Heavy Quarkonium Production in Quantum Chromodynamics

Jianwei Qiu
Theory Center, Jefferson Lab
November revolution (1974)

Experimental Observation of a Heavy Particle $J^+$


Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee

Brookhaven National Laboratory, Upton, New York 11793

(Received 12 November 1974)

November, 1974

Discovery of a Narrow Resonance in $e^+e^-$ Annihilation*


Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and


Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

(Received 13 November 1974)
Elementary particles – new periodic table:

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>$1.5 - 4.5$ MeV</td>
</tr>
<tr>
<td>$d$</td>
<td>$5.0 - 8.5$ MeV</td>
</tr>
<tr>
<td>$s$</td>
<td>$80 - 155$ MeV</td>
</tr>
<tr>
<td>$c$</td>
<td>$1.0 - 1.4$ GeV</td>
</tr>
<tr>
<td>$b$</td>
<td>$4.0 - 4.5$ GeV</td>
</tr>
<tr>
<td>$t$</td>
<td>$174.3 \pm 5.1$ GeV</td>
</tr>
</tbody>
</table>

Light quarks

Heavy quarks

Hadrons – bound states of quarks and gluons:
Outline of the rest of my talk

- Dual roles of heavy quarkonium physics
  - QCD bound states vs. “localized” probes
- Production mechanism?
  - Successes and failures of NRQCD factorization
- EIC: Inclusive, Semi-inclusive, Exclusive, …
- Gluon distribution, imaging, near threshold, …
- Summary and outlook

*If we do not understand heavy quarkonia, we do not know QCD!*
Dual roles of heavy quarkonium physics

- QCD bound states – spectroscopy, decay, production, …
Dual roles of heavy quarkonium physics

- **QCD bound states** – spectroscopy, decay, production, ...

- **“Localized”** probes – structure, medium properties, ...

![Diagram with heavy quarkonium states and decay pathways]

1. \( q \) \( \rightarrow \gamma J/\psi , \gamma , \ldots \)
Why QCD is so hard to deal with?

- It is strongly coupled – nonlinear + nonperturbative!
- It is relativistic – nontrivial QCD vacuum!
- No localized heavy mass/charge center – nucleus in an atom!
- Gluons are “dark” and carry “color” – intellectual challenge!

How to probe the quark-gluon dynamics, quantify the hadron structure, study the emergence of hadrons, ..., if we cannot see quarks and gluons?
Why QCD is so hard to deal with?

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Heavy quarkonium:

- Heavy quark as relatively localized heavy mass/charge center
- Heavy quark in the pair’s rest frame is almost non-relativistic
- Production of heavy quark pair could be perturbative
- Top decays too quickly, strange is too light, …

Charmonium (c\bar{c}) + Bottomonium (b\bar{b})

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1.0 – 1.4 GeV</td>
</tr>
<tr>
<td>b</td>
<td>4.0 – 4.5 GeV</td>
</tr>
</tbody>
</table>
Heavy quarkonium

- One of the simplest QCD bound states:
  - Localized color charges (heavy mass), non-relativistic relative motion
  - Charmonium: $v^2 \approx 0.3$
  - Bottomonium: $v^2 \approx 0.1$

- Well-separated momentum scales – effective theory:

  - Perturbative
    - $P_T$
    - $m_Q$
  - Non-Perturbative
    - $m_Q v$
    - $m_Q v^2$

  - Hard — Production of $Q\bar{Q}$ [pQCD]
  - Soft — Relative Momentum $\Lambda_{QCD}$ [NRQCD]
  - Ultrasoft — Binding Energy [pNRQCD]
Heavy quarkonium

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- Well-separated momentum scales – effective theory:
  \[
  \begin{array}{c|c|c}
  P_T & m_Q & \text{Perturbative} \\
  m_Q v & \text{Non-Perturbative} \\
  m_Q v^2 & \text{Non-Perturbative} \\
  \end{array}
  \]
  - Hard — Production of \( Q \bar{Q} \) \([pQCD]\)
  - Soft — Relative Momentum \([NRQCD]\)
  - Ultrasoft — Binding Energy \([pNRQCD]\)

