Semi-Inclusive DIS at low to moderate $Q$

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• Overview of TMD factorization
• SIDIS
• Issues at small/moderate $Q$

INT workshop, October 8, 2018
TMD Example: Drell-Yan

$$\sigma \sim \int \mathcal{H} \otimes f_{q/P}(x_1) \otimes f_{\bar{q}/\bar{P}}(x_2)$$

(Scale Dependence: DGLAP)
TMD Example: Drell-Yan

\[ \int \mathcal{H} \otimes F_{q/P}(x_1, k_{1T}) \otimes F_{\bar{q}/\bar{P}}(x_2, q_T - k_{1T}) \]

(Scale Dependence: TMD Evolution)
Example: SIDIS
Example: SIDIS
Large and Small Transverse Momentum

\[ q_T \sim \Lambda_{QCD} \quad \Lambda_{QCD} \ll q_T \ll Q \quad q_T \sim Q \]

\[
\frac{d\sigma}{dq_T^2} = \frac{q_T^2}{Q^2}, \frac{m^2}{Q^2}, \frac{q_T^2}{q_T^2}, \frac{m^2}{q_T^2}, \frac{q_T}{m^2}, \frac{m^2}{q_T^2}
\]

\[ q_T \equiv -\frac{P_{B,T}}{z} \]
### Taxonomy

<table>
<thead>
<tr>
<th>Proton Quark</th>
<th>Unpolarized</th>
<th>Longitudinally polarized</th>
<th>Transversely polarized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unpolarized</td>
<td>( f_1(x, k_T) )</td>
<td>( x )</td>
<td>( f_{1T}^\perp(x, k_T) )</td>
</tr>
<tr>
<td>Longitudinally polarized</td>
<td>( x )</td>
<td>( g_{1L}(x, k_T) )</td>
<td>( g_{1T}(x, k_T) )</td>
</tr>
<tr>
<td>Transversely polarized</td>
<td>( h_{1\perp}^1(x, k_T) )</td>
<td>( h_{1L}(x, k_T) )</td>
<td>( h_{1T}(x, k_T) )</td>
</tr>
</tbody>
</table>

- **Boer-Mulders**
- **Worm Gear**
- **Pretzelocity**

**Remarks:**
- \( f_{1T}^\perp(x, k_T) \)
- \( g_{1T}(x, k_T) \)
- \( h_{1T}(x, k_T) \)
- \( h_{1T}^\perp(x, k_T) \)
\[
\sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, k_{1T}, S_1) \otimes F_{\bar{q}/\bar{P}}(x_2, q_T - k_{1T}, S_2)
\]

\[
\sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, k_{1T}, S_1) \otimes D_{H/q}(z, q_T + k_{1T})
\]

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<tr>
<td>Unpolarized</td>
<td>(f_1(x, k_T))</td>
<td>(\times)</td>
<td>(-f_{1T}^+(x, k_T))</td>
</tr>
<tr>
<td>Longitudinally polarized</td>
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Non-Zero!
Collins-Soper / Light-cone Renormalization

• Collinear PDFs:

\[ f_{j/p}(\xi; \mu) = \sum_i \int \frac{dz}{z} Z_{ji}(z, \alpha_s(\mu)) f_{0,i/p}(\xi/z) = Z_{ji} \otimes f_{0,i/p} \]

• TMD PDFs, CS Equation:

\[ \tilde{F}_{f/P}(x_1, b_T; \mu, y_s) = \lim_{\text{WL Raps} \to \infty} \left( \tilde{F}_{f/P}^{\text{unsub.}}(x_1, b_T; \mu) \times Z_{CS}(b_T; y_s) \right) \]

Independent of hadron

X UV renormalization
Collins-Soper / Light-cone Renormalization

• Collinear PDFs:

\[ f_{j/p}(\xi; \mu) = \sum_i Z_{ji} f_{0,i/p} \]

