PROTON POLARIZABILITY IN THE MIT BAG MODEL ☆

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The polarizability of the proton is calculated in a simplified MIT bag model and compared with an analysis of Compton scattering. Qualitative agreement is obtained for the sum of the electric and magnetic polarizabilities, which is found to be within 30% of the experiment.

The determination of the electric and magnetic proton polarizabilities α and β from experiment and their connection with sum rules has been controversial for some time. The combination $\alpha + \beta$ is well determined by the sum rule [‡1]

$$\alpha + \beta = \frac{1}{2\pi^2} \int_0^\infty \frac{\sigma(\omega)}{\omega^2} \mathrm{d}\omega = 14.1 \pm 0.3 . \qquad (1)$$

The difference $\alpha - \beta$, however, is model dependent. Values of -9.7 [1] -6 [2] and $+12.5$ [3] have been obtained, depending on the type of dispersion relation and the channels taken into account. The most recent analysis [1] of Compton scattering at 80–110 MeV gives values

$$\alpha = 20 \pm 1.1, \beta = -(6 \pm 1.6) .$$

The authors of ref. [1] claim that dropping the ω^4 -terms in former analyses [4,5] has yielded incorrect values of $\alpha = 9 \pm 2(10.7 \pm 1.1)$ and $\beta = 2 \pm 2(-0.7 \pm 1.6)$, respectively.

In this work we calculate the polarizabilities in the MIT bag model [6,7]. This model has been very successful in the prediction of ground-state properties of hadrons; it is generally not quite so good for dynamical quantities, such as transition probabilities.

In a model of non-interacting quarks, the polarizabilities are given by:

$$\alpha = 2 \sum_{j=1}^{3} \sum_{\lambda} e_j^2 \frac{|\langle \phi_{0j} | \gamma_0 z | \phi_{\lambda j} \rangle|^2}{E_j^\lambda - E_j} , \qquad (2a)$$

$$\beta = 2 \sum_{j=1}^{3} \sum_{\lambda} e_j^2 \frac{|\langle \phi_{0j} | \boldsymbol{\gamma} \cdot \frac{1}{2}(\hat{z} \times \boldsymbol{r}) | \phi_{\lambda j} \rangle|^2}{E_j^\lambda - E_j} . \qquad (2b)$$

Here $|\phi_{0j}\rangle$ and $|\phi_{\lambda j}\rangle$ are normalized single-quark wavefunctions, and γ_0 and $\boldsymbol{\gamma}$ are the usual Dirac matrices; $|\phi_{0j}\rangle$ is the $1s_{1/2}$ wavefunction of the nucleon ground state, and $|\phi_{\lambda j}\rangle$ are excited quark- or antiquark states. The charge of the jth quark is e_j. In the case of the proton, the sum over e_j reduces to $\Sigma_{j=1}^{3} e_j^2 = e^2(\frac{1}{9} + \frac{4}{9} + \frac{4}{9}) = e^2$. The wavefunctions $|\phi_{0j}\rangle$ and $|\phi_{\lambda j}\rangle$ are assumed to be the same for all three quarks in a particular space-spin state. Only a few terms are important in the sum over the space-spin states $|\phi_{\lambda j}\rangle$. The 1p and 1s quark and antiquark states are taken into account. The other states lie further away in the energy spectrum, and thus give much smaller contributions. Of these, the $1p_{1/2}$, $1p_{3/2}$ quark and the $1s_{1/2}$ antiquark state contribute to the electric polarizability, whereas for the magnetic polarizability the intermediate states are the $1s_{1/2}$ quark and the $1p_{1/2}$, $1p_{3/2}$ antiquark. The antiquark contribution can be viewed as an exchange effect.

