Accessing Generalized Parton Distributions through the photoproduction of a photon-meson pair

Lech Szymanowski

National Centre for Nuclear Research
Warsaw, Poland

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in collaboration with
B. Pire (CPhT, Palaiseau), R. Boussarie (IFJ Cracow), S. Wallon (LPT, Orsay)

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with G. Duplančić, K. Passek-Kumerički (IRB, Zagreb)
Transversity of the nucleon using hard processes

What is transversity?

- Transverse spin content of the proton:
  \[ |\uparrow(x)\rangle \sim |\rightarrow\rangle + |\leftrightarrow\rangle \]
  \[ |\downarrow(x)\rangle \sim |\rightarrow\rangle - |\leftrightarrow\rangle \]
  spin along \(x\) helicity states

- Observables which are sensitive to helicity flip thus give access to transversity \(\Delta_T q(x)\). Poorly known.

- Transversity GPDs are completely unknown experimentally.

For massless (anti)particles, chirality = (-)helicity

Transversity is thus a chiral-odd quantity

Since (in the massless limit) QCD and QED are chiral-even \((\gamma^\mu, \gamma^\mu\gamma^5)\), the chiral-odd quantities \((1, \gamma^5, [\gamma^\mu, \gamma^\nu])\) which one wants to measure should appear in pairs
Transversity of the nucleon using hard processes: using a two body final state process?

How to get access to transversity GPDs?

- The dominant DA of $\rho_T$ is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- Unfortunately $\gamma^* N^\uparrow \rightarrow \rho_T N' = 0$
  - This cancellation is true at any order: such a process would require a helicity transfer of 2 from a photon.

- Lowest order diagrammatic argument:

$$\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha \rightarrow 0$$

[Diehl, Gousset, Pire], [Collins, Diehl]
Transversity of the nucleon using hard processes: using a two body final state process?

Can one circumvent this vanishing?

- This vanishing only occurs at twist 2.

- At twist 3 this process does not vanish [Ahmad, Goldstein, Liuti], [Goloskokov, Kroll].

- However processes involving twist 3 DAs may face problems with factorization (end-point singularities) can be made safe in the high-energy $k_T$–factorization approach [Anikin, Ivanov, Pire, Sz., Wallon].

- One can also consider a 3-body final state process [Ivanov, Pire, Sz., Teryaev], [Enberg, Pire, Sz.], [El Beiyad, Pire, Segond, Sz., Wallon].
Probing GPDs using $\rho$ meson + photon production

- We consider the process $\gamma N \rightarrow \gamma \rho N'$
- Collinear factorization of the amplitude for $\gamma + N \rightarrow \gamma + \rho + N'$ at large $M_{\gamma \rho}^2$

large angle factorization à la Brodsky Lepage
Probing chiral-even GPDs using $\rho$ meson + photon production

Processes with 3 body final states can give access to chiral-even GPDs

![Diagram showing chiral-even twist 2 GPD](image)

$t'$

$T_H$

$M_{\gamma\rho}^2$

$x + \xi$

$x - \xi$

$N$

$N'$

$\rho_L$

chiral-even twist 2 DA

$t$ (small)

chiral-even twist 2 GPD
Probing chiral-odd GPDs using $\rho$ meson + photon production

Processes with 3 body final states can give access to chiral-odd GPDs

![Diagram showing processes with 3 body final states giving access to chiral-odd GPDs.](image-url)

chiral-odd twist 2 DA

chiral-odd twist 2 GPD
Probing chiral-odd GPDs using $\rho$ meson + photon production

Processes with 3 body final states can give access to chiral-odd GPDs

How did we manage to circumvent the no-go theorem for $2 \rightarrow 2$ processes?

Typical non-zero diagram for a transverse $\rho$ meson

the $\sigma$ matrices (from DA and GPD sides) do not kill it anymore!
Master formula based on leading twist 2 factorization

\[ A \propto \int_{-1}^{1} dx \int_{0}^{1} dz \, T(x, \xi, z) \times H(x, \xi, t) \Phi_\rho(z) + \cdots \]

- Both the DA and the GPD can be either chiral-even or chiral-odd.
- At twist 2 the longitudinal \( \rho \) DA is chiral-even and the transverse \( \rho \) DA is chiral-odd.
- Hence we will need both chiral-even and chiral-odd non-perturbative building blocks and hard parts.
Kinematics

