclei 10), it would be of interest to see how strong the contribution is to this from core polarization. Qualitatively, the mass-dependence of the second order perturbation $V(1/T)V$ is the same as the first order interaction, $V \approx 1/A$. On the other hand, heavy nuclei have a preponderance of neutrons, which dampen the $S = 0$ neutron-neutron interaction through the core. A preliminary calculation with all of the harmonic oscillator approximations indicates that the second order force is still of the same relative importance as in the lighter nuclei considered.

The author is indebted to G. E. Brown for suggesting the problem and for many conversations.
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