- Cross sections and observed mass scales:
  \[
  \frac{d\sigma_{AB \rightarrow H(P)X}}{dy dP_T^2} \quad \sqrt{S}, \quad P_T, \quad M_H,
  \]
  PQCD is “expected” to work for the production of heavy quarks

**Difficulty:** Emergence of a quarkonium from a heavy quark pair?
Double $c\bar{c}$ production in $e^+e^-$ collisions

- **Inclusive production:**
  \[
  \sigma(e^+e^- \rightarrow J/\psi c\bar{c})
  \]
  - **Belle:** $(0.87^{+0.21}_{-0.19} \pm 0.17) \text{ pb}$
  - **NRQCD:** $0.07 \text{ pb}$

  [Kiselev, et al 1994,
  Cho, Leibovich, 1996,
  Yuan, Qiao, Chao, 1997]

- **Ratio to light flavors:**
  \[
  \frac{\sigma(e^+e^- \rightarrow J/\psi c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi X)}
  \]
  - **Belle:** $0.59^{+0.15}_{-0.13} \pm 0.12$

**Message:**

- Production rate of $e^+e^- \rightarrow J/\psi c\bar{c}$ is larger than all these channels: $e^+e^- \rightarrow J/\psi gg, e^+e^- \rightarrow J/\psi q\bar{q}, \ldots$ combined?
NLO theory fits – Butenschoen et al.

- ATLAS data: $\sqrt{s} = 7$ TeV, $|y| < 0.75$
- CDF data: $\sqrt{s} = 1.96$ TeV, $|y| < 0.6$

- BELLE data: $\sqrt{s} = 10.6$ GeV

- H1 data: HERA1, HERA2

- CS+CO, NLO: Butenschoen et al.

PRL, 2011
NLO theory fits – Gong et al.

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PRL, 2012
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PRL, 2012

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- $p\bar{p} \rightarrow J/\psi + X$, helicity frame
- CDF data: $\sqrt{s} = 1.96$ TeV, $|y| < 0.6$
- CS+CO, NLO: Chao et al.

- $60$ GeV < W < $240$ GeV
  - $0.3 < z < 0.9$
  - $Q^2 < 2.5$ GeV$^2$
  - $\sqrt{s} = 319$ GeV

- $e^+e^- \rightarrow J/\psi + X$
  - CS+CO, NLO: Chao et al.

- $H1$ data: HERA1
- $H1$ data: HERA2
- CS+CO, NLO: Chao et al.
Production in medium, cold or hot?

(a) $J/\psi$ scaling?

(b) $X_2$ scaling?

Energy loss?

Regeneration?
Basic production mechanism

- Factorization is likely to be valid for producing the pairs:
  - Momentum exchange is much larger than $1/fm$
  - Spectators from colliding beams are “frozen” during the hard collision

Approximation:

$$
\sigma_{AB \rightarrow h} \propto \int dQ^2 \hat{\sigma}_{AB \rightarrow [Q\bar{Q}]}(m_Q^2, q^2) F_{[Q\bar{Q} \rightarrow h]}(q^2) + \ldots
$$
Basic production mechanism

- **Factorization is likely to be valid for producing the pairs:**
  - Momentum exchange is much larger than $1/fm$
  - Spectators from colliding beams are “frozen” during the hard collision

- **Naïve factorization:** on-shell pair + hadronization

\[
\sigma_{AB\to J/\psi} = \sum_{[Q\bar{Q}(n)\to Q\bar{Q}(n)]} \int d\Gamma_{[Q\bar{Q}]} \hat{\sigma}_{AB\to [Q\bar{Q}(n)]}(p_Q, p_{\bar{Q}}) F_{[Q\bar{Q}(n)\to J/\psi]}(p_Q, p_{\bar{Q}}, P_{J/\psi})
\]

**Models & Debates**

- Different assumptions/treatments on how the heavy quark pair becomes a quarkonium?

- Perturbative
- Non-perturbative
- Coherent soft interaction
A long history for the production

- **Color singlet model: 1975 –**
  
  Only the pair with right quantum numbers
  Effectively No free parameter!