Kinematics to handle GPD in a 3-body final state process

- use a Sudakov basis:
  - light-cone vectors $p$, $n$ with $2p \cdot n = s$
- assume the following kinematics:
  - $\Delta_\perp \ll p_\perp$
  - $M^2, m^2_\rho \ll M^2_\gamma \rho$
- initial state particle momenta:
  $$q^\mu = n^\mu, \quad p_1^\mu = (1 + \xi) p^\mu + \frac{M^2}{s(1 + \xi)} n^\mu$$
- final state particle momenta:
  $$p_2^\mu = (1 - \xi) p^\mu + \frac{M^2 + \tilde{p}_t^2}{s(1 - \xi)} n^\mu + \Delta_\perp^\mu$$
  $$k^\mu = \alpha n^\mu + \frac{(\tilde{p}_t - \tilde{\Delta}_t/2)^2}{\alpha s} p^\mu + p_\perp^\mu - \frac{\Delta_\perp^\mu}{2},$$
  $$p^\mu_\rho = \alpha_\rho n^\mu + \frac{(\tilde{p}_t + \tilde{\Delta}_t/2)^2 + m^2_\rho}{\alpha_\rho s} p^\mu - p_\perp^\mu - \frac{\Delta_\perp^\mu}{2},$$
Non perturbative chiral-even building blocks

- Helicity conserving GPDs at twist 2:

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^- \right) \gamma^+ \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle = \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H^q(x, \xi, t) \gamma^+ + E^q(x, \xi, t) \frac{i\sigma^\alpha \Delta_\alpha}{2m} \right]
\]

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^- \right) \gamma^+ \gamma^5 \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle = \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ \tilde{H}^q(x, \xi, t) \gamma^+ \gamma^5 + \tilde{E}^q(x, \xi, t) \frac{\gamma^5 \Delta^+}{2m} \right]
\]

- We will consider the simplest case when $\Delta_\perp = 0$.

- In that case and in the forward limit $\xi \to 0$ only the $H^q$ and $\tilde{H}^q$ terms survive.

- Helicity conserving (vector) DA at twist 2:

\[
\langle 0 | \bar{u}(0) \gamma^\mu u(x) | \rho^0(p, s) \rangle = \frac{p^\mu}{\sqrt{2}} f_\rho \int_0^1 du \, e^{-iup \cdot x} \phi_\parallel(u)
\]
Non perturbative chiral-odd building blocks

- Helicity flip GPD at twist 2:

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^- \right) i\sigma^+ \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle \\
= \frac{1}{2P+} \bar{u}(p_2, \lambda_2) \left[ H^q_T(x, \xi, t)i\sigma^+ + \tilde{H}^q_T(x, \xi, t) \frac{P^+ \Delta^i - \Delta^+ P^i}{M_N^2} \right. \\
+ \left. E^q_T(x, \xi, t) \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2M_N} + \tilde{E}^q_T(x, \xi, t) \frac{\gamma^+ P^i - P^+ \gamma^i}{M_N} \right] u(p_1, \lambda_1)
\]

- We will consider the simplest case when $\Delta_\perp = 0$.

- In that case and in the forward limit $\xi \to 0$ only the $H^q_T$ term survives.

- Transverse $\rho$ DA at twist 2:

\[
\langle 0 | \bar{u}(0) \sigma^{\mu\nu} u(x) | \rho^0(p, s) \rangle = \frac{i}{\sqrt{2}} (\epsilon_\mu^p p^\nu - \epsilon_\nu^p p^\mu) f_{\rho \perp} \int_0^1 du e^{-iuP \cdot x} \phi_\perp(u)
\]
Models for DAs

Asymptotical DAs

We take the simplistic asymptotic form of the (normalized) DAs:

\[ \phi_{\parallel}(z) = 6z(1 - z), \]
\[ \phi_{\perp}(z) = 6z(1 - z). \]
Model for GPDs: based on the Double Distribution ansatz

Realistic Parametrization of GPDs

- GPDs can be represented in terms of Double Distributions [Radyushkin]
  - based on the Schwinger representation of a toy model for GPDs which has the structure of a triangle diagram in scalar $\phi^3$ theory

  \[
  H^q(x, \xi, t = 0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) f^q(\beta, \alpha)
  \]

