Higher Order calculation of SIDIS Y term

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Transverse spin and TMDs
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SIDIS overview
Semi inclusive deep inelastic scattering (SIDIS)

**Lab frame**
- Incoming lepton $l^\mu$
- Target $P^\mu$
- Identified hadron $p^\mu_h$
- Outgoing lepton $l'^\mu$

**Breit frame**
- Identified hadron $p^\mu_h$
- Incoming proton $P^\mu$
- Exchanged photon $q = l - l'$
- Outgoing lepton $l'^\mu$

**Key question**: How is $p^\perp_h$ generated at short distances?
Kinematic regions

\[ p_h^\perp \]

\[ y_h = \frac{1}{2} \ln \left( \frac{p_h^+}{p_h^-} \right) \]

- Different regions are sensitive to distinct physical mechanisms
Factorization in the current region

- $q_T$ integrated cross sections
  - 2 non perturbative ingredients: $f_1(\xi), d_1(\zeta)$

- $q_T$ differential cross sections
  - 4 non perturbative ingredients: $f_1(\xi), d_1(\zeta), f_1(\xi, k_\perp), d_1(\zeta, k_\perp)$

- How to relate the two methods?
  - Collins, Gamberg, Prokudin, Rogers, NS, Wang

$$\frac{d\sigma}{dxdzdQ^2} = \int dq_T \frac{d\sigma}{dxdzdQ^2 dq_T}$$

- Can we validate the formalism in nature?
The formalism for $q_T$ differential cross section
Theory framework for current fragmentation

small transverse momentum

large transverse momentum
Theory framework for current fragmentation

small transverse momentum

large transverse momentum

matching region

ASY
The theory framework for current fragmentation

- The formulation is based on a scale separation governed by the ratio

\[ q_T/Q \]

- where

\[ z = \frac{P \cdot p_h}{P \cdot q}, \quad q_T = p_h^\perp / z \]

- The cross section is built as

\[
\frac{d\sigma}{dx dQ^2 dz dp_h^\perp} = W + FO - ASY + O(\frac{m^2}{Q^2})
\]

\[ \sim W \quad \text{for } q_T \ll Q \]

\[ \sim FO \quad \text{for } q_T \sim Q \]
Why $q_T/Q$? (J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang)

- Lets define

$$k \equiv k_1 - q$$

- Propagators in the blob

$$\frac{1}{k^2 + O(\Lambda^2_{QCD})}, \quad \frac{1}{k^2 + O(Q^2)}$$

- Two extreme regions

  - $|k^2| \sim \Lambda^2_{QCD} \rightarrow k$ is part of PDF
  - $|k^2| \sim Q^2 \rightarrow k$ is part of hard blob

- $|k^2|/Q^2$ is the relevant Lorentz invariant measure of transverse momentum size
In terms of partonic variables

\[ \left| \frac{k^2}{Q^2} \right| = (1 - \hat{z}) + \hat{z} \frac{q_T^2}{Q^2} \]

For \( q_T < Q \) one can write

\[ \frac{q_T^2}{Q^2} < \left| \frac{k^2}{Q^2} \right| < 1 - z \left(1 - \frac{q_T^2}{Q^2}\right) \]

One can conclude that

- \( q_T \ll Q \) signals the onset of TMD region
- \( q_T \sim Q \) signals the large transverse momentum region
Phenomenology
These analyzes used only W (Gaussian, CSS)

Samples with $q_T/Q \sim 1.63$ has been included

BUT TMDs are only valid for $q_T/Q \ll 1$!
At LO:

\[
\frac{d\sigma}{dx dQ^2 dz dp_T} \sim \sum_q e_q^2 \int_{Q^2}^{z_2} \frac{d\xi}{\xi - x} \left( \frac{xz}{1 - x} + x \right) f_q(\xi, \mu) \, d_q(\zeta(\xi), \mu) \, H(\xi)
\]

For collinear distributions we use

- PDFs: CJ15
- FFs: DSS07
FO @ LO predictions (DSS07)

COMPASS 17 \( h^+ \)

data/theory(LO) vs. \( q_T \) (GeV)

PDF : CJ15 FF : DSS07

\( q_T > Q \)

\(< z > = 0.24\)
\(< z > = 0.34\)
\(< z > = 0.48\)
\(< z > = 0.68\)
The large $\rho_T$ puzzle

\[ \frac{d\sigma}{dxdzdQ^2} \equiv \int dq_T \left[ \boxed{W} + \boxed{\text{FO}} - \boxed{\text{ASY}} \right] + \mathcal{O}(m^2/Q^2) \]
How important is the $P_T$ tail for the integrated SIDIS multiplicities?

