NUCLEON TUNNELLING MODEL OF MASS DIFFUSION IN DEEP INELASTIC HEAVY ION COLLISIONS ☆

C.M. KO, G.F. BERTSCH and D. CHA
Department of Physics and Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA

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We derive a simple expression for the mass diffusion coefficient in deep inelastic collisions, based on a proximity formulation of nucleon tunnelling. The predicted value of the coefficient is consistent with empirical data. The mass diffusion coefficient has a negligible dependence on excitation energy in the physically interesting domain.

Deep inelastic collisions between heavy ions occur at incident energies well above the Coulomb barrier. Besides the large cross section for energy damped events, these collisions are also characterized by many nucleons exchanged between the two ions. These latter features are evidenced by the broad mass distribution of the final fragments. Models for describing these phenomena are based on diffusion. In a simple model used by Norenberg [1] and Moretto et al. [2], the nucleon diffusion is assumed to occur when the two ions are in contact and rotating about each other; the dependence of the diffusion rate on the relative separation of the ions is treated discontinuously. Diffusion coefficients have also been extracted from the experimental data with this model by Schröder et al. [3], Sventek et al. [4] and Wolschin et al. [5]. A typical value for the empirical mass diffusion coefficient is

$$D_A \sim 10^{22}\ \text{nucleon}^2/\text{s},$$

and it tends to increase as the combined system becomes heavier.

Determination of the mass diffusion coefficient from a microscopic model has been attempted by Ayik et al. [6], and by Randrup [7]. We shall use a proximity function to approximate the mass flux, as does Randrup. Unlike Randrup, we calculate the flux from the quantum mechanical barrier penetration, rather than from a Thomas–Fermi approximation. There are no free parameters in the model, and furthermore the dependence of the diffusion coefficient on the relative separation of the two ions is also determined from the model. The dependence on separation is needed in more detailed descriptions of the reaction, such as used by Agassi et al. [8] and Schröder et al. [9].

We assume that the deep inelastic collision is peripheral and fast so that the sudden approximation is valid. This assumption has been made by most people in the field except the work of Mustafa [10]. For simplicity, we consider only a one-dimensional geometry neglecting completely the effect of angular momentum. We describe the two ions by the Fermi gas model and neglect the single-particle Coulomb potential. We take the average single-particle potential as a Woods–Saxon well characterized by the three parameters: depth V_0, radius R, and thickness a. Then the combined potential felt by the nucleons, when the two ions are separated by the distance s measured between the half potential points, is given by

$$V(x,s) = V_0 - \frac{V_0}{1+\exp[(|x+R_1+0.5s|-R_1)/a]} - \frac{V_0}{1+\exp[(|x-R_2-0.5s|-R_2)/a]}. \tag{1}$$

This model is only applicable for $s \geqslant 0$. When $s < 0$, the

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model gives a potential pocket in the overlap region, which is inconsistent with the saturation of nuclear matter. We therefore restrict ourselves to the situation $s \geqslant 0$. With this potential the transmission probability $P(k_x, s)$ for a nucleon in the first well with momentum $\hbar k_x$ in the x-direction tunnelling through the barrier to the second well can be obtained by numerically solving the one-dimensional Schrödinger equation. These will be the same transmission probability for nucleons tunnelling in the other direction. A simple way to estimate $P(k_x, s)$ is to use the WKB approximation, i.e.

$$P(k_x, s) = \exp\left[-2\int_{x_1}^{x_2} (2mV(x,s)/\hbar^2 - k_x^2)^{1/2} \mathrm{d}x\right], \tag{2}$$

where x_1 and x_2 are the two turning points determined by $V(x_1, s) = V(x_2, s) = \hbar^2 k_x^2/2m$. The WKB method overestimates the transmission coefficient at the barrier, but underestimates it for large separations or deeply bound particles. In the range of interest, the accuracy is $\approx 50\%$.

