Studies on the ππ scattering in I < 2 channel with HAL QCD method

All-to-all propagator is necessary for 4-pt correlation function.  Large computational cost

We considered the combination of HAL QCD method and LapH smearing (distillation)  

\[ R(\mathbf{r}, t - t_0) = e^{2m_\pi(t - t_0)} \sum_\mathbf{x} \langle 0 | \pi(\mathbf{x}, t) \pi(\mathbf{x} + \mathbf{r}, t) \pi(\mathbf{P}, t_0) \pi(-\mathbf{P}, t_0) | 0 \rangle \]

\[ \text{LapH smeared sink} \quad \text{LapH smeared src.} \]

\[ \frac{1}{4m_\pi} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \]  

\[ R(\mathbf{r}, t) = \int d^3r' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t) \]

First, we checked the sink operator (scheme) independence of the potential method.

Point sink  \quad \text{LapH-smeared sink}

\[ U(\mathbf{r}, \mathbf{r}') : \text{non-local potential} \]

\[ U(\mathbf{r}, \mathbf{r}') \simeq \{ V_0(\mathbf{r}) + V_1(\mathbf{r}) \nabla^2 + \mathcal{O}(\nabla^4) \} \delta(\mathbf{r} - \mathbf{r}') \]

\text{LO} \quad \text{NLO}

\[ \text{derivative expansion} \]

\[ \text{truncation} \]

\delta_0(k) : \text{phase shift} \quad \text{scheme independent}

\delta'_0(k) : \text{phase shift from truncated potential} \quad \text{Accuracy depend on source/sink operator}
Studies on the ππ scattering in I < 2 channel with HAL QCD method

All-to-all propagator is necessary for 4-pt correlation function. Large computational cost

We considered the combination of HAL QCD method and LapH smearing (distillation)

M. Peardon et al. (Hadron Spectrum Collaboration) (2009)

R-correlator

\[ R(r, t - t_0) = e^{2m_\pi(t-t_0)} \sum_x <0|\pi(x, t)\pi(x + r, t)\pi(P, t_0)\pi(-P, t_0)|0> \]

LapH smeared sink LapH smeared src.

potential \( \left( \frac{1}{4m_\pi} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right) R(r, t) = \int d^3r' U(r, r') R(r', t) \)

Second, we applied this method to ππ scattering in I = 1 channel

• Resonant behavior in the phase shift

  First result of conventional resonance with HAL QCD method

• Direct search of S-matrix pole is performed

  Pole is found in the second Riemann sheet.
$k \cot \delta_0(k)$ plot of phase shift in $l = 2 \pi$ scattering

$N_f = 2+1$ gauge config., $a = 0.12$ fm, $16^3 \times 32$, $m_\pi = 870$ MeV


Point sink (Conventionally used in HAL QCD)

- NLO term is negligible at this energy region.
  ($\therefore$ source momentum dependence is negligible)

• The phase shift in low energy region
  The deviation of phase shift in smeared-sink scheme from $V_{\eta_{\max}}^{LO}(r;0,16)$ and the finite volume method.
  Controlled by smearing level

• The phase shift in high energy region
  The deviation is dominated by the contribution from the NLO analysis.
Point sink (Conventionally used in HAL QCD)

- NLO term is negligible at this energy region. ($\because$ source momentum dependence is negligible)

- The phase shift in **low energy region**
  The deviation of phase shift in smeared-sink scheme from $V^{\text{LO}}_{\text{max}}(r; 0, 16)$ and the finite volume method.
  - controlled by smearing level

- The phase shift in **high energy region**
  The deviation is dominated by the contribution from the NLO analysis.
Phase shift I = 1 \pi scattering from HAL QCD method with LapH smearing

The phase shift in the LO analysis for LapH smeared-sink
(# of conf. = 10 for Level = 256, so it is preliminary) Nr = 2+1 gauge config., a = 0.09 fm, 32^3 \times 64, m_\pi = 410 MeV

Results

- Phase shift given by the potential crosses 90 degrees at $\sqrt{s} \sim 900$ MeV
  
  Consistent with known value for this conf.
  
  ($\therefore m_\rho \simeq 890$ MeV)

Open problem

- It largely deviates from PACS-CS result for $\sqrt{s} > 900$ MeV
  
  [S. Aoki et al., (PACS-CS Collaboration), (2011)]
- increase of phase shift stops around 120°.

NLO analysis and larger number of $n_s$ is necessary as I = 2 case.

Preliminary

\[ 2\sqrt{m^2 + k^2} \ [\text{MeV}] \]
Phase shift $I = 1$ \textbf{m} scattering from HAL QCD method with LapH smearing

The phase shift in the LO analysis for LapH smeared-sink
(# of conf. = 10 for Level = 256, so it is preliminary)

\textbf{Results}

- Phase shift given by the potential
  crosses 90 degrees at $\sqrt{s} \sim 900$ MeV

  $\implies$ Consistent with known value for this conf.

  ($\because m_{\rho} \simeq 890$ MeV)

\textbf{Open problem}

- It largely deviates from PACS-CS result for $\sqrt{s} > 900$ MeV
  
  [S. Aoki et al., (PACS-CS Collaboration), (2011)]

- Increase of phase shift stops around 120°.