• TMD PDFs:

\[ \tilde{F}_{f/P}(x_1, b_T; \mu, y_s) = \lim_{\mu \to \infty} \left( \tilde{F}_{f/P}(x_1, b_T; \mu) \times Z_{CS}(b_T; y_s) \right) \]

\[ \sqrt{\frac{\tilde{S}(b_T; +\infty, y_s)}{\tilde{S}(b_T; +\infty, -\infty) \tilde{S}(b_T; y_s, -\infty)}} \]

Independent of hadron
Transverse Momentum Dependent Evolution

• Collinear / DGLAP, Evolution with Scale:
  \[
  \frac{d}{d \ln \mu} f_{j/P}(x; \mu) = 2 \int P_{jj'}(x') \otimes f_{j'/P}(x/x'; \mu)
  \]

• TMD Case:
  \[
  \frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)
  \]
  \[
  \frac{d \tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu))
  \]
  \[
  \frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d \ln \mu} = \gamma(g(\mu); \zeta/\mu^2)
  \]
Transverse Momentum Dependent Evolution

Evolution with Scale:

\[ \partial \ln \tilde{F}(x, b_T; \mu, \zeta) \quad \partial \ln \sqrt{\zeta} = \tilde{K}(b_T; \mu) \]

\[ \frac{d \tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu)) \]

\[ \frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d \ln \mu} = \gamma(g(\mu); \zeta/\mu^2) \]

\[ 2 \int P_{j,j'}(x') \otimes f_{j'/P}(x/x'; \mu) \]
One TMD PDF: Solution to Evolution

\[ \tilde{F}_{f/P}(x, b_T; Q, Q^2) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*, \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times \]

\[ \times \exp \left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \]

\[ \times \exp \left\{ -g_{f/P}(x, b_T; b_{\text{max}}) - g_K(b_T; b_{\text{max}}) \ln \frac{Q}{Q_0} \right\} \]

\[ \text{Ex: Cutoff Prescription:} \]

\[ b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}} \]

\[ \mu_b \equiv C_1 / |b_*(b_T)| \]

\[ \text{Collinear PDFs} \]

\[ \text{Nonperturbative parts large } b_T \]
Combining Results in TMD Factorization

Translation of results: Collins, TCR (2017)

Sudakov Form Factor: (Moch, Vermaseren (2005), Vogt, Gehrmann et al (2014))

\[ \frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2 \alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{DY}_{j,j_A,j_B}(Q, \mu_Q, a_s(\mu_Q)) \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T \cdot b_T} \]

\[ \times e^{-g_{j/A}(x_A, b_T; b_{max})} \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} f_{j/A}(\xi_A; \mu_{b_*}) \tilde{C}^{PDF}_{j/j_A} \left( \frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*}) \right) \]

\[ \times e^{-g_{j/B}(x_B, b_T; b_{max})} \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} f_{j/B}(\xi_B; \mu_{b_*}) \tilde{C}^{PDF}_{j/j_B} \left( \frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*}) \right) \]

\[ \times \exp \left\{ -g_K(b_T; b_{max}) \ln \frac{Q^2}{Q_0^2} + \tilde{K}(b_*; \mu_{b_*}) \ln \frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[ 2\gamma_j(a_s(\mu')) - \ln \frac{Q^2}{(\mu')^2} \gamma_K(a_s(\mu')) \right] \right\} \]

+ suppressed corrections.

Ex: Konychev, Nadolsky (2006)
ResBos extractions (and others)

Li, Zhu (2017)
Vladimirov (2017)

\[ \alpha_s^2 \] Wilson Coefficients from Collinear Factorization: (Catani et al, (2012)), and SCET (Echevarria, Scimemi, Vladimirov (2016))
Low-to-Moderate Q SIDIS: Motivation

• Sensitivity to intrinsic non-perturbative effects.

• Many SIDIS measurements are at low/moderate Q.