We use the same size bag for all states, and assume that the quarks have zero rest mass. Then the wavefunctions to be considered are:

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‡1 α and β are given in units of 10^{-43} cm^3.

$$\langle\phi_{0j}| = \frac{N_0}{\sqrt{4\pi}}\begin{pmatrix} ij_0(x_0R^{-1}r)\chi_m \\ -j_1(x_0R^{-1}r)(\sigma\hat{r})\chi_m \end{pmatrix},$$

$$\langle\phi_{1j}| = \frac{N_1}{\sqrt{4\pi}}\begin{pmatrix} -ij_1(x_1R^{-1}r)(\sigma\hat{r})\chi_m \\ j_0(x_1R^{-1}r)\chi_m \end{pmatrix},$$

$$\langle\phi_{2j}| = \frac{N_2}{\sqrt{4\pi}}\begin{pmatrix} ij_1(x_2R^{-1}r)\phi_m \\ -j_2(x_2R^{-1}r)(\sigma\hat{r})\phi_m \end{pmatrix}, \qquad (3)$$

$$\langle\phi_{0j}|_{\text{anti}} = \frac{N_0}{\sqrt{4\pi}}\begin{pmatrix} j_1(-x_0R^{-1}r)(\sigma\hat{r})\chi_m \\ ij_0(-x_0R^{-1}r)\chi_m \end{pmatrix},$$

$$\langle\phi_{1j}|_{\text{anti}} = \frac{N_1}{\sqrt{4\pi}}\begin{pmatrix} j_0(-x_1R^{-1}r)\chi_m \\ ij_1(-x_1R^{-1}r)(\sigma\hat{r})\chi_m \end{pmatrix},$$

$$\langle\phi_{2j}|_{\text{anti}} = \frac{N_2}{\sqrt{4\pi}}\begin{pmatrix} j_2(-x_2R^{-1}r)(\sigma\hat{r})\phi_m \\ ij_1(-x_2R^{-1}r)\phi_m \end{pmatrix}.$$

The χ_m are the conventional Dirac spinors, and the ϕ_m are the $j = \frac{3}{2}$ spherical harmonics. The normalization factors N_λ are determined by

$$1 = \frac{N_\lambda^2}{4\pi}\int_0^R \phi_{\lambda j}^\dagger \phi_{\lambda j}\, d^3r, \quad \lambda = 0, 1, 2, \quad j = 1, 2, 3. \qquad (4)$$

The energy eigenvalues for the quark, respectively, antiquark states are given by $\pm x_\lambda/R$, where $x_0 = 2.04$, $x_1 = 3.81$ and $x_2 = 3.2$.

We determine the bag radius R by fitting it to the experimental r.m.s. radius of the proton:

$$\frac{N_0^2}{4\pi}\int_0^R \phi_{0j}^\dagger\phi_{0j} r^2\, d^3r = \langle r^2\rangle_{\exp} = 0.88^2\ \text{fm}^2, \qquad (5)$$

which gives $R = 1.2$ fm. This value is 20% larger than the one which is obtained by fitting the hadronic mass spectrum [6], However, it is important for our purposes to fit $\langle r^2\rangle$, since both polarizabilities are proportional to the square of a matrix element, which is linear in r. The energy of this bag ($3 \times 2.04/1.2$ fm $= 1005$ MeV) is in good agreement with the nucleon mass.

The sum over the single-quark states in eqs. (2a) and (2b) does not entirely coincide with the physically existing excited hadronic states. For example, spurious states of the $L = 1$ [56] (in the SU(6)-language) have to be projected off the sum over the 1p quarks in eq. (2a) in order to obtain pure $L = 1$ [70] states which involve the $N^*(1/2^-)$ and $N^*(3/2^-)$ resonances, which give the most important contributions. Sound discussions of the cavity approximation and its refinements can be found in refs. [7–9]. Also, the proton component in the sum over the three $1s_{1/2}$ -quarks in eq. (2b) has to be projected off, since only the Δ^+ -contribution is relevant. The $p\Delta^+$ matrix element is related to the quark matrix elements by

$$\langle p|\sum_j e_j \tfrac{1}{2}\gamma(\hat{z}\times r)_j|\Delta^+\rangle$$

$$= (8/9)^{1/2} e^2 \langle\phi_0|\tfrac{1}{2}\gamma(\hat{z}\times r)|\phi_0\rangle. \qquad (6)$$

Thus the corresponding terms in the sum in eq. (2b) must be multiplied with $\frac{8}{9}$.