- **Color evaporation model: 1977 –**
  
  All pairs with mass less than open flavor heavy meson threshold
  One parameter per quarkonium state

- **NRQCD model: 1986 –**
  
  All pairs with various probabilities – NRQCD matrix elements
  Infinite parameters – organized in powers of $v$ and $\alpha_s$

- **QCD factorization approach: 2005 –**
  
  $P_T \gg M_H$: $M_H/P_T$ power expansion + $\alpha_s$ – expansion
  Unknown, but universal, fragmentation functions – evolution

- **Soft-Collinear Effective Theory + NRQCD: 2012 –**
  
  Fleming, Leibovich, Mehen, …
NRQCD – most successful so far

NRQCD factorization:
\[ d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X}\langle O^H(n) \rangle \]

Phenomenology:

- 4 leading channels in \( v \): \( 3S_{1}^{[1]}, 1S_{0}^{[8]}, 3S_{1}^{[8]}, 3P_{J}^{[8]} \)
- Full NLO in \( \alpha_s \)

Fine details – shape – high at large \( p_T \)?
Production at collider energies

- **NRQCD factorization:**
  \[ d\sigma_{A+B\rightarrow H+X(p_T)} = \sum_n d\hat{\sigma}_{A+B\rightarrow [QQ(n)]+X(p_T)} \langle O^H_n \rangle \]

- **Expansion in powers of both \( \alpha_s \) and \( \nu \).**

- **Hadronization**

- **Re-organization is needed when \( p_T >> m_Q \):**
  
  - **LO in \( \alpha_s \):**
  
  - **NLO in \( \alpha_s \):**
  
  - **NNLO in \( \alpha_s \):**

  - **CS channel as a case study**

  - **PT Power!**

  - **When \( p_T >> m_Q \), the expansion in powers of \( \alpha_s \) is not reliable!**
  
  - **Leading order in \( \alpha_s \)-expansion =\(\neq \) leading power in \( 1/p_T \)-expansion!**
QCD factorization + NRQCD factorization

- Color singlet as an example:

\[ \sigma_{\text{NRQCD}}^{(NLO)} \propto \left[ d\hat{\sigma}_{ab\rightarrow[Q\bar{Q}(v8)]}^{(LO)} \otimes D_{[Q\bar{Q}(v8)]\rightarrow J/\psi}^{(LO)} \right. \]

\[ \left. + d\hat{\sigma}_{ab\rightarrow[Q\bar{Q}(a8)]}^{(LO)} \otimes D_{[Q\bar{Q}(a8)]\rightarrow J/\psi}^{(LO)} \right] \]

- Reproduce NLO CSM for \( p_T > 10 \text{ GeV} \)!
- Cross section + polarization

Different kinematics, different approximation, Dominance of different production channels!
Production at collider energies

NRQCD factorization:

$$d\sigma_{A+B\rightarrow H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B\rightarrow [QQ(n)]+X}(p_T) \langle O^H_n \rangle$$

PQCD factorization:

$$d\sigma_{A+B\rightarrow H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\rightarrow i+X}(p_T/z, \mu) \otimes D_{H/i}(z, \mu)$$

$$+ \sum_n d\tilde{\sigma}_{A+B\rightarrow [QQ(n)]+X}(p_T/z, \zeta_1, \zeta_2, \mu) \otimes \mathcal{D}_{H/[QQ(n)]}(z, \zeta_1, \zeta_2, \mu)$$

Model: Using NRQCD factorization for the INPUT fragmentation functions

$$D_{H/i}(z, \mu_0) = \sum_{n} d_{i\rightarrow [QQ(n)]}(z, \mu_0) \langle O^H_n \rangle$$

$$\mathcal{D}_{H/[QQ(m)]}(z, \zeta_1, \zeta_2, \mu_0) = \sum_{n} d_{[QQ(m)]\rightarrow [QQ(n)]}(z, \zeta_1, \zeta_2, \mu_0) \langle O^H_n \rangle$$