Consider the cumulative distribution function (CDF)

\[
\text{CDF} = \int_0^{P_T^2} dP_T^2 \frac{1}{M(x, z)} \frac{dM}{dP_T^2}(x, z, P_T^2)
\]
From $q_T$ differential to $q_T$ integrated

$Q^2$
- $0.24 < z < 0.30$
- $0.30 < z < 0.40$
- $0.40 < z < 0.50$
- $0.65 < z < 0.70$
- $q_T = Q$

CDF vs. $q_T$

COMPASS 17 $h^+$

$x_{bj}$
Revisiting charged hadron FFs (JAM)
Revisiting charged hadron FFs (in JAM)

For $q_T$ integrated cross section @ NLO:

$$\frac{d\sigma}{dx dQ^2 dz} = \sum_q H_q \otimes f_q \otimes d_q(x, z)$$

Data sets:

- SIDIS($h^+, h^-$) $q_T$ integrated data from COMPASS
- $e^+e^- \rightarrow h^\pm + X$ (work with the $0.2 < z < 0.8$ samples)
- PDFs: JAM18
The gluon fragmentation is significantly different → recently observed by the NNPDF.
Revisiting charged hadron FFs (in JAM)

\[ \frac{1}{\sigma_T} \frac{d\sigma}{dz} \]

TPC
TASSO
ALEPH
DELPHI
SLD
OPAL

\[ \chi^2/npts = 0.53 \]
Revisiting charged hadron FFs (in JAM)

\[
pd \rightarrow h^+ + X
\]

\[
pd \rightarrow h^- + X
\]

\[
\chi^2 / \text{npts} = 0.48
\]
FO @ LO predictions (DSS07)

COMPASS 17 $h^+$

data/theory(LO) vs. $q_T$ (GeV)

PDF : CJ15  FF : DSS07

$q_T > Q$

< $z$ >= 0.24
< $z$ >= 0.34
< $z$ >= 0.48
< $z$ >= 0.68
FO @ LO predictions (JAM18)

COMPASS 17 $h^+$
data/theory(NLO) vs. $q_T$ (GeV)

PDF : JAM18  FF : JAM18

$q_T > Q$

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$< z >= 0.68$
The large $\rho_T$ puzzle

\[ \frac{d\sigma}{dx dz dQ^2} \equiv \int dq_T \left[ \begin{array}{c} W \\ \sqrt{\cdot} \\ ? \end{array} \right] + FO - \left[ \begin{array}{c} \sqrt{\cdot} \\ ? \end{array} \right] + O\left(\frac{m^2}{Q^2}\right) + \left[ \begin{array}{c} \sqrt{\cdot} \\ ? \end{array} \right] + O\left(\frac{m^2}{Q^2}\right) 

\]
order $\alpha_S^2$ corrections to FO

- There are strong indications that order $\alpha_S^2$ corrections are very important.

- An order of magnitude of corrections at small $p_T$.

- As a sanity check, we need to have an independent calculation.
O(\alpha_S^2) \text{ calculation} \quad (\text{J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang})

\[ W^{\mu\nu}(P, q, P_H) = \int_{x-}^{1+} \frac{d\xi}{\xi} \int_{z-}^{1+} \frac{d\zeta}{\zeta^2} \hat{W}_{ij}^{\mu\nu}(q, x/\xi, z/\zeta) f_{i/P}(\xi) d_{H/j}(\zeta) \]

\[ \left\{ P_g^{\mu\nu} \hat{W}^{(N)}_{\mu\nu}; P_{PP}^{\mu\nu} \hat{W}^{(N)}_{\mu\nu} \right\} \equiv \frac{1}{(2\pi)^4} \int \left\{ |M_{g}^{2\rightarrow N}|^2; |M_{PP}^{2\rightarrow N}|^2 \right\} d\Pi^{(N)} - \text{Subtractions} \]

\[ \equiv \left\{ P_g^{\mu\nu} \hat{W}^{(N)}_{\mu\nu}; P_{PP}^{\mu\nu} \hat{W}^{(N)}_{\mu\nu} \right\}_{\text{unsub}} - \text{Subtractions} \]

✓ Compute 2 → 2 virtual graphs (Passarino-Veltman)
✓ Compute 2 → 3 real graphs
✓ Integrate 3-body PS analytically using dim reg
✓ Cancel double and single IR poles
Our setup for $O(\alpha_s^2)$ contribution

- Dots indicates the fragmenting parton
COMPASS 17 $h^+$

$$\frac{d\sigma}{dx_{bj}dQ^2dzdP_T^2}/\frac{d\sigma}{dx_{bj}dQ^2} (\text{GeV}^{-2}) \text{ vs. } q_T (\text{GeV})$$
FO @ NLO (JAM FFs)

COMPASS 17 $h^+$

data/theory (NLO) vs. $q_T$ (GeV)

PDF : JAM18  FF : JAM18

$Q^2$ (GeV$^2$)

$x_{bj}$

$< z >= 0.24$

$< z >= 0.34$

$< z >= 0.48$

$< z >= 0.68$
The large $\rho_T$ puzzle

\[ \frac{d\sigma}{dxdzdQ^2} \equiv \int dq_T \left[ W + FO - ASY \right] + O(m^2/Q^2) \]
Summary and outlook

\[
\frac{d\sigma}{dx \ dy \ d\Psi \ dz \ d\phi_h \ dP^2_{hT}} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1 - \varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \sum_{i=1}^{18} F_i(x, z, Q^2, P^2_{hT}) \beta_i
\]

- The apparent disagreement between data and FO can be resolved by tuning FFs+NLO
- Maybe it might be possibility to describe \( F_{UU} \) in the full \( W + FO - ASY \)
- This is important as all the structure functions that are typically provided in a form of asymmetries \( A_i = F_i/F_{UU} \)