NLO analysis and larger number of $n_s$ is necessary as $I = 2$ case.
Direct search of S-matrix pole

Pole is located in the second Riemann sheet.

Invariant mass

\[ \sqrt{s} = 886.4(4.4) - \frac{i}{2} 82.1(10.2) \text{ MeV} \]
(NLevel = 128)

\[ \sqrt{s} = 883.9(17.4) - \frac{i}{2} 57.6(26.8) \text{ MeV} \]
(NLevel = 256, preliminary)

c.f.) PACS-CS result

\[ \sqrt{s} = 892.8(5.5) - \frac{i}{2} 11.2(1.7) \text{ MeV} \]

- Real part (= mass) is consistent with the result with Lüscher’s method by PACS-CS collaboration.
- The deviation in imaginary part (= decay width) will be reduced by increasing \( n_s \) and employing NLO analysis.

This method can be applied for exotic hadrons and resonance with large width such as \( \sigma \).
Summary

**Sink operator independence (I=2 channel)**

- The **truncated potential** with **LapH smeared sink** has large sink operator dependence.
  - The height of repulsive core drastically changes.
- Even with the dependence, phase shift can be correctly obtained by considering higher order terms.
  - We show that phase shifts in high energy are improved by considering the NLO term.

**I=1 ππ scattering**

- Resonant behavior in the phase shift
  - Peak point is consistent with configuration data.
    - However, improvement will be necessary to get correct behavior in higher energy.
- Direct search of S-matrix pole without any fitting such as Breit-Wigner form is possible.
  - Resonant pole in the second Riemann sheet
Back up
Spatial distribution of smearing operator

How is the dependence of smearing operator on the number of eigenvalue included?

A gauge invariant measure

\[ \Phi_{n_s}(r) = \sum_{x,t} \sqrt{\text{Tr} \{ S_{n_s}(x, x+r, t) S_{n_s}(x+r, x, t) \}} \]

(normalized at \( r = 0 \))

The smeared quark is more localized as \( n_s \) increases.

Note: The long tail structure might be the origin of systematic change in physical observables.

\[ \Phi_{16}(r) \quad \Phi_{32}(r) \quad \Phi_{64}(r) \]

\( N_f = 2+1 \) gauge config., \( a = 0.12 \text{ fm}, \ 16^3 \times 32, \ m_\pi = 870 \text{ MeV} \)

Numerical setup

• 2 + 1 flavor gauge configuration by CP-PACS & JLQCD

• Wilson clover fermion and Iwasaki gauge action

• \( a = 0.1214 \text{ fm} , 16^3 \times 32 \text{ lattice} \)

• \( m_\pi \approx 870 \text{ MeV} \)

• 60conf \( \times 32 \) time slices

• Calculated on Cray XC40 in YITP

• No gauge fixing is used

\[
C^4_M(r, t; l_0) = \sum_{x-y_1-y_2} \langle 0 | \pi^+(x, t) \pi^+(x + r, t) \pi^- (y_1, l_0) \pi^- (y_2, l_0) | 0 \rangle
\]

Remark : the sum over source space improves statistics.
Details of HAL QCD potentials with LapH smearing

4-pt correlator: \[ C_{\nu_{a},\nu_{b}}^{4,A_{i}^{+},1}(r,t;\mathbf{P},t_{0}) = \sum_{x} \langle 0|\overline{\nu}_{\nu_{a}}(x,t)\overline{\nu}_{\nu_{b}}(x+r,t)(\pi_{\nu_{b}}\pi_{\nu_{b}})^{A_{i}^{+},1}(\mathbf{P},t_{0})|0 \rangle \]

\[ \text{(effective) leading order potential:} \quad V_{\pi c}^{LO}(r;|\mathbf{P}|,n_{b}) = \frac{1}{4m_{\pi}} \frac{\partial^{2}}{\partial t^{2}} - \frac{\partial}{\partial t} - H_{0} \] \[ \frac{R_{\nu_{a},\nu_{b}}^{A_{i}^{+},1}(r,t;|\mathbf{P}|,t_{0})}{R_{\nu_{a},\nu_{b}}^{A_{i}^{+},1}(r,t;|\mathbf{P}|,t_{0})} \]

Notation

“point-sink scheme” : \( n_{a} = N_{c}N_{x}N_{y}N_{z} \equiv n_{\text{max}} \) = Conventionally used sink

“smeared-sink scheme” : \( n_{a} < n_{\text{max}} \)
Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS collaboration
  [PACS-CS Collaboration: S. Aoki et. al., (2009)]
- Wilson clover fermion and Iwasaki gauge action
- $a = 0.0907 \text{ fm} , \ 32^3 \times 64 \text{ lattice}$
- $m_\pi = 410 \text{ MeV}, \ m_\rho = 890 \text{ MeV}$
  - $\rho$ meson will appear as a resonant state
- 64 time slices are fully used for the average value.
- Periodic boundary condition is used for all direction.
- No gauge fixing is used.