RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 38, NUMBER 3 SEPTEMBER 1988

Superfluid tunneling in the restoration of parity conservation in octupole deformed nuclei

F. Barranco* and E. Vigezzi†
The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark

R. A. Broglia
Dipartimento di Fisica, Università di Milano, and Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy and The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark

G. F. Bertsch
Department of Physics, Michigan State University, East Lansing, Michigan 48824
(Received 23 May 1988)

There is strong evidence that the nucleus ^{222}Ra displays both quadrupole and octupole permanent deformations. The observed energy splitting between the associated even- and odd-parity rotational bands can be described in terms of superfluid tunneling in the space of nuclear shapes.

The nucleus is a finite many-body system which displays spontaneous symmetry breaking. In particular, nuclei exhibit superfluidity and quadrupole and octupole deformations, phenomena in which the wave function violates conservation laws of particle number, angular momentum, and parity. The potential energy surface of a superfluid nucleus with an even multipole deformation generally has a single minimum as a function of the pairing gap and deformation parameters.[1,2] For odd multipole deformations, there will be two minima with mirror image wave functions.[3]

In the present paper we will discuss a model in which tunneling between the two octupole minima restores parity conservation, leading to an energy splitting between odd and even parity bands. The model will be applied to ^{222}Ra, a nucleus that displays parity deformation with two nearly degenerate rotational bands of opposite parity. It is also superfluid, which is exhibited by the reduced moment of inertia of the bands as compared to the rigid inertia.

Several mean-field calculations[4-14] have found octupole-deformed wave functions to be most stable for nuclei in the radium region. The shapes are conventionally parametrized by deformation parameters ϵ_2 and ϵ_3, which are related to the shape of the density or the single particle potential by

$$R=R_0\left[1+\frac{2}{3}\left(\frac{4\pi}{5}\right)^{1/2}\epsilon_2 Y_{20}+\left(\frac{4\pi}{7}\right)^{1/2}\epsilon_3 Y_{30}\right], \quad (1)$$

where R_0 is the half-density radius and $Y_{\lambda 0}$ are the spherical harmonics of multipolarity λ. The mean-field calculations predict minima at $\epsilon_3=\pm(0.07-0.1)$ with a saddle point connecting the two minima at $\epsilon_2\approx 0.11$ and $\epsilon_3=0$.

We shall calculate the tunneling using a collective Hamiltonian of the form

$$\left[-\frac{\hbar^2}{2D_2}\frac{\partial^2}{\partial\epsilon_2^2}-\frac{\hbar^2}{2D_3}\frac{\partial^2}{\partial\epsilon_3^2}+V(\epsilon_2,\epsilon_3)\psi(\epsilon_2,\epsilon_3)\right]=E\psi(\epsilon_2,\epsilon_3). \quad (2)$$

While much effort has been concentrated on the study of the potential energy surfaces, and they are quite well understood, much less is known about the inertial parameters D_λ. In this work, we apply a simple expression to determine these quantities, following a method that is successful in describing the dynamics of exotic decay via large amplitude deformations.[15,16] The main steps in deriving the inertia expressions are given below (for more details see Ref. 17).

We consider a linear sequence of nuclear wave functions corresponding to a sequence of Slater determinants of different shapes, and assume that the residual interaction connects only nearest neighbors with a constant matrix element v. The Hamiltonian operator for this system is a tridiagonal matrix:

$$\begin{bmatrix} E_{i-1} & v & \\ v & E_i & v \\ & v & E_{i+1}\end{bmatrix}, \quad (3)$$

where E_i are the Hartree energies associated with the different configurations. The collective Hamiltonian [see Eq. (2)] may be approximated on a mesh in ϵ space, with the second derivative operator replaced by the difference operator

$$-\frac{\hbar^2}{2D}\frac{d^2\psi}{d\epsilon^2}=-\frac{\hbar^2}{2D}\frac{\psi(\epsilon_{i+1})+\psi(\epsilon_{i-1})-2\psi(\epsilon_i)}{(\Delta\epsilon)^2}. \quad (4)$$

Here $\Delta\epsilon$ is the step interval between states. Comparing the off-diagonal elements of Eqs. (3) and (4) we make the identification

$$v=-\frac{\hbar^2}{2D(\Delta\epsilon)^2}. \quad (5)$$

The most important part of the nuclear interaction responsible for the off-diagonal matrix elements is the pairing force, which in the BCS approximation is written as

$$H=-G\sum_{\nu,\nu'}a_\nu^\dagger a_{\bar\nu}^\dagger a_{\bar\nu'}a_{\nu'}. \quad (6)$$

