The Nuclear Fragmentation Region of High Energy Nucleus-Nucleus and Hadron-Nucleus Collisions

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When can we begin to use a high energy description for nucleus-nucleus collision?

When there is a central region
Particle not formed within the nucleus:

\[ \gamma t_{formation} \gg R \quad E/M_T^2 \gg R \]
\[ \delta y > \ln A^{1/3} \quad E_{CM} > 30 \text{ GeV} \]

This is the conventional region of CGC studies where the nuclei can be treated as two Lorentz contracted sheets with particles produced by the color fields that connect them
There is another region where the High Energy Limit Works:

$$\gamma_{proj} \gg A_{target}^{1/3}, \quad 15 \text{ GeV} \ll E_{lab} \ll 100 \text{ GeV}$$

or center of mass energies greater than about 5 GeV

Physical picture, which we shall review is that the high energy projectile compresses the baryon number and heat the target. Anishetty, Koehler and McLerran argued that the energy densities were sufficient to make quark matter. I put this in a modern context with saturation physics ideas, so that theory of fragmentation region works at high energy. Set up problem of Baryonic CGC.
Want to:
Review computation of energy density in modern saturation context
(see McLerran 2016)
Develop a theory of the classical fields and baryon density produced in the collision
(in spirit similar to the recent work of Schenke and Shen)

I will consider the fragmentation region at extremely high energies, where there is a fully developed central region. In principle such a description might also be applied at lower energies where the nuclear fragments do not separate, but where a high energy description is valid, but that problem is more complicated.

In the fragmentation region there is an asymmetry between the saturation momentum of the target and projectile.

Target saturation momentum is evaluated at $\Delta y$

Projectile saturation momentum is evaluated at $y_{target} - \Delta y$

$Q_{sat}^{projectile} \gg Q_{sat}^{target}$
Saturation momenta are

\[ Q_{\text{target}}^{\text{sat}} \sim A^{1/3} \Lambda_{QCD}^{2} e^{\kappa \Delta y} \]

\[ Q_{\text{projectile}}^{\text{sat}} \sim A^{1/3} \Lambda_{QCD}^{2} e^{\kappa (y_{\text{projectile}} - y_{\text{target}} + \Delta y)} \]

\[ \kappa \sim 0.2 - 0.3 \]

At LHC energies, the target saturation momenta is of the order of a GeV but the projectile is of order 5-10 GeV. This means that the projectile is “black” to the partons in the target up to a scale of momentum which is the projectile saturation momentum. The dominant particle production occurs in the region of momentum between these two saturation momenta. This is a region where the color sources produce a weak field \( A << 1/g \) and there is not much interaction of produced particles in this kinematic region, at least when the degrees of freedom correspond to classical fields. At momenta scales less than the saturation momenta of the target, there are strong fields and classical time evolution of classical fields. This latter region is that of the Glasma.
The multiplicity at low $p_T$ is not much changed due to very high energy

\[ \frac{dN}{dy d^2 p_T} \sim \text{const}, \quad p_T < Q_{\text{sat}}^{\text{target}} \]

But the dominant contribution comes from intermediate momenta

\[ \frac{dN}{dy d^2 p_T} \sim \frac{Q_{\text{sat}}^{\text{target}}}{p_T^2}, \quad Q_{\text{sat}}^{\text{target}} < p_T < Q_{\text{sat}}^{\text{proj}} \]

And at very high momenta the distribution smoothly goes to a perturbative dependence

\[ \frac{dN}{dy d^2 p_T} \sim \frac{Q_{\text{sat}}^{\text{target}} 2 Q_{\text{sat}}^{\text{proj}} 2}{p_T^4}, \quad Q_{\text{sat}}^{\text{proj}} < p_T \]

\[ \frac{dN}{dy} \sim Q_{\text{target}}^2 \]

Does not change up to logarithms

\[ < p_T^2 > \sim Q_{\text{proj}}^2 \]

Is about 100 times bigger at LHC than at RHIC since

\[ x_{\text{RHIC}}^{\text{proj}} \sim 10^{-2} \quad \text{and} \quad x_{\text{LHC}}^{\text{proj}} \sim 10^{-9} \]
The CGC and Limiting Fragmentation:

Empirically, limiting fragmentation works quite well.

The projectile nucleus is dark up to a resolution scale of order the inverse saturation momentum. Therefore the target nucleus is stripped of sea quarks and gluons up to a momentum scale of order this inverse resolution scale. As beam energy increases, there is smaller x probed of the projectile, and momentum scale increases, so there should be some weak breaking of scaling for multiplicity distributions.
In addition there is baryon number compression

Why is high energy fragmentation regions somewhat simple?