• Transition to partonic degrees of freedom.
  – E.g., quark-hadron duality
Low-to-moderate $Q$

\[ q_T \equiv -\frac{P_{B,T}}{z} \]

**Pion production**

\[ W_{\text{SIDIS}}^2 = (P + q - P_\pi)^2 \]

\[ Q \text{ (GeV)} \quad q_T = 0, \quad z = 0.25 \]

\[ Q \text{ (GeV)} \quad q_T = 2 \text{ GeV}, \quad z = 0.25 \]

H. Avakian, A. Bressan, and M. Contalbrigo, “Experimental results on TMDs” (2016)

Help From: Sterling Gordon
Low-to-moderate $Q$

$$q_T \equiv -\frac{P_{B,T}}{z}$$

**Kaon production**

$$W_{\text{SIDIS}}^2 = (P + q - P_K)^2$$

**Help From**:
Sterling Gordon

H. Avakian, A. Bressan, and M. Contalbrigo, “Experimental results on TMDs” (2016)
Large Q

Candidate from NC sample

<table>
<thead>
<tr>
<th>H1</th>
<th>Run 122145</th>
<th>Event 69506</th>
<th>Date 19/09/1995</th>
</tr>
</thead>
</table>

\[ Q^2 = 25030 \text{ GeV}^2, \quad y = 0.56, \quad M = 211 \text{ GeV} \]
Low-to-moderate Q

CLAS

Cerenkov

beam

NH₃, ND₃

drift chambers

Time of flight scintillators

calorimeter

CLAS Large Acceptance Spectrometer (CLAS)
drift chambers
beam
Cerenkov
e-
NH₃, ND₃
Time of flight scintillators
CLAS calorimeter p+
Low-to-moderate Q
Challenges at moderate scales

• Non-zero hadron masses.

• Constituents have non-zero virtuality, mass, etc.

• The separation between regions gets squeezed.
Cartography of SIDIS
Cartography of SIDIS

$\delta$ $(q_{\text{ms}}(e \cdot m, m))$

Hard Transverse Momentum – Current Region

$\bar{e}'_{YB,b} \cdot \frac{m}{Q'} \frac{q_{\text{f}}}{Q}$

TMD - Current Region

$e^{-Y_{B,b}} \cdot \frac{m}{Q'} \frac{q_{\text{f}}}{Q}$

Soft/Central Region

Target Region

$q_{T}$

$y_{P}$
Factorization: Inclusive Case

• Power expansion

\[
\frac{d\sigma}{dx_{Bj} \, dQ^2} = \int d\xi \frac{d\hat{\sigma}}{d\hat{x}_{Bj} \, dQ^2} f(\xi) + O \left( \frac{m^2}{Q^2} \right)
\]

• \(m^2 = \) parton virtuality, transverse momentum, mass...

• What about hadron masses?
Massless Target Approximation (MTA)

• Exact:

\[ P = \left( \sqrt{M^2 + P_z^2}, 0, 0, P_z \right) = \left( P^+, \frac{M^2}{2P^+}, 0_T \right) \]

• The approximation:

\[ P \rightarrow \tilde{P} = (P_z, 0, 0, P_z) = (P^+, 0, 0_T) \]

\[ 2P \cdot q \rightarrow 2\tilde{P} \cdot q \quad M^2/Q^2 \rightarrow 0 \]
MTA in Light-Cone Fractions

• Light-cone ratios:

− No MTA: \[ -\frac{q^+}{P^+} = x_N \equiv \frac{2x_{Bj}}{1 + \sqrt{1 + \frac{4x_{Bj}^2 M^2}{Q^2}}} \]

