Comment on "Spontaneous Fission: A Kinetic Approach"

The method proposed by Bonasera and Iwamoto [1] to calculate tunneling rates for fission would be of a tremendous practical value if it were justifiable. It apparently avoids the difficult problem of finding quantum many-particle closed trajectories in imaginary time [2], which has prevented the application of the imaginary time tunneling method to real fission conditions. Our Comment is to show that the heuristic method described in [1] must fail in principle. The basic problem, both classically and quantum mechanically, is to find a self-consistent time dependent mean field that connects the state describing the parent nucleus to the state describing the nascent fission daughters. In a classical treatment, and in many quantum mean field treatments, the fission axis is a symmetry axis in both the initial and final states. Quantum mechanically, this symmetry must be broken during the course of the imaginary time evolution in order to repopulate the single particle levels adiabatically. For the model example considered by Negele [2], the fission of ^{32}S into ^{16}O + ^{16}O, a path was found on which the axial symmetry was broken by a dynamically generated octupole field.

An analogous symmetry breaking must take place in the semiclassical theory based on the Vlasov equation. To see this, we define the distribution of particles $P(l_z)$ with respect to their angular momentum about the symmetry axis l_z

$$P(l_z) = \int d^3p\, d^3r\, f(\mathbf{p},\mathbf{r})\delta((\mathbf{r}\times\mathbf{p})_z - l_z),$$

where $f(\mathbf{p},\mathbf{r})$ is the one-body semiclassical phase space distribution and $\int dl_z\, P(l_z) = A$. In the simplest semiclassical approximation the function $P(l_z)$ for a spherical nucleus of radius $R = r_0 A^{1/2}$ and Fermi momentum p_f has the form

$$P(l_z) = \frac{A^{2/3}}{p_f r_0}\, g\left(\frac{l_z}{p_f r_0 A^{1/3}}\right),$$

where the "universal" function $g(x)$ is different from zero in the interval $-1 \leq x \leq 1$ only and its normalization is $\int dx\, g(x) = 1$. When the parent nucleus splits into two identical daughters close to their ground states, the combined distribution for the two daughters shrinks by the factor $2^{-1/3}$ and thus becomes higher by the factor $2^{1/3}$. If the final states of the two daughters do not have such an angular momentum distribution, the two fission fragments would be highly excited, with an excitation energy proportional to their volumes, and the tunneling probability would be vanishing for all practical purposes. However, and this is the crucial point, the distribution $P(l_z)$ cannot be changed at all by the effect of an axially symmetric mean field alone. Since there is nothing in the treatment of Ref. [1] to break the axial symmetry of the system at any time, any apparent change in $P(l_z)$ must have a purely numerical origin.

The nature of the difficulty described above can be illustrated in Hill and Wheeler's simple model [3]. Assume that the parent nucleus is a cube and the two daughters also have the shape of a cube, with sides shrunk by a factor of $2^{-1/3}$. In a symmetry-preserving scenario of the "fission" of such a system, one would have to elongate the initial cube along the fissioning direction and shrink it in the perpendicular space dimensions, always preserving the volume. The nucleon motion is fully separable in all three spatial directions and if the motion is close to being adiabatic in each direction the momentum distribution will change inversely proportional to the corresponding spatial extension of the "nucleus." Obviously the "final" state will not correspond to a spherically symmetric momentum distribution and thus the two "daughters" would end up with an excitation energy proportional to total number of particles. In a pure quantum description this would correspond to an evolution along diabatic single particle levels, rather than along adiabatic single particle levels.

The alternative to the path integral formulation with symmetry-breaking paths is to add two-particle interactions beyond the mean field approximation. For cold fission, this is commonly treated in a pairing approximation, either with a BCS pairing condensate [4] or explicit pair-hopping dynamics [5].

We thank J. W. Negele for discussions and DOE for support.

George F. Bertsch* and Aurel Bulgac†
Department of Physics
University of Washington
P.O. Box 351560
Seattle, Washington 98195-1560

Received 12 February 1997 [S0031-9007(97)04392-5]
PACS numbers: 25.85.Ca, 24.10.Cn, 24.75.+i

*Electronic address: bertsch@phys.washington.edu
†Electronic address: bulgac@phys.washington.edu

[1] A. Bonasera and A. Iwamoto, Phys. Rev. Lett. **78**, 187 (1997).
[2] J. Negele, Nucl. Phys. **A502**, 371c (1989).
[3] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).
[4] M. Brack *et al.*, Rev. Mod. Phys. **44**, 320 (1972).
[5] G. F. Bertsch, Nucl. Phys. **A574**, 169c (1994).

