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Giant $M1$ states in Zr isotopes within the simple shell model

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The newly observed $M1$ states in the (p,p') experiment on the Zr isotopes are considered within the simple shell model. The calculation with a constant strength δ function interaction reproduces the excitation energies and the slight increase of the $M1$ strength at small momentum transfer with mass number. We need about a factor of 2 quenching to reproduce the cross sections at small angles, which is in accordance with the general finding for magnetic transitions.

[NUCLEAR STRUCTURE $^{90-96}$Zr; shell model calculations, 1^+ states.]

Very recently Anantaraman *et al.* have performed a (p,p') experiment at $E_p = 200$ MeV on the Zr isotopes and found for the first time giant $M1$ states in these nuclei.[1] This is a very interesting result because before this experiment no $M1$ states had been reported in heavy nuclei ($A \gtrsim 90$) in both hadron and electron experiments.[2] If we look at shell structures of these heavy nuclei, we expect that the $l \cdot s$ particle-hole pairs, $g_{7/2}$-$g_{9/2}{}^{-1}$, $h_{9/2}$-$h_{11/2}{}^{-1}$, $i_{11/2}$-$i_{13/2}{}^{-1}$, . . . , should produce $M1$ states a few MeV above the $l \cdot s$ splitting energy. The fact that $M1$ strengths were not observed in heavy nuclei has been a puzzle for a long time. Thus, the recent (p,p') experiment[1] provides new valuable information for the discussion of $M1$ strength in nuclei. In this paper we compare the experimental data with a simple shell model calculation based on the general consideration of the single-particle wave functions and the residual interaction.

There has been some controversy recently about the appropriate single-particle model, engendered by the failure to find the $M1$ strength in ^{208}Pb.[2] Brown *et al.*[3] have argued that the single-particle energy differences obtained from an empirical analysis should be multiplied by a factor $m/m^* \sim 1/0.7$ to use in calculations of the $M1$ strength. We disagree with this prescription on two grounds. First, changes in effective mass for collective states can only arise from collective energy shifts, and cannot be applied to the $M1$ state, which has only shifted by a couple of MeV. Second, the effective mass m^* applies to the single-particle kinetic energy, and not to energy differences originated from the spin-orbit field. All of the single-particle energy in the $M1$ state is due to the spin-orbit field. In our calculation, we take single-particle wave functions from Woods-Saxon well calculations. These wave functions are identical to Hartree-Fock wave functions for practical purposes. The crucial spin-orbit interactions we adopt from the phenomenological optical model analysis of Becchetti and Greenless.[4] This results in 6 MeV spin-orbit splitting between the $g_{7/2}$ and $g_{9/2}$ orbits for ^{90}Zr.

The residual interaction can be taken to be zero range for describing excitation of long wavelength states. The noncentral interactions for shell model matrix elements, and the finite range aspects of the force can be included by a small shift in the strength parameter of the zero-range force.[5]

$$V_{\text{res}} = \bar{V}_\sigma \delta(\vec{r}_1 - \vec{r}_2)\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \ , \tag{1}$$

where $\bar{V}_\sigma = V_\sigma + V_{\sigma\tau}$ with V_σ being the strength of the non-τ-dependent interaction and with $V_{\sigma\tau}$ being that of the τ-dependent interaction. We note that the strength $V_{\sigma\tau}$ deduced from the giant Gamow-Teller state in medium and heavy nuclei is 220 MeV fm^3.[5] The known $M1$ state in ^{48}Ca can additionally be used to determine V_σ and we find that $V_\sigma \cong V_{\sigma\tau}$. Microscopic calculations based on the Brueckner theory with the Reid potential also indicate that $V_\sigma \cong V_{\sigma\tau}$.