- ansatz for these Double Distributions [Radyushkin]:

  \[
  \begin{align*}
  f^q(\beta, \alpha, t = 0) &= \Pi(\beta, \alpha) q(\beta) \Theta(\beta) - \Pi(-\beta, \alpha) \bar{q}(-\beta) \Theta(-\beta), \\
  \tilde{f}^q(\beta, \alpha, t = 0) &= \Pi(\beta, \alpha) \Delta q(\beta) \Theta(\beta) + \Pi(-\beta, \alpha) \Delta \bar{q}(-\beta) \Theta(-\beta).
  \end{align*}
  \]

  \[
  \begin{align*}
  \text{chiral-even sector:} \\
  f^q(\beta, \alpha, t = 0) &= \Pi(\beta, \alpha) q(\beta) \Theta(\beta) - \Pi(-\beta, \alpha) \bar{q}(-\beta) \Theta(-\beta), \\
  \tilde{f}^q(\beta, \alpha, t = 0) &= \Pi(\beta, \alpha) \Delta q(\beta) \Theta(\beta) + \Pi(-\beta, \alpha) \Delta \bar{q}(-\beta) \Theta(-\beta).
  \end{align*}
  \]

  \[
  \begin{align*}
  \text{chiral-odd sector:} \\
  f^q_T(\beta, \alpha, t = 0) &= \Pi(\beta, \alpha) \delta q(\beta) \Theta(\beta) - \Pi(-\beta, \alpha) \delta \bar{q}(-\beta) \Theta(-\beta), \\
  \Pi(\beta, \alpha) &= \frac{3}{4} \frac{(1-\beta)^2-\alpha^2}{(1-\beta)^3} : \text{profile function}
  \end{align*}
  \]

- simplistic factorized ansatz for the $t$-dependence:

  \[
  H^q(x, \xi, t) = H^q(x, \xi, t = 0) \times F_H(t)
  \]

  with \( F_H(t) = \frac{C^2}{(t-C)^2} \) a standard dipole form factor \( (C = .71 \text{ GeV}) \)
Model for GPDs: based on the Double Distribution ansatz

Sets of used PDFs

- $q(x)$: unpolarized PDF [GRV-98] and [MSTW2008lo, MSTW2008nnlo, ABM11nnlo, CT10nnlo]
- $\Delta q(x)$: polarized PDF [GRSV-2000]
- $\delta q(x)$: transversity PDF [Anselmino et al.]
Model for GPDs: based on the Double Distribution ansatz

Typical sets of chiral-even GPDs ($C = -1$ sector)

\[ \xi = 0.1 \leftrightarrow S_{\gamma N} = 20 \text{ GeV}^2 \text{ and } M_{\gamma\rho}^2 = 3.5 \text{ GeV}^2 \]

\[ H^{u(-)}(x, \xi) \]

\[ H^{d(-)}(x, \xi) \]

\[ H^{q(-)}(x, \xi, t) = H^q(x, \xi, t) + H^q(-x, \xi, t) \]

five Ansätze for $q(x)$: GRV-98, MSTW2008lo, MSTW2008nnlo, ABM11nnlo, CT10nnlo

\[ \tilde{H}^{u(-)}(x, \xi) \]

\[ \tilde{H}^{d(-)}(x, \xi) \]

\[ \tilde{H}^{q(-)}(x, \xi, t) = \tilde{H}^q(x, \xi, t) - \tilde{H}^q(-x, \xi, t) \]

“valence” and “standard”: two GRSV Ansätze for $\Delta q(x)$
Model for GPDs: based on the Double Distribution ansatz

Typical sets of chiral-odd GPDs \((C = -1 \text{ sector})\)

\[
\xi = 0.1 \leftrightarrow S_{\gamma N} = 20 \text{ GeV}^2 \text{ and } M_{\gamma \rho}^2 = 3.5 \text{ GeV}^2
\]

\[
H^u_T(-)(x, \xi) \quad \text{and} \quad H^d_T(-)(x, \xi)
\]

\[
H^q_T(-)(x, \xi, t) = H^q_T(x, \xi, t) + H^q_T(-x, \xi, t)
\]

“valence” and “standard”: two GRSV Ansätze for \(\Delta q(x)\)

\(\Rightarrow\) two Ansätze for \(\delta q(x)\)
Computation of the hard part

20 diagrams to compute

The other half can be deduced by $q \leftrightarrow \bar{q}$ (anti)symmetry

Red diagrams cancel in the chiral-odd case
Final computation

\[ \mathcal{A} \propto \int_{-1}^{1} dx \int_{0}^{1} dz \ T(x, \xi, z) \ H(x, \xi, t) \ \Phi_\rho(z) \]

- One performs the \( z \) integration \textit{analytically} using an asymptotic DA \( \propto z(1 - z) \)

- One then plugs our GPD models into the formula and performs the integral w.r.t. \( x \) numerically.