Anishetty, Koehler and McLerran, Ming and Kapusta

\[ \Delta z \sim 1 - \nu \quad \text{In boosted fame of struck nucleon, compression} \]

\[ \Delta z_{\text{comoving}} \sim \frac{1}{\gamma_{\text{nucleon}}} \]
The compression gamma factor should be of the order of the gamma factor for produced particles

\[
\gamma/M_T \sim R
\]

\[
\gamma \sim Q_{proj} R
\]

So the initial baryon density is of order

\[
N_B/V \sim Q_{targ}^2 Q_{proj}
\]

The number multiplicity of produce particles per unit area scales as

\[
\frac{1}{\pi R^2} \frac{dN}{dy} \sim Q_{targ}^2
\]

The initial longitudinal size scale is set by the typical transverse momenta of produced gluons

\[
Q_{proj}
\]

\[
N_{gluon}/V \sim Q_{targ}^2 Q_{proj}
\]
\[ \frac{N_B}{N_g} \sim \text{const} \]

However, gluons are not in thermal equilibrium

\[ \frac{E}{S} \sim Q_{\text{proj}} \]

but

\[ s \sim Q_{\text{larg}}^2 Q_{\text{proj}} \]

How might expansion change this?

Interactions and thermalization?

Can one set up CGC-Glasma initial conditions in the fragmentation region including the effects of baryon number density?
Consider a scalar field example

Un-scattered scalar field are “quarks” approximately at rest with spatial momenta small compared to mass

\[ \phi(x)|_{x^+<0} = Ne^{ik^+x^- + im^2 T x^+/2k^+ + ik_T x_T} \]

\[ x^+ = 0 \]
Infinitesimally on the other side of the sheet

\[ \phi(x)_{x^+ = 0^+} = e^{i\eta(x_T)} \phi(x)_{x^+ = 0^-} \xi \]

So that the scattered field is

\[ \phi_+(x) = N \int \frac{d^2 p_T}{(2\pi)^2} e^{ik^+x^- + i\{(p_T + k_T)^2 + M^2\}x^+/2k^+ + i(p_T + k_T)x_T} V(p_T) \xi \]

This allows for a computations of the induced current after scattering.

It is simplest after averaging over transverse fluctuations
After this averaging the scattered current density is uniform
\[ \langle \tilde{j}^- \rangle = \frac{1}{M} \int \frac{d^2 p_T}{(2\pi)^2 V_T} V^+(p_T) \left( \frac{(p_T + k_T)^2}{2k^+} \right) V(p_T) \]
\approx \frac{1}{M} \int \frac{d^2 p_T}{(2\pi)^2 V_T} V^+(p_T) \left( \frac{p_T^2}{2k^+} \right) V(p_T) 

\[ \langle \tilde{j}^+ \rangle = \frac{1}{M} \int \frac{d^2 p_T}{(2\pi)^2 V_T} V^+(p_T) k^+ V(p_T) = k^+ \]

\[ \langle \tilde{j}^i \rangle = \frac{1}{M} \int \frac{d^2 p_T}{(2\pi)^2 V_T} V^+(p_T) (p_T^i + k_T^i)^+ V(p_T) \]
\approx \frac{1}{M} \int \frac{d^2 p_T}{(2\pi)^2 V_T} V^+(p_T) p_T^i V(p_T) \]
The induced gamma factor and the local rest frame baryon density are

\[ e^{2Y} = \frac{\langle P^+ \rangle}{\langle P^- \rangle} = \frac{j^+}{j^-} = \frac{\langle p_T^2 \rangle}{2k'^2} \sim \frac{Q_{nuc}^2}{M^2} \]

\[ \rho_{local} \sim \frac{j^-}{\gamma} \sim \frac{Q_{nuc}}{M} \]

Results are in accord with Anishetty-Koehler-McLerran
We can compute local fluctuations
The gamma factor and compression is no longer fixed and can increase with energy
Results are more complicated to derive but same with fermion fields
Can compute baryon density as function of times and momentum and coordinate space rapidity. At late times coordinate space and momentum space rapidity are tightly correlated.
General comments about the distribution of the gluon fields:

Expect a steady state rate of gluon production

There will therefore be a spatial gradient in the produced distribution since the number density of produced gluons will be proportional to the number of struck nucleons and this increases as time increases.

The deposition of energy will therefore be different for baryons and gluons

The interactions of these two fluids will therefore might have a significant amount of shear
Defining a problem (with Kajantie and Paatelainen)

Consider a classical colored test particle at rest.

It classically hits a sheet of colored glass at $x^+=0$ and is accelerated

Compute the velocity and coordinates of the colored test particle after the collision

\[
\frac{dp^\mu}{d\tau} = T \cdot F^{\mu\nu} u_\nu
\]

Compute spatial and momentum space distribution of gluon in region where gluon-field is weak

Set up problem for large number of classical colored test particles, both in region where field is weak and strong. Two sources: the current of particle and interaction of colored fields at $x^+=0$.

Provides initial conditions for the ba-glasma
To solve the problem simply, choice of gauge is very important:

For background nuclear field: $2$-d transverse gauge for $x^- < 0$

For small fluctuation field corresponding to source: $\delta A^- = 0$

$$\delta A^- \delta J^+ = 0$$

Nuclear field does not precess

In this gauge the transverse vector potential is continuous across nucleus

Solution to first order in source and all order in nuclear field

Two contributions:

Acceleration of source as it accelerates through the nucleus

Acceleration of colored Coulomb field as it passes through nucleus

They interfere
Computation is being now being checked. Find an explicit expression in terms of the nuclear field for the space time evolution of the source, and the momentum and space time distribution of the gluon field

Believe it will be a useful for understanding high energy fragmentation region,
And some insight into lower energy scattering