Peering into the infrared with colored glass

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Outline of talk

- Gluon saturation leads to weakly coupled albeit strongly correlated glue in the Regge limit
- The CGC: a theory framework for gluon saturation
- Light from the CGC: the structure of higher order computations from a specific example
- Color memory and the infrared circle
The proton: a complex many-body system

A key lesson from the HERA DIS collider:

gluons and sea quarks dominate the proton wave-function at high energies
Boosting the proton uncovers many-body structure

Wee parton fluctuations time dilated on strong interaction time scales

Long lived gluons can radiate further small $x$ gluons...

Is the proton a runaway popcorn machine at high energies?
The runaway proton...

Nature does not like this!
The boosted proton viewed head-on

When occupancies become large ~ $1/\alpha_s$, gluons resist further close packing -- recombining and screening their color charges -- leading to \textbf{gluon saturation}

Characterized by an emergent semi-hard scale $Q_s \gg \Lambda_{QCD}$ in the Regge limit...

a weak coupling window into the infrared!
Saturation in the QCD landscape

Unique and controlled *dynamical* exploration of a fully nonlinear regime of quantum field theory
Saturation: dipole model formulation in DIS

\[ \sigma_{T,L}^{\gamma^*,P} = \int d^2 r_\perp \int dz |\psi_{T,L}(r_\perp, z, Q^2)|^2 \sigma_{q\bar{q},P}(r_\perp, x) \]

Golec-Biernat Wusthoff model

\[ \sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0 \left[ 1 - \exp \left( -r_\perp^2 Q_s^2(x) \right) \right] \]

Sophisticated dipole models give excellent fits to all HERA small x data

Parameters:
\[ Q_0 = 1 \text{ GeV}; \lambda = 0.3; \]
\[ x_0 = 3*10^{-4}; \sigma_0 = 23 \text{ mb} \]
Saturation scale from dipole model fits to DIS data

$x \leq 0.01$

Big nuclear oomph:

dipole couples coherently with color charges in different nucleons in path of its scattering: $Q_s^2 \sim A^{1/3}$
Nonlinear response of saturated matter to probes

Varying both $x$ and $Q^2$ essential to see nonlinear response of saturated gluons - a clear manifestation of the fully nonlinear character of QCD

For further discussion of some potentially striking systematics in DIS off nuclei, see e.g., Mantysaari and RV, PLB781 (2018) 664
Theory framework: the Color Glass Condensate
The nuclear wavefunction at high energies

\[ |A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q_{gg}\ldots gg\rangle \]

Higher Fock components dominate multiparticle production—construct Effective Field Theory

Born--Oppenheimer LC separation natural for EFT.

RG eqns describe evolution of wavefunction with energy
Effective Field Theory on Light Front

Poincare group on LF \( \longleftrightarrow \) Galilean sub-group of 2D Quantum Mechanics

Eg., LF dispersion relation

Large x \((P^+)\) modes: static LF (color) sources \(\rho^a\)
Small x \((k^+ \ll P^+)\) modes: dynamical fields \(A^a_\mu\)

CGC: Coarse grained many body EFT on LF

\[< P|\mathcal{O}|P> \longrightarrow \int [d\rho^a][dA^{\mu,a}] W_{\Lambda^+}[\rho] e^{is_{\Lambda^+}[\rho,A]} \mathcal{O}[\rho,A]\]

\(W_{\Lambda^+}[\rho]\) non-pert. gauge invariant “density matrix” defined at initial scale \(\Lambda_0^+\)

RG equations describe evolution of W with x

Susskind
Bardacki-Halpern
McLerran, RV

JIMWLK, BK
Inclusive DIS: dipole evolution

\[ \sigma_{\gamma^*T} = \int_0^1 dz \int d^2 r_\perp |\psi(z, r_\perp)|^2 \sigma_{\text{dipole}}(x, r_\perp) \]

\[ \sigma_{\text{dipole}}(x, r_\perp) = 2 \int d^2 b \int [D\rho] W_{\Lambda^+}[\rho] T(b + \frac{r_\perp}{2}, b - \frac{r_\perp}{2}) \]

\[ 1 - \frac{1}{N_c} \text{Tr} \left( V \left( b + \frac{r_\perp}{2} \right) V^\dagger \left( b - \frac{r_\perp}{2} \right) \right) \]
Inclusive DIS: dipole evolution

\[
\frac{\partial}{\partial Y} \langle \text{Tr}(V_x V_y^\dagger) \rangle_Y = -\frac{\alpha_s N_c}{2\pi^2} \int \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)(z_\perp - y_\perp)^2} \langle \text{Tr}(V_x V_y^\dagger) - \frac{1}{N_c} \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y
\]

