TMDs at small-x

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Probing the Weizsacker-Williams gluon distribution at EIC

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Ref: Dominguez, Xiao, Yuan, arXiv:1009.2141
Nice things about transverse momentum distributions (TMDs)

- Universality and a universal language
  - DIS/Drell-Yan
  - eA/pA/AA(?), small-x wave functions of nucleus

- QCD dynamics
  - TMD evolution
  - Small-x evolution
Among recent developments

- Spin-dependent TMD gluon at small-\(x\)
  - Related to the spin-dependent odderon, Boer-Echevarria-Mulders-Zhou, PRL 2016
  - Gluon/quark helicity distributions, Kovchegov-Pitonyak-Sievert, 2016, 2017, 2018

- Subleading power corrections in the TMD gluon/quark distributions
  - Balitsky-Tarasov, 2017, 2018

- Sudakov resummation for small-\(x\) TMDs
  - Mueller-Xiao-Yuan, PRL110, 082301 (2013); Xiao-Yuan-Zhou, NPB921, 104 (2017); Zhou 2018
  - Balitsky-Tarasov, JHEP1510,017 (2015)
TMDs: Conventional gluon distribution

- Collins-Soper, 1981

\[ xG^{(1)}(x, k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} \frac{e^{ixP^+\xi^- - ik_\perp \cdot \xi_\perp}}{P^+} \times \langle P|F^{+i}(\xi^-, \xi_\perp)\mathcal{L}_\xi^+ \mathcal{L}_0 F^{+i}(0)|P\rangle \]

- Gauge link in the adjoint representation

\[ \mathcal{L}_\xi = \mathcal{P} \exp\{-ig \int_{\xi^-}^{\infty} d\zeta^- A^+(\zeta, \xi_\perp)\} \]

\[ \mathcal{P} \exp\{-ig \int_{\xi_\perp}^{\infty} d\zeta_\perp \cdot A_\perp(\zeta^- = \infty, \xi_\perp)\} \]
Physical interpretation

- Choosing light-cone gauge, with certain boundary condition (either one, but not the principal value)

\[ A_\perp(\zeta^- = \infty) = 0 \]

- Gauge link contributions can be dropped

- Number density interpretation, and can be calculated from the wave functions of nucleus
  - McLerran-Venugopalan
  - Kovchegov-Mueller
Classic YM theory

- McLerran-Venugopalan

\[ xG^{(1)}(x, k_\perp) = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \int \frac{d^2 r_\perp}{(2\pi)^2} \frac{e^{-ik_\perp \cdot r_\perp}}{r_\perp^2} \left( 1 - e^{-\frac{r_\perp^2 Q_s^2}{4}} \right) \]

- See also, Kovchegov-Mueller
- We can reproduce this gluon distribution using the TMD definition with gauge link contribution, following BJY 02, BHPS 02
- WW gluon distribution is the conventional one
Saturation at small-\(x\)/large A

\[ k_T \phi(x, k_T^2) \]

\[ \alpha_s \sim 1 \quad \Lambda_{QCD} \quad Q_S \quad \alpha_s \ll 1 \]

\[ \sim 1/k_T \]

Small-x/large A

know how to do physics here
DIS dijet probes WW gluons

- Hard interaction includes the gluon attachments to both quark and antiquark
- The $q_t$ dependence is the gluon distribution w/o gauge link contribution at this order

Dominguez-Marquet-Xiao-Yuan 2011
QCD evolution at high energy

- BFKL/BK-JIMWLK (small-x)
- Sudakov (TMD)

Mueller-Xiao-Yuan 2013
Balistky-Tarasov 2014
Xiao-Yuan-Zhou 2016
High energy scattering

BFKL: \( \frac{\partial F}{\partial \ln(1/x)} = \kappa \otimes F \) Un-integrated gluon distribution
Non-linear term at high density

- Balitsky-Fadin-Lipatov-Kuraev, 1977-78

\[
\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x, r_T)
\]

- Balitsky-Kovchegov: Non-linear term, 98

\[
\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x, r_T) - \alpha_s [N(x, r_T)]^2.
\]
Hard processes at small-$x$

- Manifest dependence on un-integrated gluon distributions

- Dominguez-Marquet-Xiao-Yuan, 2010
Additional dynamics comes in

- BFKL vs Sudakov resummations (LL)
Sudakov resummation at small-x

- Take massive scalar particle production $p+A \rightarrow H+X$ as an example to demonstrate the double logarithms, and resummation

\[
\frac{d\sigma^{(LO)}}{dy \, d^2 k_\perp} = \sigma_0 \int \frac{d^2 x_\perp \, d^2 x'_{\perp}}{(2\pi)^2} e^{ik_{\perp} \cdot r_{\perp}} x_0 g_p(x_0) S^{WW}(x_{\perp}, x'_{\perp})
\]

\[
S_Y^{WW}(x_{\perp}, y_{\perp}) = -\left\langle \text{Tr} \left[ \partial_{\perp}^\beta U(x_{\perp}) U^\dagger(y_{\perp}) \partial_{\perp}^\beta U(y_{\perp}) U^\dagger(x_{\perp}) \right] \right\rangle_Y
\]
Sudakov leading double logs+small-x logs in hard processes