The single particle levels ν and ν' depend on the collective coordinate ϵ. One can group these levels according to whether the single-particle energies increase or decrease with deformation. Under the influence of interaction (6), pairs of particles are transferred from one type of level to the other, and the system evolves from one configuration to the next, passing through level crossings. This process can be viewed as a pick up of a pair of particles from one type of level in the pairing condensate and a stripping onto the other kind of level in the condensate, without exciting any quasiparticle. In the BCS approximation, each process is associated with an amplitude of the type

$$\langle\psi_{BCS}|\sum_\nu a_\nu^\dagger a_{\bar\nu}^\dagger|\psi_{BCS}\rangle=\langle\psi_{BCS}|\sum_{\nu'} a_{\bar\nu'}a_{\nu'}|\psi_{BCS}\rangle$$
$$=\sum_\nu U_\nu V_\nu=\frac{\Delta}{G}\,. \tag{7}$$

The effective residual interaction between configurational shapes is

$$v=-\frac{G}{4}\left[\sum_\nu U_\nu V_\nu\right]^2=-\frac{\Delta^2}{4G}\,. \tag{8}$$

The factor $\frac{1}{4}$ arises because only one of the four possible types of pairs jumps between levels of various slopes leads to a level crossing. The collective inertial parameter resulting from Eqs. (5) and (8) is $D=\hbar^2/(\Delta\epsilon)^2(2G/\Delta^2)$. The quadratic dependence of the inertia on the reciprocal pairing gap found in this formula is well known.[18] The above derivation considers only one kind of particle; with both neutron (ν) and proton (π) level crossings one finds

$$D_\lambda=(D_\pi)_\lambda+(D_\nu)_\lambda,\quad(\lambda=2,3,\ldots)\,, \tag{9}$$

where

$$(D_i)_\lambda=\hbar^2\frac{2G}{\Delta_i^2}\left(\frac{dn_i}{d\lambda}\right)^2,\quad(i=\pi,\nu)\,. \tag{10}$$

We can also write the total interaction strength as

$$v=v_\pi+v_\nu\,, \tag{11}$$

provided that neutrons and protons behave similarly.

We determine the density of level crossings by counting them in the graphed single-particle energies of Ref. 14. Reference 14 shows 2 neutron crossings and 1 proton crossing as the octupole deformation is varied from zero to the stable minimum point. Thus, six level crossings are required to go from one minimum to the other. Then the density of level crossings is $d(n_\pi+n_\nu)/d\epsilon_3\approx 6/0.15=40$; a similar value can be obtained from the Fermi gas model.[19] We also find the same crossing density for quadrupole deformation, $d(n_\pi+n_\nu)/d\epsilon_2$.[20]

The residual interaction v was estimated in Ref. 15 from pairing systematics: $\Delta\approx 12/\sqrt{A}\approx 0.8$ MeV, $G\approx 25/A$ MeV≈ 0.11 MeV, giving $v=2.9$ MeV and $D_3=276\hbar^2$ MeV^{-1}. To complete the Hamiltonian, we take the potential from Fig. 2 in Ref. 14. This potential has a barrier between the two minima with a value at zero-octupole deformation $V_B=300$ keV. We then solve the two-dimensional Schrödinger equation using a discrete mesh in the $\epsilon_2\times\epsilon_3$ plane.

The resulting wave functions of the ground state and of the first excited state are displayed in Fig. 1. The ground-state wave function is concentrated close to the saddle point ($\epsilon_2=0.11$, $\epsilon_3=0.0$) of the potential-energy surface, while the excited state is peaked around the absolute minima of the potential ($\epsilon_2=0.14$, $\epsilon_3=\pm 0.075$). These results show that the ground state is an even combination of the unperturbed solutions associated with each of the octupole minima, while the first excited state is an odd combination. In this way, the system has regained invariance with respect to parity reflections. Rotational and particle number invariance are restored by rotating the wave function in both normal and gauge spaces.

The energy difference between the two states is found to be 500 keV, exceeding the experimental band splitting by a factor of 2. The kinetical splitting can in fact be estimated in a simple way, neglecting the motion in the ϵ_2 coordinate. We may approximate the ϵ_3 dependence of the wave function by $\cos k_o i$ and $\sin k_1 i$, where i labels a Hartree-Fock configuration. Comparing with Figs. 1(c) and 1(d), we choose the parameters k_0 and k_1 to make the wave function vanish at twice the minimum deformation. This gives $k_0\approx\frac{1}{12}\pi$ and $k_1\approx\frac{1}{6}\pi$. One then obtains for the kinetic contribution $\langle T\rangle=v(k_1^2-k_0^2)=600$ keV. There is also a potential contribution, which has a magnitude between zero and V_B, say $\frac{1}{2}V_B$. Then we expect a splitting of the order of 450 keV from the one-dimensional Schrödinger equation.

The factor of 2 disagreement between theory and experiment could be lessened by (1) decreasing the pairing gap, (2) increasing the pairing strength, keeping the gap fixed, (3) increasing the number of states, and (4) increasing the potential barrier.