- Differential cross section:

\[
\left. \frac{d\sigma}{dt \ dw' \ dM_{\gamma\rho}^2} \right|_{-t=-(t)_{\text{min}}} = \frac{|\mathcal{M}|^2}{32 S_{\gamma N}^2 M_{\gamma\rho}^2 (2\pi)^3}.
\]

\( |\mathcal{M}|^2 \) = averaged amplitude squared

- Kinematical parameters: \( S_{\gamma N}^2, M_{\gamma\rho}^2 \) and \( -w' \)
Fully differential cross section

Chiral even cross section

\[ \frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \text{ (pb} \cdot \text{GeV}^{-6}) \]

\[ \frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \text{ (pb} \cdot \text{GeV}^{-6}) \]

proton

\[ S_{\gamma N} = 20 \text{ GeV}^2 \]
\[ M_{\gamma\rho}^2 = 3, 4, 5, 6 \text{ GeV}^2 \]

solid: “valence” model

dotted: “standard” model

neutron
Fully differential cross section

Chiral odd cross section

at $-t = (-t)_{\text{min}}$

\[
\frac{d\sigma_{\text{odd}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \quad \text{(pb} \cdot \text{GeV}^{-6})
\]

- **Proton**
  - "valence" and "standard" models,
  - each of them with $\pm 2\sigma$ [S. Melis]

- **Neutron**
  - "valence" model only

\[
S_{\gamma N} = 20 \text{ GeV}^2 \\
M_{\gamma\rho}^2 = 3, 4, 5, 6 \text{ GeV}^2
\]
Phase space integration

Evolution of the phase space in \((-t, -u')\) plane

large angle scattering: \(M_{\gamma\rho}^2 \sim -u' \sim -t'\)

in practice: \(-u' > 1\, \text{GeV}^2\) and \(-t' > 1\, \text{GeV}^2\) and \((-t)_{\text{min}} \leq -t \leq .5\, \text{GeV}^2\)

this ensures large \(M_{\gamma\rho}^2\)

example: \(S_{\gamma N} = 20\, \text{GeV}^2\)

\(M_{\gamma\rho} = 2.2\, \text{GeV}^2\)

\(M_{\gamma\rho}^2 = 2.5\, \text{GeV}^2\)

\(M_{\gamma\rho} = 3\, \text{GeV}^2\)

\(M_{\gamma\rho} = 5\, \text{GeV}^2\)

\(M_{\gamma\rho} = 8\, \text{GeV}^2\)

\(M_{\gamma\rho} = 9\, \text{GeV}^2\)
Variation with respect to $S_{\gamma N}$

Mapping $(S_{\gamma N}, M_{\gamma \rho}) \mapsto (\tilde{S}_{\gamma N}, \tilde{M}_{\gamma \rho})$

One can save a lot of CPU time:

- $\mathcal{M}(\alpha, \xi)$ and $GPDs(\xi, x)$

- In the generalized Bjorken limit:
  - $\alpha = \frac{-u'}{M_{\gamma \rho}^2}$
  - $\xi = \frac{M_{\gamma \rho}^2}{2(S_{\gamma N} - M_{\gamma \rho}^2) - M_{\gamma \rho}^2}$

Given $S_{\gamma N} (= 20 \text{ GeV}^2)$, with its grid in $M_{\gamma \rho}^2$, choose another $\tilde{S}_{\gamma N}$.

One can get the corresponding grid in $\tilde{M}_{\gamma \rho}$ by just keeping the same $\xi$'s:

$$\tilde{M}_{\gamma \rho}^2 = M_{\gamma \rho}^2 \frac{\tilde{S}_{\gamma N} - M^2}{S_{\gamma N} - M^2},$$

From the grid in $-u'$, the new grid in $-\tilde{u}'$ is given by just keeping the same $\alpha$'s:

$$-\tilde{u}' = \frac{\tilde{M}_{\gamma \rho}^2}{M_{\gamma \rho}^2} (-u'),$$

$\Rightarrow$ a single set of numerical computations is required (we take $S_{\gamma N} = 20 \text{ GeV}^2$)
Single differential cross section

**Chiral even cross section**

\[
\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2} \text{ (pb} \cdot \text{GeV}^{-2})
\]

- **Proton**
- **Neutron**

\[
S_{\gamma N} \text{ vary in the set } 8, 10, 12, 14, 16, 18, 20 \text{ GeV}^2 \text{ (from left to right)}
\]
Single differential cross section

Chiral odd cross section

\[
\frac{d\sigma_{\text{odd}}}{dM_{\gamma\rho}^2} \text{ (pb} \cdot \text{GeV}^{-2})
\]

\[
S_{\gamma N} = 20\text{GeV}^2
\]

Various ansätze for the PDFs $\Delta q$ used to build the GPD $H_T$:

- *dotted curves*: “standard” scenario
- solid curves: “valence” scenario
- deep-blue and red curves: central values
- light-blue and orange: results with $\pm 2\sigma$. 
Single differential cross section