Dipole factorization:

\[
\langle \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y \rightarrow \langle \text{Tr}(V_x V_z^\dagger) \rangle_Y \langle \text{Tr}(V_z V_y^\dagger) \rangle_Y \quad N_c \rightarrow \infty
\]

Resulting closed form eqn. for a large nucleus is the Balitsky-Kovchegov eqn. Widely used in phenomenological applications

The BFKL equation is the low density $V \approx 1 - ig\rho/\nabla_t^2$ limit of the BK equation
CGC Effective Theory: B-JIMWLK hierarchy of many-body correlators

Diffusion of the fuzz of “wee” partons in the functional space of colored fields can be represented as a Langevin equation that can be solved numerically to “leading logs in x” accuracy.
Lighting up the CGC: Inclusive photon+dijet production

Right moving nucleus with momentum $P_N^+$ is Lorentz contracted in $x^-$ direction

Glue fields satisfy Yang-Mills eqns.

$$[D_\mu, F^{\mu\nu}](x) = g\delta^{\nu+} \delta(x^-) \rho_A(x_-)$$

$A^{-,a} = 0$, $F_{ij}^a = 0$ with $A^{+,a}, A^{i,a}$ static (independent of $x^+$)

Suppressed at small $x$
Inclusive photon+dijet production in DIS at LO

\[ e(\tilde{l}) + A(P) \rightarrow e(\tilde{l}') + Q(k) + \bar{Q}(p) + \gamma(k_{\gamma}) + X \]

\[ \frac{d\sigma}{dx dQ^2} = \frac{2\pi y^2}{64\pi^3 Q^2} \frac{d^3k}{(2\pi)^3 2E_k} \frac{d^3p}{(2\pi)^3 2E_p} \frac{d^3k_{\gamma}}{(2\pi)^3 2E_{k_{\gamma}}} \frac{1}{2q^-} \left( \frac{1}{2} \sum_{\text{spins, } \lambda} \left| \tilde{M} \right|^2 \right) Y_A (2\pi)\delta(P^- - q^-) \]

\[ \frac{1}{2} \sum_{\text{spins, } \lambda} \left| \tilde{M} \right|^2 Y_A = L_{\mu\nu} X^{\mu\nu} \]

L_{\mu\nu} is well known-the lepton tensor

X^{\mu\nu} - the hadron tensor for inclusive photon+dijet production is what we compute
DIS inclusive cross-section at LO

\[
\frac{d\sigma}{dx dQ^2 d^2 k_{\gamma\perp} d\eta_{k\gamma}} = \frac{\alpha^2 q_f^4 y^2 N_c}{512\pi^5 Q^2} \frac{1}{2q^-} \int_0^{+\infty} \frac{dk^-}{k^-} \int_0^{+\infty} \frac{dp^-}{p^-} \int_{k_{\perp}, p_{\perp}} L^{\mu\nu} \tilde{X}_{\mu\nu}(2\pi) \delta(P^- - q^-)
\]

\[
L^{\mu\nu} = \frac{2e^2}{Q^4} \left[ (\tilde{l}^\mu l^\nu + \tilde{l}^\nu l^\mu) - \frac{Q^2}{2} g^{\mu\nu} \right]
\]

\[
\tilde{X}_{\mu\nu} = \int_{x_{\perp}, y_{\perp}, x'_{\perp}, y'_{\perp}, l_{\perp}, l'_{\perp}} e^{-i(P_{\perp} - 1_{\perp}) \cdot x_{\perp} - i l_{\perp} \cdot y_{\perp} + i (P_{\perp} - l'_{\perp}) \cdot x'_{\perp} + i l'_{\perp} \cdot y'_{\perp}} \tau_{\mu\nu}^{qq, q\bar{q}}(l_{\perp}, l'_{\perp} | P_{\perp}) \Xi(x_{\perp}, y_{\perp}; x'_{\perp}, y'_{\perp})
\]