To get the total transmitted flux we have to multiply $P(k_x, s)$ by the velocity of the nucleon $\hbar k_x/m$ and sum over all the nucleons in the first well. Since the ions are highly excited in deep inelastic collisions, the Fermi surface is diffuse with the following occupation probability

$$n(k) = \frac{1}{1 + \exp[(\hbar^2/2m)(k^2 - k_\mathrm{F}^2)/T]}, \tag{3}$$

where k_F is the Fermi momentum and has the value $1.36\ \mathrm{fm}^{-1}$; T is the temperature in MeV and is related to the excitation energy E of the ion by $T \approx \sqrt{E/a}$ with a the level density parameter (mass number/8). Typical temperatures T in deep inelastic collisions are a few MeV. Assuming full spin and isospin saturations, the total transmitted flux from first well to second well or vice versa is given by

$$\phi(s) = [4/(2\pi)^3] \int \mathrm{d}^3\boldsymbol{k}\, P(k_x, s)(\hbar k_x/m)\, n(\boldsymbol{k})$$

$$= \frac{T}{\pi^2\hbar} \int_0^\infty \mathrm{d}k_x P(k_x, s) k_x$$

$$\times \ln\{1 + \exp[(\hbar^2/2m)(k_\mathrm{F}^2 - k_x^2)/T]\}. \tag{4}$$

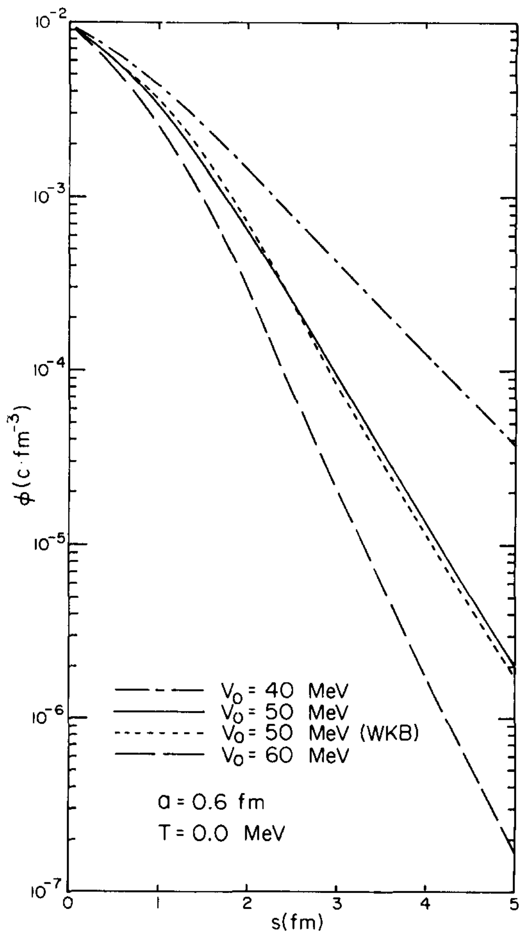

Fig. 1. Total transmitted flux from one ion to the other ion as a function of the separation distance s, calculated for different values of potential depth V_0.

In the limit $T \to 0$, it reduces to

$$\phi(s) = \frac{\hbar}{2\pi^2 m} \int_0^{k_\mathrm{F}} \mathrm{d}k_x P(k_x, s) k_x (k_\mathrm{F}^2 - k_x^2). \tag{5}$$

In figs. 1–3, we show the dependence of $\phi(s)$ on the separation distance s for different values of V_0, a and T. We observe that $\phi(0) = 9 \times 10^{-3}\ \mathrm{c\,fm}^{-3}$ and is independent of V_0, a and T. This is so because $T(k_x, 0) = 1.0$ as the potential barrier vanishes at $s = 0$ fm. Also, all curves are approximately exponential. They are more sensitive to V_0 and a than T. Fortunately the quantities V_0 and a are known to be $V_0 \sim 50$ MeV and $a \sim 0.6$ fm. For physically significant distance $s = 0$–2 fm, the T