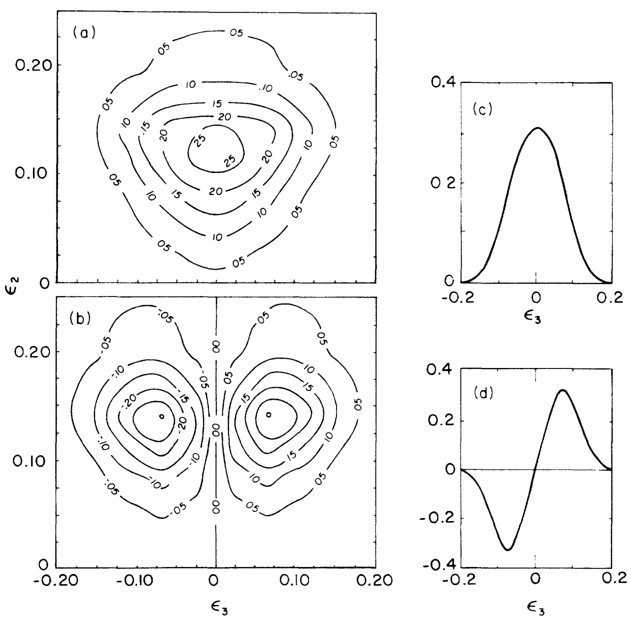

FIG. 1. Level curves for the wave functions of the ground state and the first excited state in the (ϵ_2,ϵ_3) plane are shown in (a) and (b), respectively. In (c) and (d), we show cuts of the ground-state wave function and of the first excited-state wave function, corresponding to the fixed value of the quadrupole deformation, $\epsilon_2=0.11$ in (c) and $\epsilon_2=0.14$ in (d).

Neither the pairing gap Δ nor the number of states can be changed enough to explain the disagreement. The dependence on G is a somewhat unsatisfactory aspect of the model, as there is no direct way to measure it. The potential barrier calculation might be subject to enough uncertainty to account for the difference, also.

The strong dependence of the tunneling on the pairing gap [see, for example Eq. (10)] leads to a prediction: At high angular momentum, the gap should be reduced and the two bands should become more nearly degenerate. This is probably not observable in ^{222}Ra, which is expected to lose its static octupole deformation[12,13] around spin $15\hbar$, but this might apply to other nuclei in the same mass region that keep their deformations up to very high spins.

One of us (F.B.) wants to acknowledge financial support from La Comisión Asesora de Investigación Científica y Técnica, Madrid, Spain.

*Permanent address: Escuela Superior de Ingenieros Industriales, Av.da Reina Mercedes, Sevilla, Spain.

†Permanent address: Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy.

[1] D. R. Bes, R. A. Broglia, R. Perazzo, and K. Kumar, Nucl. Phys. **A143**, 1 (1970).

[2] A. Bohr and B. R. Mottelson, *Nuclear Structure, Vol. II* (Benjamin, Reading, Mass., 1975).

[3] K. Lee and D. R. Inglis, Phys. Rev. **108**, 774 (1957).

[4] P. Möller and J. R. Nix, Nucl. Phys. **A361**, 117 (1981).

[5] A. Gyurkovich, A. Sobiczewski, B. Nerlo-Pomorska, and K. Pomorski, Phys. Lett. B **105**, 95 (1981).

[6] G. A. Leander, R. K. Sheline, P. Möller, P. Olanders, I. Ragnarsson, and A. J. Sierk, Nucl. Phys. **A388**, 452 (1982).

[7] I. Ragnarsson, Phys. Lett. **130B**, 353 (1983).

[8] R. Chasman, J. Phys. (Paris) Colloq. **45**, Suppl. 6, C6-167 (1984).

[9] W. Nazarewicz, P. Olanders, I. Ragnarsson, J. Dudek, G. A. Leander, P. Möller, and E. Ruchowska, Nucl. Phys. **A429**, 269 (1984).

[10] G. A. Leander and R. K. Sheline, Nucl. Phys. **A413**, 375 (1984).

[11] G. A. Leander, in *Nuclear Structure 1985,* edited by R. A. Broglia, G. Hagemann, and B. Herskind (North-Holland, Amsterdam, 1985), p. 249.

[12] W. Nazarewicz, in *Nuclear Structure 1985,* edited by R. A. Broglia, G. Hagemann, and B. Herskind (North-Holland, Amsterdam, 1985), p. 263.

[13] W. Nazarewicz, G. A. Leander, and J. Dudek, Nucl. Phys. **A467**, 437 (1987).

[14] P. Bonche, P. H. Heenen, H. Flocard, and D. Vautherin, Phys. Lett. B **175**, 387 (1986).

[15] F. Barranco, R. A. Broglia, and G. F. Bertsch, Phys. Rev. Lett. **60**, 507 (1988).

[16] G. F. Bertsch, F. Barranco, and R. A. Broglia, *Windsurfing the Fermi Sea,* edited by T. T. S. Kuo and J. Speth (Elsevier, Amsterdam, 1987), p. 33.

[17] G. F. Bertsch, in *Proceedings of the Enrico Fermi International School of Physics, Varenna, Italy, 1987,* edited by R. A. Broglia and R. J. Schrieffer (North Holland, Amsterdam in press).

[18] M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. **44**, 320 (1972).

[19] F. Barranco, Ph.D. thesis, University of Sevilla, 1986 (unpublished).

[20] G. F. Bertsch, Phys. Lett. **95B**, 157 (1980).