− MTA: \[ -\frac{q^+}{P^+} = x_{Bj} + O \left( \frac{x_{Bj}^2 M^2}{Q^2} \right) \]
Factorization and Parton Approximations

\[ q = \left( -x_N P^+, \frac{Q^2}{2x_N P^+}, 0_T \right) \]

\[ k_i^+ = O(Q) \]

\[ k_i^2 = O(m^2) \]

\[ (k_i + q)^2 = O(m^2) \]

\[ 2k_i^+ q^- + 2k_i^- q^+ - Q^2 + k_i^2 = O(m^2) \]

\[ 2k_i^+ q^- = Q^2 + O(m^2) \]

\[ \xi \equiv \frac{k_i^+}{P^+} = x_N + O\left(\frac{m^2}{Q^2}\right) \]

\[ = x_{Bj} + O\left(\frac{x_{Bj} M^2}{Q^2}\right) + O\left(\frac{m^2}{Q^2}\right) \]
• Normal factorization, just keeping exact mass.

  – Target mass corrected (TMC)

  \[ W^{\mu\nu} = \int_{x_N}^{1} \frac{d\xi}{\xi} \hat{W}^{\mu\nu}(x_N/\xi, q) f(\xi) + O\left(m^2/Q^2\right) \]

  – MTA

  \[ W^{\mu\nu} = \int_{x_{Bj}}^{1} \frac{d\xi}{\xi} \hat{W}^{\mu\nu}(x_{Bj}/\xi, q) f(\xi) + O\left(m^2/Q^2\right) + O\left(x_{Bj}^2 M^2/Q^2\right) \]

• Purely kinematical.
Extend AOT to SIDIS

• Light-cone fractions versus x and z:

\[
x_N = -\frac{q^+}{P^+} = \frac{2x_{Bj}}{1 + \sqrt{1 + \frac{4x_{Bj}^2 M_B^2}{Q^2}}}
\]

\[x_{Bj} = \frac{Q^2}{2P \cdot q}\]

\[
z_N = \frac{P_B^-}{q^-} = 2x_{Bj} \frac{P \cdot P_B}{Q^2}
\]

• Final state hadron mass \( (M_B) \) sensitivity:

\[
z_N = \frac{x_N z_h}{2x_{Bj}} \left( 1 + \sqrt{1 - \frac{4M_B^2 M_{B,T}^2 x_{Bj}^2}{Q^4 z_h^2}} \right)
\]

\[= z_h \left( 1 - \frac{x_{Bj}^2 M_B^2}{Q^2} \right) \left( 1 + \frac{P_{B,T}^2}{z_h^2 Q^2} \right) + \left( \frac{x_{Bj}^2 M_B^2}{Q^2} \right)^2 \left( \frac{P_{B,T}^2}{z_h^2 Q^2} - \frac{P_{B,T}^4}{z_h^4 Q^4} + 2 - \frac{M_B^2}{z_h^2 M_B^2 x_{Bj}^2} \right) + O \left( \left( \frac{x_{Bj}^2 M_B^2}{Q^2} \right)^3 \right)
\]
Light-cone fractions

\[ x_{N}/x_{Bj} \]

- \( Q = 1 \) (GeV)
- \( Q = 2 \) (GeV)

\[ z_{N}/z \]

- \( q_T = .3 \) (GeV)
- \( q_T = 2 \) (GeV)

(Pion production)\n\[ Q = 1 \text{ GeV} \]
Light-cone fractions

\[ \frac{x_N}{x_{Bj}} \]

- \( Q = 1 \) (GeV)
- \( Q = 2 \) (GeV)

\[ \frac{z_N}{z} \]

- \( q_T = 0.3 \) (GeV)
- \( q_T = 2 \) (GeV)

(Kaon production)

\( Q = 1 \) GeV
Factorization and Parton Approximations

\[ k_f = k_i + q \]

\[ \xi = x_N \left( 1 + \frac{k_f^2 + k_T^2}{Q^2} + \cdots \right) \]
Current Fragmentation

\[ q_T \equiv -\frac{P_{B,T}}{z} \]

\[ \xi \equiv \frac{k_i^+}{P^+} \]

\[ \zeta \equiv \frac{P_B^-}{k_f^-} \]
**Current fragmentation**

\[ R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i} \quad m^2/Q^2 \rightarrow 0 \quad e^{-\Delta y} \]


