Gluon TMDs and Opportunities at EIC

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INT-18-3 Workshop
Transverse Spin and TMDs

October 8 - 12 2018
Seattle (USA)
TMD factorization and color gauge invariance
TMD factorization

Two scale processes $Q^2 \gg p_T^2$

SIDIS $lp \rightarrow l' h X$

Drell-Yan $pp \rightarrow l^+ l^- X$

Factorization proven
TMD factorization

Gauge invariant definition of $\Phi$ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P, S \left| \overline{\psi}(0) \mathcal{U}^c_{[0,\xi]} \psi(\xi) \right| P, S \right\rangle$$

SIDIS

Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)

Possible effects in transverse momentum observables ($\xi_T$ is conjugate to $k_T$)
TMD factorization
Process dependence of gauge links

SIDIS

\[
\begin{align*}
&\mathbf{p} \quad \mathbf{k} \\
&\Phi \\
&\mathbf{q} \\
&\mathbf{P} \\
\end{align*}
\]

Drell-Yan

\[
\begin{align*}
&\mathbf{p} \quad \mathbf{k} \\
&\Phi \\
&\mathbf{q} \\
&\mathbf{P} \\
\end{align*}
\]

\[
\int d\xi_T \quad \xi_T = 0 \quad \text{the same in both cases}
\]

Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)
Boer, talk at RBRC Synergies workshop (2017)
TMD factorization
The quark Sivers function

Fundamental test of TMD theory

\[
f_{1T}^{[DY]}(x, k_{\perp}^2) = -f_{1T}^{[SIDIS]}(x, k_{\perp}^2) \quad h_{1}^{[DY]}(x, k_{\perp}^2) = -h_{1}^{[SIDIS]}(x, k_{\perp}^2)
\]

Collins, PLB 536 (2002)

FSI in SIDIS

ISI in DY

ISI/FSI lead to process dependence of TMDs, could even break factorization

Collins, Qiu, PRD 75 (2007)
Collins, PRD 77 (2007)
Rogers, Mulders, PRD 81 (2010)
Process dependence of gluon TMDs
Gluon TMDs
The gluon correlator

\[ \Gamma_{\alpha\beta}(p;P,S) \]

Gauge invariant definition of \( \Gamma^{\mu\nu} \)

\[
\Gamma[\mathcal{U},\mathcal{U}'][\mu\nu] \propto \langle P, S | \text{Tr}_c \left[ F^{-\nu}(0) \mathcal{U}^C_{[0,\xi]} F^{+\mu}(\xi) \mathcal{U}^{C'}_{[\xi,0]} \right] | P, S \rangle
\]

Mulders, Rodrigues, PRD 63 (2001)
Buffing, Mukherjee, Mulders, PRD 88 (2013)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

The gluon correlator depends on two path-dependent gauge links

- \( ep \rightarrow e' Q \overline{Q} X \)
- \( ep \rightarrow e' \text{jet jet} X \)
- \( pp \rightarrow \gamma\gamma X \) (and/or other CS final state)
- \( pp \rightarrow \gamma \text{jet} X \) probes an entirely independent gluon TMD: \([+-]\) links (dipole)
The gluon Sivers functions
Sign change test

Related Processes

\[ ep^\uparrow \rightarrow e'Q\bar{Q}X, \quad ep^\uparrow \rightarrow e'\text{jet}\text{jet}X \] probe GSF with \([++]\) gauge links (WW)

\[ p^\uparrow p \rightarrow \gamma\gamma X \] (and/or other CS final state) probe GSF with \([-[-]\) gauge links

Analogue of the sign change of \( f_{1T}^{\perp q} \) between SIDIS and DY (true also for \( h_1^g \) and \( h_{1T}^{\perp g} \))

\[
f_{1T}^{\perp g} [ep^\uparrow \rightarrow e'Q\bar{Q}X] = -f_{1T}^{\perp g} [p^\uparrow p \rightarrow \gamma\gamma X]
\]

Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC
Complementary Processes

\[ e^p \uparrow \rightarrow e'Q\bar{Q}X \] probes a GSF with \([++]\) gauge links (WW)

\[ p^\uparrow p \rightarrow \gamma \text{jet} X (gq \rightarrow \gamma q) \] probes a gluon TMD with \([+-]\) links (DP)

At small-\(x\) the WW Sivers function appears to be suppressed by a factor of \(x\) compared to the unpolarized gluon function, unlike the dipole one.