All the nonperturbative info about strongly correlated gluons is in

\[
\Xi(x_{\perp}, y_{\perp}; x'_{\perp}, y'_{\perp}) = 1 - D(x_{\perp}, y_{\perp}) - D(y'_{\perp}, x'_{\perp}) + Q(x_{\perp}, y_{\perp}; y'_{\perp}, x'_{\perp})
\]

Dipoles: \( D(x_{\perp}, y_{\perp}) = \frac{1}{N_c} \langle Tr \left( \tilde{U}(x_{\perp}) \tilde{U}^\dagger(y_{\perp}) \right) \rangle_{YA} \)

Quadrupoles: \( Q(x_{\perp}, y_{\perp}) = \frac{1}{N_c} \langle Tr \left( \tilde{U}(y_{\perp}) \tilde{U}^\dagger(x'_{\perp}) \tilde{U}(x_{\perp}) \tilde{U}^\dagger(y_{\perp}) \right) \rangle_{YA} \)
Interesting limits

When $k_\gamma \to 0$, the amplitude satisfies the Low-Burnett-Kroll theorem:

$$\mathcal{M}_\mu(q, k, p, k_\gamma) \to -(e_{q_f}e^*_{q_t})(k_\gamma)\left(\frac{p^\alpha}{p \cdot k_\gamma} - \frac{k^\alpha}{k \cdot k_\gamma}\right)\mathcal{M}^{NR}_\mu(q, k, p)$$

Polarization vector $\times$ Vectorial structure depending only on momenta of emitted particles $\times$ Non-radiative DIS amplitude

Recover results in soft photon limit for di-jet production - sensitive to the gluon Weizsäcker-Williams distribution for large pair momenta

Dominguez, Marquet, Xiao, Yuan, PRD83 (2011)105005

Recover in DIS
kt-factorization & collinear factorization
small $x$ limits
(sensitivity to leading twist gluon distribution)

Structure of higher order computations: Shockwave propagators

Convenient to work in the wrong light cone gauge $A^{-}=0$ for this problem (Gauge links in pdf definitions are unity in the right LC gauge $A^{+}=0$)

Dressed quark and gluon propagators: remarkably simple forms in $A^{-}=0$ gauge

$$S(p, p') = (2\pi)^{4} \delta^{(4)}(p - p') S_{0}(p) + S_{0}(p) T(p, p') S_{0}(p')$$

$$G^{\mu\nu;ab}(p, p') = (2\pi)^{4} \delta^{(4)}(p - p') G^{\mu\nu;ab}_{0} + G^{\mu\rho;ac}_{0} T_{\rho\sigma;cd} G^{\sigma\nu;db}_{0}(p')$$

Structure of vertices identical to quark-quark-reggeion and gluon-gluon-reggeon in Lipatov’s Reggeon EFT

Bondarenko,Lipatov,Pozdnyakov, Prygarin, arXiv:1708.05183
Hentschinski, arXiv:1802.06755
**DIS inclusive photo+dijet production at NLO+NLLx**

Formally NNLO: but collect NLO pieces in the photon+dijet impact factor
+ leading log pieces $\alpha_S \log(\Lambda^{-}/\Lambda_0^{-})$

Formally NNLO: but collect LO pieces in the photon+dijet impact factor
+ NLL leading log pieces $\alpha_S^2 \log(\Lambda^{-}/\Lambda_0^{-})$

Leading order JIMWLK Hamiltonian computed 20 years ago: 1997-2001
NLO JIMWLK Hamiltonian: 2013-2016

The NLO inclusive photon impact factor

Several computations exist for inclusive DIS – subtleties in choice of scheme, etc.

- Balitsky, Chirilli, arXiv:1009.4729
- Beuf, arXiv:1606.00777, 1708.06557
- Hanninen, Lappi, Paatelainen, 1711.08207
- Dijet: Boussarie, Grabovsky, Szymanowski, Wallon, 1606.00419

First computation discussed here of photon+dijet: Roy, RV, in preparation

I) Real contributions: (20 × 20 diagrams)

- Gluon rescatters (or not) along with quarks
- Gluon emitted after rescattering

II) Interference contributions:

A) Vertex corrections

- ●●● and 21 more permutations

B) Self-energy corrections:

- ●●● and likewise, 33 more
Coda: A nontrivial derivation of JIMWLK evolution

Recall

\[ X_{\mu \nu}^{\text{LO}} = C_{\mu \nu}^{\text{LO}} \otimes \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp | \Lambda_0^-) \]

\[ \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp | \Lambda_0^-) = 1 - D_{xy} - D_{y'x'} + Q_{y'x';xy} \]