PQCD improved NRQCD factorization:

$$d\tilde{\sigma}_{A+B\rightarrow [QQ(n)]+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\rightarrow i+X}(p_T/z) \otimes d_{i\rightarrow [QQ(n)]}(z)$$

$$+ \sum_m d\tilde{\sigma}_{A+B\rightarrow [QQ(m)]+X}(p_T/z, \zeta_1, \zeta_2) \otimes d_{[QQ(m)]\rightarrow [QQ(n)]}(z, \zeta_1, \zeta_2)$$

Evolution = resummation
NRQCD vs. PQCD improved NRQCD:

\[ \sigma^{(NLO)}_{NRQCD} \propto \left[ d\sigma^{A(LO)}_{ab \rightarrow [Q\bar{Q}(v8)]} \otimes D^{(LO)}_{[Q\bar{Q}(v8)] \rightarrow J/\psi} \right. \\
\left. + d\sigma^{S(LO)}_{ab \rightarrow [Q\bar{Q}(a8)]} \otimes D^{(LO)}_{[Q\bar{Q}(a8)] \rightarrow J/\psi} \right] \]

**LO analytical results reproduce NLO NRQCD calculations (numerical)**

\( p_T \) distribution is not sufficient for fixing all NRQCD matrix elements.

Need more physical observables!
Matching between different approaches

- **Expectation:**
  
  ![Graphical representation of Expectation]

- **Matching:**
  \[
  E_P \frac{d\sigma_{A+B \rightarrow H+X}}{d^3 P}(P, m_Q) \equiv E_P \frac{d\sigma_{QCD}^{A+B \rightarrow H+X}}{d^3 P}(P, m_Q = 0) 
  \]
  \[
  + E_P \frac{d\sigma_{A+B \rightarrow H+X}^{NRQCD}}{d^3 P}(P, m_Q \neq 0) - E_P \frac{d\sigma_{QCD-Asym}}{d^3 P}(P, m_Q = 0) 
  \]

Mass effect + expanded $P_T$ region ($P_T \gtrsim m_Q$)
Production at low $p_T (< M_Q)$

- Spectator interaction – always there:
  - $p_T \sim m_Q v^2, m_Q v$
  - Interfere with the formation of the quarkonium
  - Process dependence

- The Challenge:
  - Break factorization – Process dependence – Alter $p_T$ distribution, …

- Understand the factorization breaking:
  - If the breaking effect is controllable, we still have predictive power!
  - Even the Drell-Yan process is NOT fully factorizable!
Upsilon at the LHC:

No adjustment on any parameter from Tevatron to the LHC!

BUT: this does not apply for J/ψ at low PT, logarithmic contribution from the shower is not strong enough!
To understand what we could calculate, test, and learn at EIC energies!
Heavy quarkonium production at EIC

- Semi-inclusive DIS:
  - Low $P_T$: Glue TMD ?
  - CGC ?
  - Shower vs. multiple scattering
  - $P_T \sim Q$: Gluon PDF
  - High $P_T$: LP +NLP FFs

- Exclusive / Gap:
  - Imaging gluon density distribution
  - $\frac{d\sigma}{dt}(x_B, Q^2) \propto$ GPDs
  - Near threshold
  - $\propto$ Trace Anomaly?
  - On-going effort, …

One facility covers all issues of quarkonium production!
Diffraction sensitive to gluon momentum distributions.

$g(x, Q^2)$

$\gamma^* + p \rightarrow J/\psi + p$

$\gamma^* + p \rightarrow J/\psi + p$

A Golden process for imagining gluon

Allow us to ask “new” fundamental questions:

Color confining radius?

How far does glue density spread?

How fast does glue density fall?

Can A be a bigger P at small-x?
Diffractive production in e+A at EIC

- Diffractive vector meson (\(\Phi\), J/\(\psi\), ..) production:

\[
\frac{d\sigma}{dx_B dQ^2 dt}
\]

Fourier transform of the t-dependence

- as a function of t

- J/\(\psi\)-production – probe for saturation and nuclear imaging:

- Incoherent: Nucleus breaks up
- Coherent: Nucleus stays intact

Need EIC’s Energy & Luminosity to do this!
Heavy quarkonium photoproduction

- **Exclusive processes:**
  - Heavy quarkonium:
    - $^3S_1 (J/\psi, \Upsilon, \ldots)$
    - $^1P_1 (h_c, h_b, \ldots)$
    - ...