The DP gluon Sivers function at small-\(x\) is the spin dependent odderon (single spin asymmetries from a single Wilson loop matrix element)

Boer, Echevarria, Mulders, Zhou, PRL 116 (2016)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)
Gluon TMDs

<table>
<thead>
<tr>
<th>GLUONS</th>
<th>unpolarized</th>
<th>circular</th>
<th>linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>( f_1^g )</td>
<td></td>
<td>( h_{1T}^{\perp g} )</td>
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<tr>
<td>L</td>
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<td>( g_{1L}^g )</td>
<td>( h_{1L}^{\perp g} )</td>
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<td>T</td>
<td>( f_{1T}^{\perp g} )</td>
<td>( g_{1T}^g )</td>
<td>( h_{1T}^g, h_{1T}^{\perp g} )</td>
</tr>
</tbody>
</table>

Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

- \( h_{1T}^{\perp g} \): \( T \)-even distribution of linearly polarized gluons inside an unp. hadron
- \( h_{1T}^g, h_{1T}^{\perp g} \): helicity flip distributions like \( h_{1T}^q, h_{1T}^{\perp q} \), but \( T \)-odd, chiral even!
- \( h_1^g \equiv h_{1T}^g + \frac{p_T^2}{2M_p^2} h_{1T}^{\perp g} \) does not survive under \( p_T \) integration, unlike transversity

In contrast to quark TMDs, gluon TMDs are almost unknown
The distribution of linearly polarized gluons inside an unpolarized proton: $h_{1}^{\perp g}$
Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum

\[ h_{1}^\perp g \]

Interference between \( \pm 1 \) gluon helicity states

It does not need ISI/FSI to be nonzero, unlike the Sivers function. However it is affected by them \( \Longrightarrow \) process dependence
Gluon polarization and the Higgs boson $p p \rightarrow H X$ at the LHC

Higgs boson production happens mainly via $gg \rightarrow H$

Pol. gluons affect the Higgs transverse spectrum at NNLO pQCD

Catani, Grazzini, NPB 845 (2011)

The nonperturbative distribution can be present at tree level and would contribute to Higgs production at low $q_T$

Sun, Xiao, Yuan, PRD 84 (2011)
Gluon polarization and the Higgs boson

\[ p \ p \rightarrow H \ X \] at the LHC

**q_T-distribution of the Higgs boson**

\[ \frac{1}{\sigma} \frac{d\sigma}{dq_T^2} \propto 1 + R(q_T^2) \]

\[ R = \frac{h_{1g}^\perp \otimes h_{1g}^\perp}{f_1^g \otimes f_1^g} \]

\[ |h_{1g}^\perp(x, p_T^2)| \leq \frac{2M_p^2}{p_T^2} f_1^g(x, p_T^2) \]

**Gaussian Model**

**TMD evolution**

\[ m_H = 126 \text{ GeV} \]

Study of \( H \rightarrow \gamma\gamma \) and interference with \( gg \rightarrow \gamma\gamma \)

Echevarria, Kasemets, Mulders, CP, JHEP 1507 (2015) 158

Boer, den Dunnen, CP, Schlegel, PRL 111 (2013)
$C = +1$ quarkonium production

$q_T$-distribution of $\eta_Q$ and $\chi_QJ$ ($Q = c, b$) in the kinematic region $q_T \ll 2M_Q$

\[
\frac{1}{\sigma(\eta_Q)} \frac{d\sigma(\eta_Q)}{dq_T^2} \propto f_1^g \otimes f_1^g [1 - R(q_T^2)] \quad \text{[pseudoscalar]}
\]

\[
\frac{1}{\sigma(\chi_{Q0})} \frac{d\sigma(\chi_{Q0})}{dq_T^2} \propto f_1^g \otimes f_1^g [1 + R(q_T^2)] \quad \text{[scalar]}
\]

\[
\frac{1}{\sigma(\chi_{Q2})} \frac{d\sigma(\chi_{Q2})}{dq_T^2} \propto f_1^g \otimes f_1^g
\]