In the "soft gluon limit" that generates logs in \( x \), our NLO hadron tensor gives

\[ X_{\mu \nu; LLx}^{\text{NLO}} = C_{\mu \nu}^{\text{LO}} \otimes \ln(\Lambda_1^-/\Lambda_0^-) \left[ \frac{\alpha_s N_c}{2\pi^2} \left\{ K_B(x_\perp, y_\perp; z_\perp) D_{xy} + \left( \frac{x_\perp \rightarrow y'_\perp}{y'_\perp \rightarrow x_\perp} \right) \right\} - \frac{\alpha_s N_c}{2(2\pi)^2} K_1(x_\perp, y_\perp, y'_\perp, x'_\perp, z_\perp \rightarrow y'_\perp) 
\right. \\
- \left. \frac{\alpha_s N_c}{2\pi^2} \left\{ K_B(x_\perp, y_\perp; z_\perp) D_{xy} + \left( \frac{x_\perp \rightarrow y'_\perp}{y'_\perp \rightarrow x_\perp} \right) \right\} + \frac{\alpha_s N_c}{(2\pi)^2} \left\{ A(x_\perp, y'_\perp, y_\perp, x'_\perp; z_\perp) D_{xx'} D_{y'y} + x_\perp \rightarrow y'_\perp \right\} \\
+ \left\{ K_2(x_\perp, y_\perp, x'_\perp; z_\perp) D_{x'y} Q_{y'y';x'y'} + \left( \frac{x_\perp \rightarrow y'_\perp}{y'_\perp \rightarrow x_\perp} \right) \right\} + \left\{ K_2(x'_\perp, x_\perp, y'_\perp; z_\perp) D_{xx'} Q_{y'y;x'y'} + \left( \frac{x_\perp \rightarrow y'_\perp}{y'_\perp \rightarrow x_\perp} \right) \right\} \right] \]

Nontrivial combinations of dipole and quadrupole operators

Remarkably, this can be reexpressed as

\[ X_{\mu \nu; LLx}^{\text{NLO}} = C_{\mu \nu}^{\text{LO}} \otimes \ln \left( \frac{\Lambda_1^-}{\Lambda_0^-} \right) H_{\text{JIMWLK}}^{\text{LO}} \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp | \Lambda_0^-) \]

This immediately leads to the JIMWLK RG equation for the rapidity evolution of many-body gluon correlators

\[ W_{\lambda^-}[\rho_A] = \left( 1 + \ln \left( \frac{\Lambda_1^-}{\Lambda_0^-} \right) H_{\text{JIMWLK}}^{\text{LO}} \right) W_{\lambda_0^-}[\rho_A] \]
Color Memory in the CGC

In the CGC, the solutions of the YM-eqns can be represented as two pure gauges separated by a discontinuity at \( x^- = 0 \) corresponding to the shockwave and \( A_i \) satisfies

\[
A_i = \frac{-1}{ig} U \partial_i U^\dagger
\]

with the solution

\[
U = P \exp \left( i \int_y^\infty dy' \frac{\rho(x_t,y')}{\nabla^2_t} \right)
\]

This \( U \) is precisely the color memory effect corresponding to a color rotation and \( p_t \sim Q_s \) kick experienced by a quark-antiquark pair traversing the shock wave – discussed previously as a property of YM fields by Pate, Raclariu and Strominger (1707.08016)

*Its presence is ubiquitous in DIS final states*
Conjectured to be very general property of the infrared in gauge theories & gravity
In gravity, the symmetries are the BMS symmetry and the corresponding gravitational memory leads to a physical displacement of inertial detectors measurable by LIGO.
In QED, these correspond to a symmetry group of an infinite number of conserved charges at $x^- = \pm \infty$ on the celestial sphere (stereographic projection of transverse plane) and these satisfy a conservation law with the S-matrix

$$\langle \text{out}\left| (Q_+ S - S Q_-)\right|\text{in}\rangle = 0$$

This is equivalent to the soft photon theorem and also imposes a condition that the S-matrix be dressed by a soft factor that renders it infrared finite

Equivalent to Faddeev-Kulish (1970) coherent state representation of S-matrix

Kapec,Perry,Raclariu,Strominger,arXiv:1705.043011

On light front, for FK rep. see More, Misra, 1206.3097
Bold conjecture: QCD in the infrared

