Extracting Neutron Star Radii from the Tidal Deformability of GW170817

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First measurement of $\tilde{\Lambda}$ from GW170817

- Constraint is on the combined, effective tidal deformability:
  $$\tilde{\Lambda} < 800$$

- Measurement “disfavors EOS that predict less compact stars”

Abbott et al. 2017 (PRL)
What does $\tilde{\Lambda}$ measure?

\[
\tilde{\Lambda} = \frac{16(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}
\]

- $\Lambda = \frac{2}{3}k_2\left(\frac{R}{m}\right)^5$
- $k_2$ depends on the EOS and compactness
- $k_2 \sim 0.05-0.15$ (Hinderer 2008; Hinderer et al. 2010; Postnikov et al. 2010)
What does $\tilde{\Lambda}$ measure?

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2)m_1^4 (C_1)^5 + (m_2 + 12m_1)m_2^4 (C_2)^5}{(m_1 + m_2)^5} \left(\frac{2}{3} k_2\right)$$

- Expectation: $\tilde{\Lambda}$ measures a mass-weighted compactness
Evidence of a universal relation between $\tilde{\Lambda}$ and $R$

- $\Lambda_1$, $\Lambda_2$ are calculated for a range of $m_1$ and $m_2$
- All combinations obey observed chirp mass, $M_c = 1.188 M_\odot$
- GW170817 probes radius directly, not compactness!

Raithel, Özel, and Psaltis (in prep.)
**Analytic origin**

- Use Newtonian expression for each star:
  \[
  \Lambda_N = \frac{15 - \pi^2}{3\pi^2} \left( \frac{Rc^2}{Gm} \sqrt{1 - \frac{2Gm}{Rc^2}} \right)^5
  \]

- Or in terms of universal relations of YY 2017:
  \[
  C = a_0 + a_1 \ln \Lambda + a_2 (\ln \Lambda)^2
  \]

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**Newtonian approximation**

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**Universal relation of YY 2017**

Raithel, Özel, and Psaltis (in prep).
Analytic origin

- Expand combined effective deformability, assuming \( q = (1-\epsilon) \)

\[
\tilde{\Lambda}_N = \frac{15 - \pi^2}{3\pi^2} \xi^{-5} (1 - 2\xi)^{5/2} 
\times \left[ 1 - \frac{3}{108} (1 - 2\xi)^{-2} (10 - 94\xi + 83\xi^2) \epsilon^2 \right] + \mathcal{O}(\epsilon^3)
\]

\[
\xi = \frac{2^{1/5} G M_c}{R c^2}
\]

"Effective" compactness: Depends only on \( R \)

Coefficient

Deviation away from equal mass ratio
Expansion results

• Component mass dependence enters at $O(\varepsilon^2)$, and only as deviation away from equal mass ratio

• Dependence on mass even weaker for larger radii

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<tr>
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<th>Coefficient</th>
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<td>$R = 10$ km</td>
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TABLE 1
$\tilde{\Lambda}_N$ Expansion terms for the chirp mass measured from GW170817.

Raithel, Özel, and Psaltis (in prep).
Expansion results

- Component mass dependence enters at $O(\varepsilon^2)$, and only as deviation away from equal mass ratio
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\(\tilde{\Lambda}\) is a direct probe of NS radius!
How do we make further use of deformability measurements?

• To infer the underlying equation of state: need Bayesian inference
• We use parametric EOS, with 5 piecewise polytropes

\[ P(\text{EOS} | \{M_c, \tilde{\Lambda}\}) = P_{pr}(P_1, \ldots, P_5) P(\{M_c, \tilde{\Lambda}\} | \text{EOS}) \]

- Nuclear physics information
- Hydrostatic stability
- Causality
- \( 1.97 \, M_\odot < M_\text{max} < 2.33 \, M_\odot \)
- \( 0.7 < q < 1.0 \)

Rezzolla et al. 2018
Margalit & Metzger 2017
Example inferences

1. $\tilde{\Lambda}$ is a zero centered Gaussian, with $\sigma = 490$
2. $\tilde{\Lambda}$ is a Gaussian centered at 400 with $\sigma = 240$
3. No data at all (just priors)

Both give 90% interval at $\tilde{\Lambda} = 800$

Also use an asymmetric Gaussian for the chirp mass:
$M_c = 1.188^{+0.004}_{-0.002} \, M_\odot$

Abbott et al. 2017
$\tilde{\Lambda} = 400$
Marginalized to see “density” of results

i.e., integrated the priors over mass-radius volume

Λ = 400

Max L solution
Universal R-Λ relation
Marginalized results
\[ \Lambda = 0 \]

- Max L solution
- Universal R- \( \Lambda \) relation
- Marginalized results

Marginalized to see “density” of results

i.e., integrated the priors over mass-radius volume
No data at all!

Max L solution
Universal R-Λ relation
Marginalized results

Marginalized to see “density” of results
i.e., integrated the priors over mass-radius volume
Conclusions

• Ñ probes the radii of the neutron stars
• Full Bayesian inference is required to get the underlying EOS
• Additional measurements of Ñ will provide new constraints on the EOS