Boer, CP, PRD 86 (2012) 094007

Proof of factorization at NLO for $p p \rightarrow \eta_Q X$ in the Color Singlet Model (CSM)

Ma, Wang, Zhao, PRD 88 (2013), 014027; PLB 737 (2014) 103
$J/\psi$-pair production at the LHC
\(J/\psi\)-pair production at the LHC

\(J/\psi\)'s are relatively easy to detect. Accessible at the LHC: already studied by LHCb, CMS & ATLAS

\(gg\) fusion dominant, negligible \(q\bar{q}\) contributions even at AFTER@LHC energies

Lansberg, Shao, NPB 900 (2015)

No final state gluon needed for the Born contribution in the Color Singlet Model. Pure colorless final state, hence simple color structure because one has only ISI

Lansberg, Shao, PRL 111 (2013)

Negligible Color Octet contributions, in particular at low \(P_T^{\psi\psi}\)
\[
\frac{d\sigma}{dQdYd^2q_Td\Omega} \approx A f_1^g \otimes f_1^g + B f_1^g \otimes h_1^\perp g \cos(2\phi_{CS}) + C h_1^\perp g \otimes h_1^\perp g \cos(4\phi_{CS})
\]

- valid up to corrections $O(q_T/Q)$
- $Y$: rapidity of the $J/\psi$-pair, along the beam in the hadronic c.m. frame
- $d\Omega = d\cos\theta_{CS} d\phi_{CS}$: solid angle for $J/\psi$-pair in the Collins-Soper frame

Analysis similar to the one for $pp \rightarrow \gamma\gamma X$, $pp \rightarrow J/\psi \gamma(\ast) X$, $pp \rightarrow H_{\text{jet}} X$

Qiu, Schlegel, Vogelsang, PRL 107 (2011)
den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)
Lansberg, CP, Schlegel, NPB 920 (2017)
Boer, CP, PRD 91 (2015)

The three contributions can be disentangled by defining the transverse moments

\[
\langle \cos n\phi_{CS} \rangle \equiv \frac{\int_0^{2\pi} d\phi_{CS} \cos(n\phi_{CS}) \frac{d\sigma}{dQdYd^2q_Td\Omega}}{\int_0^{2\pi} d\phi_{CS} \frac{d\sigma}{dQdYd^2q_Td\Omega}} \quad (n = 2, 4)
\]

\[
\int d\phi_{CS} d\sigma \implies f_1^g \otimes f_1^g
\]
\[
\langle \cos 2\phi_{CS} \rangle \implies f_1^g \otimes h_1^\perp g
\]
\[
\langle \cos 4\phi_{CS} \rangle \implies h_1^\perp g \otimes h_1^\perp g
\]
We consider $q_T = P_T^{\psi\psi} \leq M_{\psi\psi}/2$ in order to have two different scales.

Gaussian model:

$$f_1^g(x, k_T^2) = \frac{f_1^g(x)}{\pi \langle k_T^2 \rangle} \exp \left( -\frac{k_T^2}{\langle k_T^2 \rangle} \right)$$

Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., JHEP 06 (2017)
Heavy quark pair production at an EIC
Gluon TMDs probed directly in
\[ e(\ell) + p(P, S) \rightarrow e(\ell') + Q(K_1) + \bar{Q}(K_2) + X \]
Boer, Mulders, CP, Zhou, JHEP 1608 (2016)

- the \(Q\bar{Q}\) pair is almost back to back in the plane \(\perp\) to \(q\) and \(P\)
- \(q \equiv \ell - \ell'\): four-momentum of the exchanged virtual photon \(\gamma^*\)