Chiral odd cross section

\[ \frac{d\sigma_{\text{odd}}}{dM_{\gamma\rho}^2} \text{ (pb} \cdot \text{GeV}^{-2}) \]

proton, “valence” scenario

\( S_{\gamma N} \) vary in the set 8, 10, 12, 14, 16, 18, 20 GeV\(^2\) (from left to right)
Integrated cross-section

Chiral even cross section

\[ \sigma_{\text{even}} \text{ (pb)} \]

- **proton**
- **neutron**

- **solid red**: "valence" scenario
- **dashed blue**: "standard" one
Integrated cross-section

Chiral odd cross section

\[ \sigma_{odd} \text{ (pb)} \]

- **Proton**
  - Solid red: “valence” scenario
  - Dashed blue: “standard” one

- **Neutron**
  - Solid red: “valence” scenario
  - Dashed blue: “standard” one
Counting rates for 100 days

example: JLab Hall B

- untagged incoming $\gamma \Rightarrow$ Weizsäcker-Williams distribution

- With an expected luminosity of $\mathcal{L} = 100 \text{ nb}^{-1} \text{s}^{-1}$, for 100 days of run:
  - Chiral even case: $\simeq 1.9 \times 10^5 \rho_L$.
  - Chiral odd case: $\simeq 7.5 \times 10^3 \rho_T$. 
Effects of an experimental angular restriction for the produced $\gamma$

Angular distribution of the produced $\gamma$, $\rho_L$ photoproduction (chiral-even cross section)

after boosting to the lab frame

$$\frac{1}{\sigma_{\text{even}}} \frac{d\sigma_{\text{even}}}{d\theta}$$

$S_{\gamma N} = 10 \text{ GeV}^2$

$M_{\gamma\rho}^2 = 3, 4 \text{ GeV}^2$

JLab Hall B detector equipped between $5^\circ$ and $35^\circ$

$S_{\gamma N} = 15 \text{ GeV}^2$

$M_{\gamma\rho}^2 = 3, 4, 5 \text{ GeV}^2$

$S_{\gamma N} = 20 \text{ GeV}^2$

$M_{\gamma\rho}^2 = 3, 4, 5 \text{ GeV}^2$

⇒ this is safe!
Effects of an experimental angular restriction for the produced $\gamma$

Angular distribution of the produced $\gamma$, $\rho_L$ photoproduction (chiral-even cross section)

$$\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2} \text{ (pb} \cdot \text{GeV}^{-2})$$

$S_{\gamma N} = 10 \text{ GeV}^2$

$S_{\gamma N} = 15 \text{ GeV}^2$

$S_{\gamma N} = 20 \text{ GeV}^2$

$\theta_{\text{max}} = 35^\circ, 30^\circ, 25^\circ, 20^\circ, 15^\circ, 10^\circ$

JLab Hall B detector equipped between $5^\circ$ and $35^\circ$

$\Rightarrow \text{this is safe!}$
Effects of an experimental angular restriction for the produced $\gamma$

Angular distribution of the produced $\gamma$, $\rho_T$ photoproduction (chiral-odd cross section)

after boosting to the lab frame

\[
\frac{1}{\sigma_{odd}} \frac{d\sigma_{odd}}{d\theta}
\]

\[
\begin{align*}
S_{\gamma N} &= 10 \text{ GeV}^2 \\
S_{\gamma N} &= 15 \text{ GeV}^2 \\
S_{\gamma N} &= 20 \text{ GeV}^2 \\
M^2_{\gamma\rho} &= 3, 4 \text{ GeV}^2 \\
M^2_{\gamma\rho} &= 3.5, 5, 6.5 \text{ GeV}^2 \\
M^2_{\gamma\rho} &= 4, 6, 8 \text{ GeV}^2
\end{align*}
\]

JLab Hall B detector equipped between 5° and 35°

$\Rightarrow$ this is safe!
Effects of an experimental angular restriction for the produced $\gamma$

Angular distribution of the produced $\gamma$, $\rho_T$ photoproduction (chiral-odd cross section)

\[ \frac{d\sigma_{\text{odd}}}{dM_{\gamma\rho}^2} \text{ (pb} \cdot \text{GeV}^{-2}) \]

\[ S_{\gamma N} = 10 \text{ GeV}^2 \]

\[ \theta_{max} = 35^\circ, 30^\circ, 25^\circ, 20^\circ, 15^\circ, 10^\circ \]

JLab Hall B detector equipped between $5^\circ$ and $35^\circ$

⇒ this is safe!
Conclusion (1)