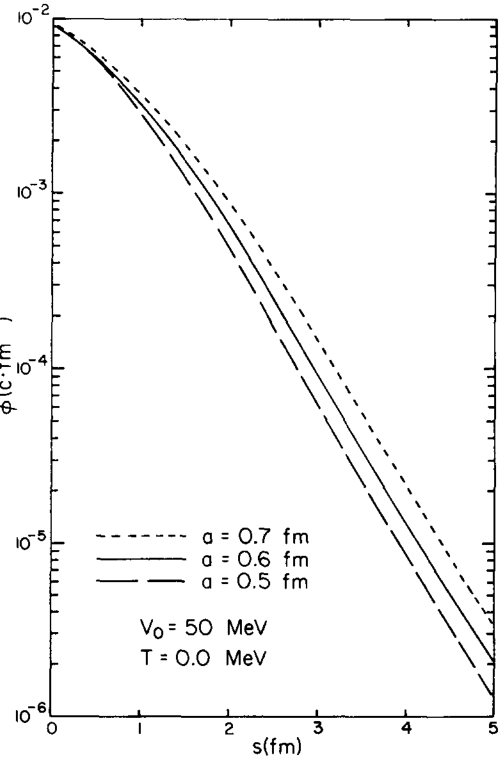

Fig. 2. Same as fig. 1 for different values of potential thickness a.

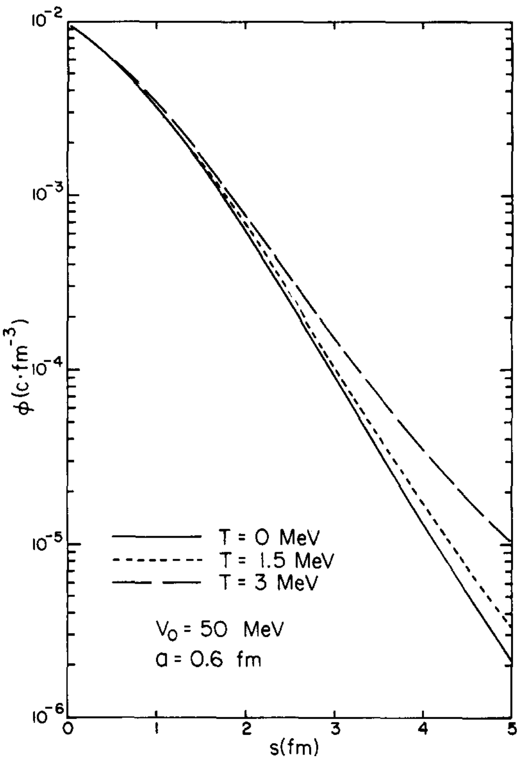

Fig. 3. Same as fig. 1 for different excitation energies of the ion.

dependence of $\phi(s)$ is almost negligible. Therefore $\phi(s)$ is essentially a universal function. Using the above values for V_0 and a, we can reasonably approximate $\phi(s)$ by

$$\phi(s) \approx \alpha_1 e^{-\beta_1 s} + \alpha_2 e^{-\beta_2 s} ,$$

$$\alpha_1 = 0.027\,\mathrm{c\,fm^{-3}}, \quad \alpha_2 = -0.018\,\mathrm{c\,fm^{-3}}, \qquad (6)$$

$$\beta_1 = 1.90\,\mathrm{fm^{-1}}, \qquad \beta_2 = 2.98\,\mathrm{fm^{-1}} .$$

In fig. 1, we also show one calculation using the WKB approximation and observe that it agrees very well with the exact calculation.

The rate of nucleon transferred from first ion to the second ion $\mathrm{d}N_{1\to 2}/\mathrm{d}t$ can be obtained by integrating $\phi(s)$ over the facing surfaces of the two ions. We use a polar coordinate system for the surface between the two nuclei, shown in fig. 4. The integral of the flux can then be evaluated as follows:

$$\frac{\mathrm{d}N_{1\to 2}}{\mathrm{d}t} = \int \mathrm{d}A\,\phi(x) \approx 2\pi \int_0^\infty \mathrm{d}Z\,Z\,\phi\left(s + \frac{Z^2}{2R_1} + \frac{Z^2}{2R_2}\right)$$

$$= 2\pi \frac{R_1 R_2}{R_1 + R_2} \left\{ \frac{\alpha_1}{\beta_1} e^{-\beta_1 s} + \frac{\alpha_2}{\beta_2} e^{-\beta_2 s} \right\} . \qquad (7)$$

This equation has the familiar form of the proximity force [11], proportional to the mean curvature of the two ions.