Correlation limit:
\[ |q_T| \ll |K_\perp|, \quad |K_\perp| \approx |K_{1\perp}| \approx |K_{2\perp}| \]
Heavy quark pair production in DIS
Angular structure of the cross section

$\phi_T, \phi_\perp, \phi_S$ azimuthal angles of $q_T, K_\perp, S_T$

At LO in pQCD: only $\gamma^* g \rightarrow Q\bar{Q}$ contributes

$$d\sigma(\phi_S, \phi_T, \phi_\perp) = d\sigma^U(\phi_T, \phi_\perp) + d\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

Angular structure of the unpolarized cross section for $ep \rightarrow e' Q\bar{Q}X$, $|q_T| \ll |K_\perp|

$$\frac{d\sigma^U}{d^2 q_T d^2 K_\perp} \propto \left\{ A_0^U + A_1^U \cos \phi_\perp + A_2^U \cos 2\phi_\perp \right\} f_1^g(x, q_T^2) + \frac{q_T^2}{M_p^2} h_1^{\perp g}(x, q_T^2)$$

$$\times \left\{ B_0^U \cos 2\phi_T + B_1^U \cos(2\phi_T - \phi_\perp) + B_2^U \cos 2(\phi_T - \phi_\perp) + B_3^U \cos(2\phi_T - 3\phi_\perp) + B_4^U \cos 2(\phi_T - 2\phi_\perp) \right\}$$

The different contributions can be isolated by defining

$$\langle W(\phi_\perp, \phi_T) \rangle = \frac{\int d\phi_\perp d\phi_T W(\phi_\perp, \phi_T) d\sigma}{\int d\phi_\perp d\phi_T d\sigma}, \quad W = \cos 2\phi_T, \cos 2(\phi_\perp - \phi_T), \ldots$$
$h_{1}^{g}$ in $ep \rightarrow e'Q\bar{Q}X$

Maximal asymmetries

Positivity bound for $h_{1}^{g}$: \[ |h_{1}^{g}(x, p_T^2)| \leq \frac{2M_p^2}{p_T^2} f_1^g(x, p_T^2) \]

It can be used to estimate maximal values of the asymmetries

Asymmetries usually larger when $Q$ and $\bar{Q}$ have same rapidities

Upper bounds on $R \equiv |\langle \cos 2(\phi_T - \phi_\perp) \rangle|$ and $R' \equiv |\langle \cos 2\phi_T \rangle|$ at $y = 0.01$

CP, Boer, Brodsky, Buffing, Mulders, JHEP 1310 (2013)
Boer, Brodsky, Mulders, CP, PRL 106 (2011)
Spin asymmetries in $ep^\uparrow \rightarrow e' Q\bar{Q}X$

Angular structure of the single polarized cross section for $ep^\uparrow \rightarrow e' Q\bar{Q}X$, $|q_T| \ll |K_\perp|$

$$\begin{align*}
d\sigma^T &\propto \sin(\phi_S - \phi_T) \left[ A_0^T + A_1^T \cos \phi_\perp + A_2^T \cos 2\phi_\perp \right] f_{1T}^\perp g + \cos(\phi_S - \phi_T) \left[ B_0^T \sin 2\phi_T \\
&\quad + B_1^T \sin(2\phi_T - \phi_\perp) + B_2^T \sin 2(\phi_T - \phi_\perp) + B_3^T \sin(2\phi_T - 3\phi_\perp) + B_4^T \sin(2\phi_T - 4\phi_\perp) \right] h_{1T}^\perp g \\
&\quad + \left[ B_0'^T \sin(\phi_S + \phi_T) + B_1'^T \sin(\phi_S + \phi_T - \phi_\perp) + B_2'^T \sin(\phi_S + \phi_T - 2\phi_\perp) \\
&\quad + B_3'^T \sin(\phi_S + \phi_T - 3\phi_\perp) + B_4'^T \sin(\phi_S + \phi_T - 4\phi_\perp) \right] h_{1T}^g
\end{align*}$$