- Asymptotic symmetries from gravity/gauge theories-soft theorems
- Faddeev-Kulish coherent state formalism for asymptotic scattering states
- QCD computations – up to 3 loops soft gluon exponentiation, cusp anomalous dimensions,
- Simplification in high energy limit – allowing for clean connection to Reggeization
- Boundary conditions at null-infinity
  Memory (including color memory)
- Color Glass Condensate

Soft pion theorems
Thank you for your attention!
Classical field of a large nucleus

\[ < AA >_\rho = \int [d\rho] A_{cl.}(\rho) A_{cl.}(\rho) W_{\Lambda+}[\rho] \]

For a large nucleus, \( A >> 1 \),

\[ W_{\Lambda+} = \exp \left( - \int d^2x_\perp \left[ \frac{\rho^a \rho^a}{2 \mu_A^2} - \frac{d_{abc} \rho^a \rho^b \rho^c}{\kappa_A} \right] \right) \]

A_{cl} from \( (D_\mu F^{\mu \nu})^a = J^{\nu,a} \equiv \delta^{\nu+} \delta(x^-) \rho^a(x_\perp) \)

\( 1/\Lambda_{QCD} \quad 2R / \gamma \)

Wee parton dist. : \( \frac{1}{\Lambda_{QCD}} e^{-\lambda \Delta Y/2} \)

determined from RG evolution

\[ \phi_A \]

\[ \Lambda_{QCD} \quad Q_4(A_1) \quad Q_4(A_2) \quad k_L \]

McLerran, RV
Kovchegov
Jeon, RV
The NLO inclusive photon impact factor

Inclusive photon cross-section: NLO-real * NLO*real + LO * NLO virtual

<table>
<thead>
<tr>
<th>Wilson line factor</th>
<th>Real emission</th>
<th>Virtual: Vertex</th>
<th>Virtual: Self-energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{N_c}{2} \left( 1 - D_{xy} D_{zy} - D_{y'z} D_{zx'} \right) + D_{y'y} D_{zx'} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_1, x'_1)$</td>
<td>$T_R^{(1)*} T_R^{(1)}$</td>
<td>$T_{LO} T_V^{(3)} + c.c$</td>
<td>$T_{LO} T_S^{(3)} + c.c$</td>
</tr>
<tr>
<td>$C_F N_c \Xi(x_\perp, y_\perp; y'_1, x'_1)$</td>
<td>$T_R^{(2)<em>} T_R^{(2)} + T_R^{(3)</em>} T_R^{(3)}$</td>
<td>$T_{LO} T_V^{(4)} + c.c$</td>
<td>$T_{LO} T_V^{(4)} + c.c$</td>
</tr>
<tr>
<td>$\frac{N_c}{2} \left[ (1 - D_{xy}) (1 - D_{y'z}) \right] - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_1, x'_1)$</td>
<td>$T_R^{(2)*} T_R^{(3)} + c.c$</td>
<td>$T_{LO} T_V^{(1)}$</td>
<td>$T_{LO} T_S^{(1)}$</td>
</tr>
<tr>
<td>$\frac{N_c}{2} \left( 1 + (Q_{xy:z} - D_{y'z}) D_{zx} - D_{y'z} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_1, x'_1)$</td>
<td>$T_R^{(2)*} T_R^{(1)}$</td>
<td>$T_V^{(1)*} T_L O$</td>
<td>$T_{S}^{(1)*} T_{LO}$</td>
</tr>
<tr>
<td>$\frac{N_c}{2} \left( 1 + (Q_{y':z} - D_{y'z}) D_{zx'} - D_{y'y} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_1, x'_1)$</td>
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<td>$T_{LO} T_V^{(2)}$</td>
<td>$T_{ LO} T_S^{(2)}$</td>
</tr>
<tr>
<td>$\frac{N_c}{2} \left( 1 + (Q_{xz:z} - D_{zx}) D_{xy} - D_{y'y} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_1, x'_1)$</td>
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<td>$T_{S}^{(2)*} T_{LO}$</td>
</tr>
</tbody>
</table>

For each Wilson line structure, collinear divergences cancel between real and interference contributions.

Rapidity and UV divergent pieces: these can be absorbed, in a subtraction scheme, into the NLLx JIMWLK expressions.