- High statistics for the chiral-even component: enough to extract $H$ ($\tilde{H}$?) and test the universality of GPDs.
- In this chiral-even sector: analogy with Timelike Compton Scattering, the $\gamma\rho$ pair playing the role of the $\gamma^*$.  
- Relative dominance of the chiral-even component w.r.t. the chiral-odd one: $\sigma_{odd}/\sigma_{even} \sim 1/25$.  
  - possible separation $\rho_L/\rho_T$ through an angular analysis of its decay products. Cuts in $\theta_\gamma$ might help to increase this ratio (allowed by the huge statistics).
- Future: study of polarization observables $\Rightarrow$ sensitive to the interference of these two amplitudes: very sizable effect expected, of the order of 20%.
- The Bethe Heitler component (outgoing $\gamma$ emitted from the incoming lepton) is:
  - zero for the chiral-odd case
  - suppressed for the chiral-even case
- Our result can also be applied to electroproduction ($Q^2 \neq 0$) after adding Bethe-Heitler contributions and interferences.
- Possible measurement at JLab (Hall B, C, D).
- A similar study could be performed at COMPASS, EIC, LHC in UPC?
Collaboration with Goran Duplančić, Kornelija Passek-Kumerički (IRB, Zagreb), Bernard Pire (CPhT), Samuel Wallon (LPT, Orsay)

- We are planning to investigate the process $\gamma N \rightarrow \gamma \pi^{\pm, 0} N'$ at one loop

- the processes $\gamma N \rightarrow \gamma \pi^0 N'$ and $\gamma N \rightarrow \gamma \eta^0 N'$ are of particular interest: they give an access to the gluonic GPDs at Born order.
Chiral-even cross section

**Contribution of \( u \) versus \( d \), \( \rho_L \) photoproduction**

\[
\frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \quad (\text{pb} \cdot \text{GeV}^{-6})
\]

\( M_{\gamma\rho}^2 = 4 \text{ GeV}^2 \). Both vector and axial GPDs are included.

- **proton**
- **neutron**

\( u + d \) quarks  \quad u \) quark  \quad d \) quark

Solid: “valence” model
dotted: “standard” model

- \( u \)-quark contribution dominates due to the charge effect
Chiral-even cross section

Contribution of vector versus axial amplitudes, $\rho_L$ photoproduction

\[ \frac{d\sigma_{\text{even}}}{dM_{\gamma\rho}^2 d(-u')d(-t)} \text{ (pb} \cdot \text{GeV}^{-6}) \]

(proton)

\[ M_{\gamma\rho}^2 = 4 \text{ GeV}^2. \text{ Both } u \text{ and } d \text{ quark contributions are included.} \]

vector + axial amplitudes / vector amplitude / axial amplitude

solid: “valence” model

dotted: “standard” model

- dominance of the vector GPD contributions
- no interference between the vector and axial amplitudes
Hard photoproduction of a diphoton with a large invariant mass


\[ \gamma(q, \epsilon) + N(p_1, s_1) \rightarrow \gamma(k_1, \epsilon_1) + \gamma(k_2, \epsilon_2) + N'(p_2, s_2) \]

Figure: Feynman diagrams contributing to the coefficient function of the process
\[ \gamma N \rightarrow \gamma \gamma N' \]
Hard photoproduction of a diphoton with a large invariant mass

- Purely electromagnetic process at Born order - as are deep inelastic scattering (DIS), deeply virtual Compton scattering (DVCS) and timelike Compton scattering (TCS).
-Insensitive to gluon GPDs.
- No contribution from the badly known chiral-odd quark distributions.
- This study enlarges the range of $2 \rightarrow 3$ reactions analyzed in the framework of collinear QCD factorization. Simplest - great tool to study factorization.
Coefficient functions and generalized Form Factors

\[ iCF^V_q = Tr[iM \not{p}] = \]
\[-ie_q^3 \left[ A^V \left( \frac{1}{D_1(x)D_2(x)} + \frac{1}{D_1(-x)D_2(-x)} \right) \right. \]
\[ \left. + B^V \left( \frac{1}{D_1(x)D_3(x)} + \frac{1}{D_1(-x)D_3(-x)} \right) \right] \]
\[ \left. + C^V \left( \frac{1}{D_2(x)D_3(-x)} + \frac{1}{D_2(-x)D_3(x)} \right) \right], \]

\[ iCF^A_q = Tr[iM\gamma^5 \not{p}] = \]
\[-ie_q^3 \left[ A^A \left( \frac{1}{D_1(x)D_2(x)} - \frac{1}{D_1(-x)D_2(-x)} \right) \right. \]
\[ \left. + B^A \left( \frac{1}{D_1(x)D_3(x)} - \frac{1}{D_1(-x)D_3(-x)} \right) \right] \]

where \( A^V, \ldots, A^A, \ldots \) depend on photons polarizations and final photons \( p_T \).
Denominators read:

\[ D_1(x) = s(x + \xi + i\varepsilon), \quad D_2(x) = s\alpha_2(x - \xi + i\varepsilon), \quad D_3(x) = s\alpha_1(x - \xi + i\varepsilon) \]
Generalized form factors