Assuming no interaction between quarks, the energy denominators for the antiquark states are given by the difference of the $1s_{1/2}$ ground state with positive energy and the antiquark states with negative energy. They are $-2x_0/R$ for the $1s_{1/2}^{\text{anti}}$, $-(x_0 + x_1)/R$ for the $1p_{1/2}^{\text{anti}}$ and $-(x_0 + x_2)/R$ for the $1p_{3/2}^{\text{anti}}$ states. We use these tree particle energy denominators for the antiquark states, but the free particle description is too crude for the positive energy states. For quark particle states, we modify the formulas by using energy denominators of the known physical states. For the $1p_{1/2}$ and $1p_{3/2}$ quark states, we use the average excitation energy of the $N^*_{1/2^-}$, $N^*_{3/2^-}$ doublet, $\Delta E \approx 600$ MeV. For the $1s_{1/2}$-quark state, the excitation energy of the Δ-resonance $\Delta E \approx 300$ MeV is taken.

The evaluation of α and β is now straightforward. The results are

$$\alpha = 7.1 = 9.6 - 2.5, \quad \beta = 2.6 = 5.0 - 2.4. \qquad (7)$$

decomposed into the contribution from the quark- and antiquark-states. The electric polarizability is dominated by the $N^*_{1/2^-}$, $N^*_{3/2^-}$ doublet, and the magnetic one by the Δ^+-resonance. These values disagree strongly with the recent analysis of the data [1], they are however in fair agreement with earlier results [4,5]. The sum rule (1) is understimated by 30%. The missing enclosure of higher lying states partly accounts for this difference. The value for $\alpha - \beta$ is 4.5, thus lying in the spectrum of various predictions [1–3]. A negative magnetic polarizability (diamagnetism) is not consistent with the given quark model. Possible explanations for this discrepancy are, that the quark–quark interactions change the sign of β, or that the analysis [1] of the data is insufficient. Given the still remaining uncertainties in the interpretation of the experimental results, we conclude that our result at least does not clearly contradict the experiment and is in fair agreement with the sum rule. Clearly, new data and a more rigorous treatment of the dispersion relations yielding $\alpha - \beta$ are necessary.

The corresponding results for the neutron can be obtained quite easily. The sum $\Sigma_{j=1}^{3} e_j^2$ is $\frac{2}{3} e^2$ for the neutron instead of e^2. The overlap between the neutron and the Δ^0, however, is the same as for the corresponding proton-Δ^+ matrix element. The values for the neutron polarizabilities are $\alpha_n = 4.7$ and $\beta_n = 3.4$.

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References

[1] D.M. Akhmedov and L.V. Filkov, Nucl. Phys. B125 (1977) 530.
[2] J. Bernabeu, T.E.O. Ericson and C. Ferro Fontan, Phys. Lett. 49B (1974) 381.
[3] I. Guiasu and E.E. Radescu, Phys. Lett. 62B (1976) 193.
[4] V.I. Goldansky et al., Nucl. Phys. 18 (1960) 473.
[5] P.S. Baranov et al., Yad. Fiz. 5 (1967) 1221; 7 (1968) 100.
[6] T.A. DeGrand et al., Phys. Rev. D12 (1976) 2060.
[7] T.A. DeGrand and R.J. Jaffe, Ann. Phys. 100 (1976) 425; 101 (1976) 396.
[8] C. Rebbi, Phys. Rev. D14 (1976) 2362.
[9] J. Donoghue and K. Johnson, Phys. Rev. D21 (1980) 1975.