Using the above model Hamiltonian, we calculated the excitation energies and the (p,p') cross sections at 0°. In order to obtain the cross sections, we used the empirically obtained distortion factor N_D and the $\sigma\tau$ interaction strength $J_{\sigma\tau}$ from the work by Goodman *et al.*[6] Furthermore, the small isospin independent spin-spin term is added to the $\sigma\tau$ interaction strength for the neutron particle hole excitation by the scattering proton. The isospin independent strength J_σ is estimated from the work by Love and Franey[7] to be

$$(J_\sigma + J_{\sigma\tau})/J_{\sigma\tau} \simeq 1.15 \ . \tag{2}$$

A further assumption is that the distortion factor does not change greatly from $E_p = 120$ to 200 MeV. Although we believe this to be qualitatively correct, it needs to be checked quantitatively. The calculated results along with the experimental data are depicted in Table I. The almost constant excitation energies

TABLE I. Experimental and calculated results for the excitation energies and the cross sections at $\theta=4°$. Calculated results with and without the residual interaction, $\bar{V}_\sigma=370$ and 0 MeV fm^3, respectively, are shown in the second and the third columns.

	Expt.		$\bar{V}_\sigma=370$ MeV fm^3		$\bar{V}_\sigma=0$ MeV fm^3	
	E_x (MeV)	$\frac{d\sigma}{d\Omega}$ (mb/sr)	E_x (MeV)	$\frac{d\sigma}{d\Omega}$ (mb/sr)	E_x (MeV)	$\frac{d\sigma}{d\Omega}$ (mb/sr)
^{90}Zr	8.90±0.15	2.8±0.3	8.9	5.9	6.2	5.9
^{92}Zr	8.8 ±0.2	2.8±0.3	8.9	6.3	6.2	5.9
			2.6	0.6	2.1	1.0
^{94}Zr	8.63±0.15	3.1±0.3	8.9	6.7	6.2	5.9
			3.1	1.1	2.1	2.0
^{96}Zr			8.9	7.2	6.1	5.9
			3.5	1.5	2.2	3.0

with the mass number are reproduced with the strength $\bar{V}_\sigma=370$ MeV fm^3, when only the s-d-g shells are considered. We have also checked the effect of the higher oscillator shells, but E_x is decreased by only ~2%. This strength $\bar{V}_\sigma=370$ MeV fm^3 indicates that $V_\sigma \lesssim V_{\sigma\tau}$.

As for the strength, the experimental cross sections are the ones at $\theta=4°$ and we need to further extrapolate the calculated cross sections at $\theta=0°$ to those at $\theta=4°$. The DWBA (distorted-wave Born approximation) angular distribution is helpful for this extrapolation, which has been provided by Anantaraman *et al.*[1] They state that $d\sigma/d\Omega$ at 0° is about a factor of 2.5 larger than $d\sigma/d\Omega$ at 4°. This factor is used to obtain the theoretical cross sections at $\theta=4°$. Now let us look at the theoretical numbers which correspond to $E_x \sim 8.9$ MeV. The calculated cross section increases slightly with mass number and there seems to be a slight indication of this in the experiment. This increase of the cross section is due to the small admixture of the $d_{3/2}$-$d_{5/2}^{-1}$ strength which increases with the occupation number of the $d_{5/2}$ shell relative to the main $g_{7/2}$-$g_{9/2}^{-1}$ configuration due to the residual interaction. If our assumption that $J_{\sigma\tau}$ and N_D are not much different at $E_p=200$ MeV from the values at $E_p=120$ MeV, we need about a factor of 2 quenching effect. This value is reasonable as the quenching factor for the magnetic strength in nuclei.[2,8] Furthermore, our simple analysis predicts some $M1$ strength at $E_x \sim 3$ MeV in the Zr isotopes $A \geqslant 92$. For information, the calculated results without the residual interaction are also shown on the right of Table I.

In conclusion, the newly observed $M1$ states are readily reproduced in terms of the simple shell model. The strength of the residual interaction is reasonable and the quenching factor needed to reproduce the cross section at small angles is about 2, which is again consistent with the systematics in this region. Therefore, our study indicates that simple shell model considerations account for the $M1$ states in nuclei as heavy as the Zr isotopes. It may mean that in fact $M1$ states with large $M1$ strength do exist in ^{208}Pb but so far they have not been detected due to the high level densities. Our present study suggests the need for further studies: Experimentally, we should carefully look for $M1$ strengths at $E_x=6 \sim 8$ MeV in ^{208}Pb, and, theoretically, we need to reexamine the concept of the effective mass for the $M1$ states in heavy nuclei.

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