The scattering amplitude is written in terms of generalized Compton form factors $\mathcal{H}^q(\xi)$, $\mathcal{E}^q(\xi)$, $\tilde{\mathcal{H}}^q(\xi)$ and $\tilde{\mathcal{E}}^q(\xi)$ as

$$
\mathcal{T} = \frac{1}{2s} \left[ \left( \mathcal{H}(\xi) \bar{U}(p_2) \not{n} U(p_1) + \mathcal{E}(\xi) \bar{U}(p_2) \frac{i\sigma^{\mu\nu}\Delta_{\nu\mu}}{2M} U(p_1) \right) + \left( \tilde{\mathcal{H}}(\xi) \bar{U}(p_2) \not{n} \gamma^5 U(p_1) + \tilde{\mathcal{E}}(\xi) \bar{U}(p_2) \frac{i\gamma_5(\Delta \cdot n)}{2M} U(p_1) \right) \right]
$$

$$
\mathcal{H}(\xi) = \sum_q \int_{-1}^{1} dx \, CF_q^V(x, \xi) H^q(x, \xi), \quad \tilde{\mathcal{H}}(\xi) = \sum_q \int_{-1}^{1} dx \, CF_q^A(x, \xi) \tilde{H}^q(x, \xi),
$$

$$
\mathrm{Re} \, \mathcal{H}(\xi) \sim \sum_q e_q^3 P.V. \int_{-1}^{1} dx \, \frac{H^q(x, \xi) + H^q(-x, \xi)}{x - \xi}
$$

$$
\mathrm{Im} \, \mathcal{H}(\xi) \sim \sum_q e_q^3 [H^q(\xi, \xi) + H^q(-\xi, \xi)]
$$

$$
\mathrm{Re} \, \tilde{\mathcal{H}}(\xi) \sim 0
$$

$$
\mathrm{Im} \, \tilde{\mathcal{H}}(\xi) \sim \sum_q e_q^3 \left[ \tilde{H}^q(\xi, \xi) - \tilde{H}^q(-\xi, \xi) \right]
$$
Differential cross section

Choosing as independent kinematical variables \{t, u', M_{\gamma\gamma}^2\}, the fully unpolarized differential cross section reads

\[
\frac{d\sigma}{dM_{\gamma\gamma}^2 dt d(-u')} = \frac{1}{2} \frac{1}{(2\pi)^3} \frac{32 S_{\gamma N}^2 M_{\gamma\gamma}^2}{\lambda,\lambda_1\lambda_2,s_1,s_2} \sum \frac{|T|^2}{4}
\]

**Figure:** the \(M_{\gamma\gamma}^2\) dependence of the unpolarized differential cross section on a proton at \(t = t_{\text{min}}\) and \(S_{\gamma N} = 20\text{GeV}^2\) (full curves) and \(S_{\gamma N} = 100\text{GeV}^2\) (dashed curve). The bounds in \(u'\) are chosen so that both \(-u'\) and \(-t'\) are larger than 1 GeV^2.
Polarization asymmetries

- **Circular** initial photon polarization cross-section difference reads:

\[ \mathcal{T}_+ \mathcal{T}_+^* - \mathcal{T}_- \mathcal{T}_-^* \sim |\Delta t||p_t|, \]

so circular polarization asymmetry is of \( O(\frac{\Delta T}{Q}) \).