- Each incoming parton contributes to a half of the associated color factor in Sudakov

  □ Initial gluon radiation, aka, TMDs

\[
\frac{d\sigma}{dy_1 dy_2 dP^2 \perp d^2 k_\perp} \propto H(P^2_\perp) \int d^2x_\perp d^2y_\perp e^{i k_\perp \cdot (x_\perp - y_\perp)} \widetilde{W}_{x_A}(x_\perp, y_\perp)
\]

\[
H(P^2_\perp) \int d^2x_\perp d^2y_\perp e^{i k_\perp \cdot R} e^{-S_{sud}(P_\perp, R_\perp)} \widetilde{W}_{x_A}(x_\perp, y_\perp)
\]

Mueller-Xiao-Yuan 2013
Sudakov vs BFKL (BK)

- Start with the factorized TMDs, with full operator definitions
- Calculate the high order corrections in dipole formalism
  - With proper subtraction
- Solve the TMD evolution with BK-evolved dipole (quadrupole) amplitude
QCD Evolution: Soft vs Collinear gluons

- Radiated gluon momentum
  \[ k_g = \alpha_g p_1 + \beta_g p_2 + k_{g\perp} \]
- Soft gluon, \( \alpha \sim \beta \ll 1 \)
- Collinear gluon, \( \alpha \sim 1, \beta \ll 1 \)
- Small-\( x \) collinear gluon, \( 1 - \beta \ll 1, \alpha \to 0 \)
  - Rapidity divergence
Subtracted TMD at small-x

\[ f_g^{(\text{sub.})}(x, r_\perp, \mu_F, \zeta_c) = f_g^{\text{unsub.}}(x, r_\perp) \sqrt{\frac{S_{\bar{n},v}(r_\perp)}{S_{n,\bar{n}}(r_\perp) S_{n,v}(r_\perp)}} \]

WW-gluon Dipole gluon

Subtract the endpoint Singularity (Collins 2011)

\[ \zeta_c^2 = x^2(2v \cdot P)^2 / v^2 \]
- TMD evolution follows Collins 2011
  - with resummation, doesn’t depend on scheme
  - Beta_0 term missing though
- Small-x evolution follows the relevant BK-evolution, respectively
  - Dipole: BK
  - WW: DMMX
Final results

\[ xG^{(1)}(x, k_\perp, \zeta_c = \mu_F = Q) \]

Hard scale entering TMD
Factorization, e.g., Higgs

\[ -\frac{2}{\alpha_S} \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp \cdot r_\perp} \mathcal{H}^{WW}(\alpha_S(Q)) e^{-S_{sud}(Q^2,r_\perp^2)} \times \mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp), \]

Small-x evolution
Pert. corrections
Sudakov resum.
One-loop examples

Gauge link goes to $-\infty$

Gauge link goes to $+\infty$
Virtual is the same

(a)

(b)
One-loop result

- **WW-gluon (universal)**

\[ x G^{(WW)}_{-\infty}(x, r_\perp) \big|^{(1)} = \]

\[ \frac{\alpha_s}{2\pi} C_A \left\{ \left( \frac{-2}{\alpha_s} \right)^2 \mathcal{F}^{(WW)}(r_\perp) \left[ \frac{1}{2} \left( \ln \frac{\zeta^2}{\mu^2} \right)^2 - \frac{1}{2} \left( \ln \frac{\zeta^2 r_\perp^2}{c_0^2} \right)^2 \right] \right\} + \ln \left( \frac{1}{x} \right) \left( \frac{-2}{\alpha_s} \right) \int K_{\text{DMMX}} \otimes \mathcal{F}^{(WW)}(x_g, r_\perp) \]

Sudakov double logs

Small-x logs (BK-type of evolution)
TMD quark at small-x

- Can be calculated from the dipole amplitude, and can be applied to DIS and Drell-Yan processes

\[ q(x, k_\perp) = \frac{N_c}{8\pi^4} \int \frac{dx'}{x'r^2} \int d^2b d^2q_\perp F(q_\perp, x') A(q_\perp, k_\perp) \]

- Universality

McLerran-Venugopalan 98
Marquet-Xiao-Yuan 2009
TMD quark at small-x

Kazuhiro Watanabe's calculations

solid line: rcBK-MV
dashed line: GBW
What we know the TMD quarks (not small-x)

See also, BLNY 2002
TMD quark at small-x: CGC vs Collinear

- Realistic comparison will shed light on the TMD quarks at small-x (work in progress)
We need more data at small-x

**Drell-Yan from PHENIX**

1805.02448

LHCb, pp and pA: Drell-Yan and Upsilon

EIC: SIDIS and di-hadron
Sudakov Resummation

Di-hadron azimuthal Correlations at the Electron-ion Collider

Compare to RHIC Data

Saturation and Sudakov resummation in a single formula to describe both pp and dAu, Stasto-Wei-Xiao-Yuan, 1805.0571
Conclusions

- Theory developments since the last INT program provide solid ground to study TMDs at small-x
- Looking forward to new data from RHIC/LHC, and of course, EIC