- **Necessary condition for collinear factorization:**
  \[
  P = p - p',
  \]
  \[
  \vec{P}/2 + \vec{l} \quad \rightarrow \quad (P/2 + l)^2 + i\epsilon
  \]
  \[
  \vec{P}/2 - \vec{l} \quad \rightarrow \quad (P/2 - l)^2 + i\epsilon
  \]
  Heavy quark to ensure:
  \[
  P^+ \gg \langle l^2 \rangle
  \]

- **Polarization states of the quarkonium:**
  Help select various twist-2 and twist-3 GPDs

Heavy quarkonium photoproduction

\[ W^2 = (P + q)^2 \]
\[ t = (P' - P)^2 \]

\[ \gamma \rightarrow J/\psi, \ Upsilon, \ldots \]

Collinear factorization works
Good for gluon GPDs

Gluon GPDs at twist-2

Operator for Trace Anomaly is twist-4

Large W region: \( W^2 \gg |t| \)
Heavy quarkonium photoproduction

Threshold region: $W^2 \geq |t|$

- What dominates the exchange?

Holographic J/ψ production:

\[
\frac{d\sigma}{dt} = \frac{\alpha_{em}}{4(W^2 - M_p^2)^2} \sum_{pol} \sum_{spin} \left| \langle P|\vec{\epsilon} \cdot \vec{J}(0)|P'p\rangle \right|^2
\]

- How to calculate the scattering amplitude?

\[
\langle P|\vec{\epsilon} \cdot \vec{J}(q)|P'p\rangle = (2\pi)^4 \delta^4(P + q - P' - p)\langle P|\vec{\epsilon} \cdot \vec{J}(0)|P'p\rangle
\]

- Y. Hatta, D.L. Yang (1801.02163) – gauge/string duality:

\[
\langle P|\epsilon \cdot J(0)|P'k\rangle = -\frac{2\kappa^2}{f^3} \int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta g_{\mu\nu}} \frac{z^2 R^2}{4} \langle P|T_{\mu\nu}^{g^{TT}}|P'\rangle \\
+ \frac{2\kappa^2}{f^3} \frac{3}{8} \int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta \phi} \frac{z^4}{4} \langle P|\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a|P'\rangle
\]

$2\kappa^2 = \frac{8\pi^2}{N_c^2} R^3$ – 5D gravitational constant

Y. Hatta, D.L. Yang, 1808.02163
Also see Sterman’s talk

Gluon from Traceless $T^{\mu\nu}$
Trace Anomaly
Dilaton field
The Role of Trace Anomaly

Numerical estimate:

\[ M_a = \left( \frac{\langle P|H_a|P\rangle}{\langle P|P\rangle} \right)_{\text{at rest}} = (1 - b) \frac{1}{4} M_p \]

Max. contribution from Tracd Anomaly
\[ b = 0 \]

Min. contribution from Tracd Anomaly
\[ b = 1 \]

- Normalization is fitted
- Not expected to work for large \( W^2 \)
The Role of Trace Anomaly

Numerical estimate:

As expected, the Trace Anomaly contribution is the most relevant near threshold regime!

\[ M_a = \frac{\langle P | H_a | P \rangle}{\langle P | P \rangle} \bigg|_{\text{at rest}} \]

\[ = (1 - b) \frac{1}{4} M_p \]

Max. contribution from Tracd Anomaly

\[ b = 0 \]

Min. contribution from Tracd Anomaly

\[ b = 1 \]
It has been over 40 years since the discovery of J/Ψ, but, still not completely sure about its production mechanism.

EIC kinematics covers all potential issues/physics of heavy quarkonium production + opportunity for the threshold production

Connection to the trace anomaly (proton mass), XYZ states, ...

NRQCD factorization is expected to work for $P_T \sim Q$, and QCD factorization works for both LP and NLP at high $P_T$

Challenge for low $P_T$ region or near the threshold

Exclusive production could be a golden process for GPDs

Nuclear medium could be a good “filter” or a fermi-scale “detector” for studying the emergence of a quarkonium from a heavy quark pair

Special role of the rapidity!

Thank you!