- **Linear** initial photon polarization defines the \( x \) axis:

\[ \epsilon(q) = (0, 1, 0, 0) \]

and hence the azimuthal angle \( \phi \) through

\[ p_T^\mu = (0, p_T \cos \phi, p_T \sin \phi, 0). \]
Azimuthal dependence

Figure: the azimuthal dependence of the differential cross section \( \frac{d\sigma}{dM_{\gamma\gamma}^2 dt du' d\phi} \) at \( t = t_{\text{min}} \) and \( S_{\gamma N} = 20 \text{ GeV}^2 \). \( (M_{\gamma\gamma}^2, u') = (3, -2) \text{ GeV}^2 \) (solid line), \( (M_{\gamma\gamma}^2, u') = (4, -1) \text{ GeV}^2 \) (dotted line) and \( (M_{\gamma\gamma}^2, u') = (4, -2) \text{ GeV}^2 \) (dashed line). \( \phi \) is the angle between the initial photon polarization and one of the final photon momentum in the transverse plane.
Summary - diphoton photoproduction

- Purely electromagnetic process at Born order
- Insensitive to gluon GPDs
- Cross section of the order of TCS which is measurable at JLAB
- Strong azimuthal dependence for linearly polarized photon beam

To be done:
- The $O(\alpha_s)$ corrections to the amplitude need to be calculated. They are particularly interesting since they open the way to a perturbative proof of factorization.
- Importance of the timelike vs spacelike nature of the probe with respect to the size of the NLO corrections; since the hard scales at work in our process are both the timelike one $M_{\gamma\gamma}^2$ and the spacelike one $u'$, we are facing an intermediate case between timelike Compton scattering (TCS) and spacelike DVCS.
- Leptoproduction needs to be complemented by the analysis of the Bethe Heitler processes where one or two photons are emitted from the lepton line. Probably dominating and leading to interesting interference effects.
Fully differential cross section: $\pi^{\pm}$

**Chiral even sector: $\pi^{\pm}$**

At $-t = (-t)_{\text{min}}$

$$\frac{d\sigma_{\gamma\pi}^{+}}{dM_{\gamma\pi}^{2}d(-u')d(-t)} \quad (\text{pb} \cdot \text{GeV}^{-6})$$

$$\frac{d\sigma_{\gamma\pi}^{-}}{dM_{\gamma\pi}^{2}d(-u')d(-t)} \quad (\text{pb} \cdot \text{GeV}^{-6})$$

$\pi^{+}$ photoproduction (proton target)  \hspace{1cm} $\pi^{-}$ photoproduction (neutron target)

$$S_{\gamma N} = 20 \text{ GeV}^2$$  \hspace{1cm} $$M_{\gamma\rho}^2 = 4 \text{ GeV}^2$$

**vector GPD / axial GPD / total result**

- solid: “valence” model
- dotted: “standard” model
Fully differential cross section: $\pi^\pm$

Chiral even sector: $\pi^\pm$

at $-t = (-t)_{\text{min}}$

$\pi^+$ photoproduction (proton target)  
$
\pi^- \text{ photoproduction (neutron target)}$

$S_{\gamma N} = 20$ GeV$^2$

$M^2_{\gamma \rho} = 3, 4, 5, 6$ GeV$^2$

solid: "valence" model

dotted: "standard" model

$\frac{d\sigma_{\gamma \pi^+}}{dM^2_{\gamma \pi^+} d(-u') d(-t)} \ (\text{pb} \cdot \text{GeV}^{-6})$

$\frac{d\sigma_{\gamma \pi^-}}{dM^2_{\gamma \pi^-} d(-u') d(-t)} \ (\text{pb} \cdot \text{GeV}^{-6})$
Single differential cross section: $\pi^\pm$

Chiral even sector: $\pi^\pm$

\[
\frac{d\sigma_{\pi^+}}{dM_{\gamma\pi^+}^2} \text{ (pb \cdot GeV}^{-2})
\]

\[
\frac{d\sigma_{\pi^-}}{dM_{\gamma\pi^-}^2} \text{ (pb \cdot GeV}^{-2})
\]

$\pi^+$ photoproduction (proton target)  $\pi^-$ photoproduction (neutron target)

$S_{\gamma N}$ vary in the set 8, 10, 12, 14, 16, 18, 20 GeV$^2$ (from left to right)

solid: “valence” model  
dotted: “standard” model
Integrated cross-section: $\pi^\pm$

Chiral even sector: $\pi^\pm$

$\sigma_{\gamma\pi^+}$ (pb) vs $S_{\gamma N}$ (GeV$^2$)

$\sigma_{\gamma\pi^-}$ (pb) vs $S_{\gamma N}$ (GeV$^2$)

$\pi^+$ photoproduction (proton target)  $\pi^-$ photoproduction (neutron target)

solid red: "valence" scenario

dashed blue: "standard" one
Counting rates for 100 days: $\pi^\pm$

example: JLab Hall B

- untagged incoming $\gamma \Rightarrow$ Weizsäcker-Williams distribution

- With an expected luminosity of $\mathcal{L} = 100 \text{ nb}^{-1} \text{s}^{-1}$, for 100 days of run:
  - $\pi^+ : \simeq 10^4$
  - $\pi^- : \simeq 4 \times 10^4$