Comment on "Method for observation of neutrinos and antineutrinos"

G. F. Bertsch and Sam M. Austin

Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory,
Michigan State University, East Lansing, Michigan 48824

(Received 21 January 1986)

The proposed supercoherent elastic neutrino scattering is not compatible with basic assumptions about wave mechanics. The arguments leading to supercoherence would also predict an analogous effect in x-ray scattering, contrary to observation.

In a recent paper\(^1\) Weber argued that neutrino cross sections in material media might be enhanced many orders of magnitude by previously unrecognized coherence effects. This possibility has stimulated experimental proposals requiring substantial resources, so a critical comment is in order. Weber speculates that neutrinos may scatter from extended objects with cross sections proportional to \(N^2\), where \(N\) is the number of scatterers, even when the momentum transfer is large compared to the inverse linear dimension of the object. We will make two arguments why this cannot happen, one based on very general theoretical grounds and the other based on empirical properties of x-ray scattering.

The theoretical argument depends only on translational invariance and the linearity of the wave equation. Let us suppose that coherence occurs in a situation with a plane wave of wave number \(k\) incident along the z axis on a cubic lattice containing \(N\) scatterers. The scattered wave is observed at a large distance from the cube, located at an angle \(\theta\) in the x-z plane. Suppose the amplitude of the wave at the observation point is \(\phi\). By translational invariance, a displacement of the detector is equivalent to an opposite displacement of the scatterer. If the cube is moved a distance \(a\) in the x direction, the wave amplitude at the detector is changed to \(e^{iak\sin \theta}\). If \(a\) is chosen to be the lattice spacing, then all of the atoms are in old lattice sites except those on the y-z surfaces and by linearity, the difference in wave amplitudes is entirely attributable to the two surface layers. The number of surface atoms is proportional to \(N^{2/3}\), so that the amplitude change is \((e^{iak\sin \theta} - 1)\phi \sim N^{2/3}\). Then if \(\phi\) is to be proportional to \(N\), the prefactor must vary as \(N^{-1/3}\), i.e., \(\sin \theta \sim 1/kaN^{1/3}\). Thus no large angle coherent scattering is possible away from the Bragg peaks. As Weber shows, in this limit the total coherent cross section is proportional to \(N\), not \(N^2\).

The speculation, as argued in Ref. 1, can also be refuted on empirical grounds. The only properties of the neutrino scattering that are invoked in Ref. 1 are that

(a) the scattering is weak enough to apply the Born approximation;

(b) The scattering is recoilless in the Mössbauer sense.

These requirements are also fulfilled by the scattering of long-wavelength x-ray from the atomic nuclei. For example, consider x rays of 10 keV energy scattering from a lattice of silicon atoms. The photon momentum is low enough to satisfy the recoilless condition for all scattering angles. The photon interaction with the nuclei is dominated by the Compton amplitude at these low energies. Then a particle of silicon having a linear dimension \(R \sim 1\) \(\mu\)m would have \(N \sim 5.0 \times 10^{10}\) atoms and a coherent cross section \(2.5 \times 10^{-8}\) \(\text{cm}^2\) [obtained by scaling the cross section, \(\sigma = 0.665\) \(\text{b}\) for an electron by \(N^2Z^4/(4 \times 1830)^2\)]. This is larger than its geometric cross section. As a result the dominant interaction process would not be the attenuation by the photoelectric effect that gives the usual mass absorption coefficient,\(^2\) but rather, elastic scattering by nuclei. In fact there are negligible differences in the attenuation of x rays by amorphous or crystalline silicon, for thin films of the same areal mass density.\(^3\)

This research was supported by the National Science Foundation.

---

3. S. Solin, private communication.