- Estimate of non-perturbative scales needed.

\[ y_i = \ln \frac{Q}{M_{i,T}} ; \quad y_f = -\ln \frac{Q}{M_{f,T}} \]
"The overlap of kinematic coverage of COMPASS, HERMES and JLab (see fig. 1) would allow studies of $Q^2$-dependence in the range of Bjorken $x \sim 0.1–0.2$, where the effects related to orbital motion of quarks are expected to be significant."

-H. Avakian, A. Bressan, and M. Contalbrigo, “Experimental results on TMDs” (2016)
Current fragmentation

\[ R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i} \]

Help From:
Andrew Dotson
&
Sterling Gordon

Pion production

\( Q \text{ (GeV)} \quad q_T = 0.3 \text{ GeV} \)

\( Q \text{ (GeV)} \quad q_T = 2.0 \text{ GeV} \)

\[ z = 0.25, \xi = 0.3, \chi = 0.2, m = m_\pi \]
Current fragmentation

\[ R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i} \]

Help From:
Andrew Dotson
&
Sterling Gordon

Kaon production

\( Q \) (GeV) \hspace{1cm} q_T = 0.3 \text{ GeV} \\
\( Q \) (GeV) \hspace{1cm} q_T = 2.0 \text{ GeV}

\( z = 0.25, \ \zeta = 0.3, \ x_{Bj} = 0.2, \ m = m_K \)
Large and Small Transverse Momentum

\[
\frac{1}{k^2 + O(Q^2)} - \frac{1}{k^2 + O(m^2)}
\]

\[k_f - q = k \]

\[R_2 \equiv \frac{|k^2|}{Q^2} \approx 1 - \frac{z}{\zeta} + \frac{z}{\zeta} \frac{q_T^2}{Q^2} \]
Large and Small Transverse Momentum

\[ R_2 \equiv \frac{|k^2|}{Q^2} \]

Help From:
Andrew Dotson
&
Sterling Gordon

\[ Q \text{ (GeV)} \]

**Pion production**

\[ z = 0.25, \ z, = 0.3, \ x_{Bj} = 0.2, \ m = m_\pi \]

\[ Q \text{ (GeV)} \]

**Kaon production**

\[ z = 0.25, \ z, = 0.3, \ x_{Bj} = 0.2, \ m = m_K \]
Extra Emissions

\[ R_3 = \left| \frac{k_X^2}{Q^2} \right| \]

The momentum labeling in the partonic case is \( \delta k_T \) when \( q_T = 0 \). When \( \delta k_T = 0 \), the partons are, up to power suppressed corrections, exactly aligned with the target. The diagram illustrates the momentum transfers involved in the process, with arrows indicating the directions of the momenta \( q, k_i, k_f, k_f, k_X, H \).
Region Diagnostics

• From model assumptions of underlying partonic picture, generate:
  
  \[- W_{SIDIS}^2 \]
  
  \[- x_N / x_{Bj}, z_N / z \]
  
  \[- R_1 \]
  
  \[- R_2 \]
  
  \[- R_3 \]

• Make a region map.

• Compare with measurements to constrain underlying picture.
Summary

• TMD factorization: Basics are well-established.

• SIDIS is important for TMD and related studies.

• Low-to-moderate Q opportunities: Access to interesting non-perturbative phenomena.

• Standard physical picture cannot be taken for granted.
  – Mass effects need to be accounted for.
  – Systematic diagnostic tools needed.
Small to large transverse momentum

Daleo, de Florian, Sassot (2005)

Data: H1 (2004)
Small to large transverse momentum
Small to large transverse momentum
Small to large transverse momentum
Small to large transverse momentum