Boer, Mulders, CP, Zhou, JHEP 1608 (2016)

The $\phi_S$ dependent terms can be singled out by means of azimuthal moments $A_N^{W}$$\begin{align*}
A_N^{W(\phi_S, \phi_T)} &\equiv 2 \frac{\int d\phi_T \, d\phi_\perp \, W(\phi_S, \phi_T) \, d\sigma_T(\phi_S, \phi_T, \phi_\perp)}{\int d\phi_T \, d\phi_\perp \, d\sigma_U(\phi_T, \phi_\perp)}\\
A_N^{\sin(\phi_S - \phi_T)} &\propto \frac{f_{1T}^\perp g}{f_1^g} \quad A_N^{\sin(\phi_S + \phi_T)} \propto \frac{h_1^g}{f_1^g} \quad A_N^{\sin(\phi_S - 3\phi_T)} \propto \frac{h_{1T}^\perp g}{f_1^g}
\end{align*}$$

Same modulations as in SIDIS for quark TMDs ($\phi_T \rightarrow \phi_h$)
Spin asymmetries in $e p^\uparrow \to e' Q \bar{Q} X$

Upper bounds

Maximal values for $|A_N^W|$, $W = \sin(\phi_S + \phi_T)$, $\sin(\phi_S - 3\phi_T)$ ($|K_\perp| = 1$ GeV)

\begin{align*}
|A_N^W|_{\text{max}} & \quad Q^2 = 100$ GeV$^2 \\
& \quad Q^2 = 10$ GeV$^2 \\
& \quad Q^2 = 1$ GeV$^2 \\

M_Q = M_c & \quad y \\

\end{align*}

\begin{align*}
|A_N^W|_{\text{max}} & \quad Q^2 = 100$ GeV$^2 \\
& \quad Q^2 = 10$ GeV$^2 \\
& \quad Q^2 = 1$ GeV$^2 \\

M_Q = M_b & \quad y \\

\end{align*}
Spin asymmetries in $ep^\uparrow \rightarrow e' Q\overline{Q} X$

Sivers asymmetry

The Sivers asymmetry in open heavy quark production is bounded by

$$A^{\sin(\phi_S - \phi_T)} = \frac{|q_T|}{M_p} \frac{f_{1T}^g(x, q_T^2)}{f_1^g(x, q_T^2)}$$

If $f_{1T}^g$ is 10% of the positivity bound, then the measurement of $A_N^{\sin(\phi_S - \phi_T)}$ is problematic because of statistics.

The situation for dijets is more promising, but theoretically less clean.

$L_{\text{int}} = 10 \text{ fb}^{-1}$

Asymmetries in e⁰ → e'jet jet X

Upper bounds

Contribution to the denominator also from γ⁺q → gq, negligible at small-x

Asymmetries much smaller than in c̅c case for Q² ≤ 10 GeV²

Upper bounds for A_N^W for K⊥ ≥ 4 GeV
Asymmetries in $eA \rightarrow e'\text{jet jet } X$ small-$x$ framework

Also in a $eA$ collisions polarization shows itself through a $\cos 2\phi$ distribution

$\langle \cos 2\phi \rangle$ has opposite signs for $L$ and $T$ $\gamma^*$-polarization, large effects

Monte-Carlo Generator: measurement feasible at the EIC
Quarkonium production at the EIC

Bacchetta, Boer, CP, Tael, arXiv:1809.02056
\[ e p^\uparrow \rightarrow e' Q X \text{ with } Q \text{ either a } J/\psi \text{ or a } \Upsilon \text{ meson, with } P_{Q,T}^2 \ll M_Q^2 \sim Q^2 \]

Color octect (CO) production dominates

\[
Q\overline{Q} \left[^{2S+1}L_J^{(8)}\right]
\]

\[ q \quad P_Q \quad p \]

Theoretically described by Color Evaporation Model or NRQCD

Godbole, Misra, Mukherjee, Rawoot, PRD 85 (2012)
Mukherjee, Rajesh, EPJC 77 (2017)
Rajesh, Kishore, Mukherjee, PRD 98 (2018)