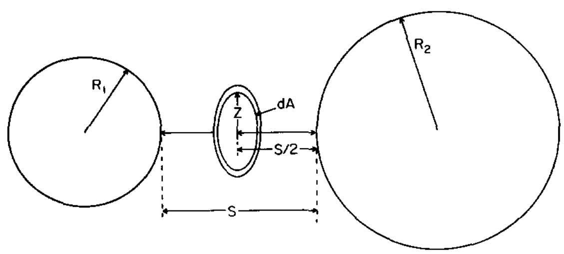

Fig. 4. Geometry involved in the integration of the area.

Table 1
Experimental and theoretical values for the mass diffusion coefficients in units of 10^{22} s^{-1}.

Reaction	E_{lab}(MeV)	D_A^{th} eq. 8	D_A^{th} Nörenberg et al.	D_A empirical ref. [5]
$^{40}Ar + ^{108}Ag$	288	3.0	1.63	1.3
$^{86}Kr + ^{197}Au$	620	3.7	2.43	2.0
$^{84}Kr + ^{165}Ho$	714	3.6	2.40	2.4
$^{84}Kr + ^{209}Bi$	714	3.8	2.48	3.7
$^{136}Xe + ^{209}Bi$	1130	4.1	2.83	4.0

The same expression holds for $dN_{2\to 1}/dt$, the rate of nucleon transferred from the second ion to the first ion. The mean number of nucleon transferred per second is given by the difference of $dN_{1\to 2}/dt$ and $dN_{2\to 1}/dt$ and is therefore zero. In the language of the diffusion model, the mass drift coefficient is zero from our model. This is because we take the two ions as identical. Both are described by the same Fermi gas model with same single-particle potential. A net drift would occur if the Fermi levels were not equal on both sides, as would be the case if the nuclei had different neutron–proton ratio, or if the nuclei differed greatly in size.

One basic underlying assumption of any diffusion model is that the process is statistical. We assume that this is indeed so for nucleon transfer in deep inelastic heavy ion collisions. Then the rate of change of the half variance of the mass distribution, defined as $\Delta \equiv \frac{1}{2}(N-\bar{N})^2$ has the same form as eq. (7), which can be easily justified from the random walk problem. The mass diffusion coefficient D_A is therefore given by

$$D_A(s) = 2\pi \frac{R_1 R_2}{R_1 + R_2} \left\{ \frac{\alpha_1}{\beta_1} e^{-\beta_1 s} + \frac{\alpha_2}{\beta_2} e^{-\beta_2 s} \right\} . \qquad (8)$$

This formula is radically different from that of Randrup, who assumes strong correlations between transfers in the two directions.

On the basis of our model, the diffusion rate of neutrons and protons would be the same, providing the densities and potential well parameters were the same. The Coulomb potential by itself plays a negligible role; the fact that the nuclear potential for protons is somewhat deeper in heavy nuclei implies that proton diffusion should be somewhat reduced. In order to compare with the empirical values, we assume that nucleon exchanges occur when the two ions are in contact at $s = 0$ fm. Using $R = 1.25 A^{1/3}$ for the radius, where A is the mass number, we obtain table 1. We observe that our predictions are systematically larger than those predicted by Nörenberg et al. [6]. Our results agree with experiment better for the heavier systems, but not as well for the lighter system, being off by a factor of two. This is perhaps as good as can be expected for a model which does not treat the relative motion.

We conclude by saying that our model is physically appealing and does give results consistent with empirical data. Furthermore, it offers a concise formula for more complete dynamical calculations. Certainly, it will be interesting to pursue further by relaxing the simplifications we have so far made. This can be carried out following the approach of Bertsch and Schaeffer [12].

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