Results depend on the quite uncertain CO \( ^1S_0 \) and \( ^3P_J \) LDMEs
Upper bounds for $\langle \cos 2\phi_T \rangle$ and $A_{N}^{W}$ with $W = \sin(\phi_S + \phi_T), \sin(\phi_S - 3\phi_T)$

$\langle \cos 2\phi_T \rangle$ still sizeable at small $x$, using the MV model as a starting input for $h_{1g}^{\perp}$ and $f_{1g}$ at $x = 10^{-2}$ and evolve them with the JIMWLK equations

Bacchetta, Boer, CP, Taels, arXiv:1809.02056
Marquet, Rosnel, Taels, PRD 97 (2018)
The Sivers asymmetry does not depend on the CO NRQCD LDMEs

\[ A^{\sin(\phi_S - \phi_T)} = \frac{|q_T|}{M_p} \frac{f_{1T}^{\perp g}(x, q_T^2)}{f_g^1(x, q_T^2)} \]

The other asymmetries depend on them, but one can consider ratios of asymmetries to cancel them out

\[ \frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S + \phi_T)}} = \frac{q_T^2}{M_p^2} \frac{h_1^{\perp g}(x, q_T^2)}{h_1^g(x, q_T^2)} \]

\[ \frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S - 3\phi_T)}} = -\frac{1}{2} \frac{h_1^{\perp g}(x, q_T^2)}{h_1^{g T}(x, q_T^2)} \]

\[ \frac{A^{\sin(\phi_S - 3\phi_T)}}{A^{\sin(\phi_S + \phi_T)}} = -\frac{q_T^2}{2M_p^2} \frac{h_1^{\perp g}(x, q_T^2)}{h_1^g(x, q_T^2)} \]

Same relations hold for $e p \rightarrow e' Q \bar{Q} X$
Quarkonium production at the EIC
CO NRQCD LDMEs @EIC

One can consider ratios where the TMDs cancel out and one can obtain new experimental information on the CO NRQCD LDMEs

It requires comparison with $e p^\uparrow \rightarrow e' Q \overline{Q} X$

$$\mathcal{R}^{\cos 2\phi} = \frac{\int d\phi_T \cos 2\phi_T d\sigma^Q(\phi_S, \phi_T)}{\int d\phi_T d\phi_\perp \cos 2\phi_T d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

$$\mathcal{R} = \frac{\int d\phi_T d\sigma^Q(\phi_S, \phi_T)}{\int d\phi_T d\phi_\perp d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

Two observables depending on two unknowns:

$$\mathcal{O}_8^S \equiv \langle 0 | O_8^{Q}(1 S_0) | 0 \rangle$$

$$\mathcal{O}_8^P \equiv \langle 0 | O_8^{Q}(3 P_0) | 0 \rangle$$

$$\mathcal{R}^{\cos 2\phi} = \frac{27\pi^2}{4} \frac{1}{M_Q} \left[ O_8^S - \frac{1}{M_Q^2} O_8^P \right]$$

$$\mathcal{R} = \frac{27\pi^2}{4} \frac{1}{M_Q} \left[ 1 + (1 - y)^2 \right] O_8^S + \frac{(10 - 10y + 3y^2) O_8^P / M_Q^2}{26 - 26y + 9y^2}$$

Plus similar (but different) equations for polarized quarkonium production
Azimuthal asymmetries in heavy quark pair and dijet production in DIS could probe WW-type gluon TMDs (similar to SIDIS for quark TMDs)

Quarkonia are also good probes for gluon TMDs: first extraction of unpolarized gluon TMD from LHC data on di-$J/\Psi$ production

Asymmetries maximally allowed by positivity bounds of gluon TMDs can be sizeable in specific kinematic region

Different behavior of WW and dipole gluon TMDs accessible at RHIC, AFTER@LHC and at EIC, overlap of both spin and small-$x$ programs