TMDs from low to high energies
and
the parton branching method

INT Program 18-3 “Probing Nucleons and Nuclei in High Energy Collisions” - EIC Symposium

Institute for Nuclear Theory, Seattle, October 2018
Overview

TRANSVERSE MOMENTUM DEPENDENT (TMD) PARTON DISTRIBUTION FUNCTIONS

- Parton correlation functions at non-lightlike distances:

\[ \tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle, \quad y = (0, y^-, y_\perp) \]

\[ V_y(n) = \mathcal{P} \exp \left( ig_s \int_0^\infty d\tau \ n \cdot A(y + \tau n) \right) \]

- TMD pdfs:

\[ f(x, k_\perp) = \int \frac{dy^-}{2\pi} \frac{d^{d-2}y_\perp}{(2\pi)^{d-2}} e^{-ixp^+y^- + ik_\perp \cdot y_\perp} \tilde{f}(y) \]
Evolution equations for TMD parton distribution functions

low \( q_T : q_T \ll Q \)

\[ \alpha_s^n \ln^m Q/q_T \]

CSS evolution equation

high \( \sqrt{s} : \sqrt{s} \gg M \)

\[ (\alpha_s \ln \sqrt{s}/M)^n \]

CCFM evolution equation

TMD distributions (unpolarized and polarized)

Parton Branching (PB) approach


PB evolution equation motivated by

– applicability over large kinematic range from low to high transverse momenta

– applicability to exclusive final states and Monte Carlo event generators
Outline of this talk

- TMDs at high $\sqrt{s}$ and at low $q_T$
- The parton branching (PB) method
- New results and applications
I. INTRODUCTION

TMDs at high energies

Ex.: heavy flavor electroproduction for $s \gg M^2 \gg \Lambda_{QCD}^2$

\[ \gamma + h \rightarrow Q + \bar{Q} + X \]

\[ 4M^2 \sigma(x, M^2) = \int d^2k_\perp \int_x^1 \frac{dz}{z} \hat{\sigma}_{\gamma g}(x/z, k^2_\perp/M^2, \alpha_s(M^2)) A_{g/h}(z, k_\perp) \]

where TMD gluon distribution is given by Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution:

\[ A_{g/h}(x, k_\perp) \sim \frac{1}{2\pi} e^{-\lambda \ln x} \left( k^2_\perp \right)^{-\frac{1}{2}}, \quad \lambda = 4 C_A \frac{\alpha_s}{\pi} \ln 2 \]
TMDs at high energies

\[ 4M^2 \sigma(x, M^2) \sim x^{-\lambda} \left( M^2 \right)^{1/2} h(1/2), \]

where \( h(1/2) = \frac{1}{2} \int_0^\infty \frac{d\mathbf{k}_\perp^2}{k_\perp^2} \left( \frac{k_\perp^2}{M^2} \right)^{1/2} \int_0^1 \frac{dx}{x} \hat{\sigma} g(x, k_\perp^2/M^2, \alpha_S) \)

asymptotic resummation effect

realistic effects at EIC, LHeC, VHEeP?

NB: - incorporate sub-asymptotic, finite-x terms \( \rightarrow \) CCFM evolution
   - dense-medium modifications in nucleons and nuclei \( \rightarrow \) nonlinear evolution
TMDs for low $q_T$

Ex.: Drell-Yan production $q_T$ spectra for $Q \gg q_T$

\[
\frac{d\sigma}{d^2q_T dq^2 dy} = \sum_{i,j} \frac{\sigma^{(0)}}{s} H(\alpha_s) \int \frac{d^2b}{(2\pi)^2} e^{i q_T \cdot b} A_i(x_1, b, \mu, \zeta) A_j(x_2, b, \mu, \zeta) + \{q_T-\text{finite}\} + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{Q^2}\right)
\]

where

\[
\frac{\partial \ln A}{\partial \ln \sqrt{\zeta}} = K(b, \mu)
\]

Collins-Soper-Sterman (CSS) evolution

and

\[
\frac{d \ln A}{d \ln \mu} = \gamma_f(\alpha_s(\mu), \zeta/\mu^2), \quad \frac{dK}{d \ln \mu} = -\gamma_K(\alpha_s(\mu))
\]

RG evolution

cusp anomalous dimension

\[
\Rightarrow -\gamma_K = \frac{2}{3} \gamma_f
\]

i.e., $\gamma_f(x_1(\mu), \frac{3}{2} \mu^2) = \gamma_f(x_1(\mu), 1) - \frac{1}{2} \gamma_f \ln 3$

- Soft Collinear Effective Theory (SCET) provides alternative approach leading to same results

F Hautmann: Institute for Nuclear Theory, University of Washington, October 2018
TMDs for low $q_T$

- **Outcome:**
  
  $$d_s \propto \frac{1}{2\alpha_s^2}$$

- **Physical Vector Boson Spectrum!**

- **Resummed**

**Note:** Shower Monte Carlo generators do this “effectively.”

- A more “parton-like” formulation is achieved by decomposing the TMD PDF in terms of ordinary PDF’s (“OPE”)

  $$S = \mu - Q \sim b^{-1}$$

  $$f_j (b, \mu) = \sum \phi (Q, b) \ c_{j \alpha} (b, \mu) \ f_{\alpha} (\mu)$$

  **Sudakov Form Factor** (soft radiation)

  **Evolution Kernels** (collinear radiation)

  **Inclusive PDF**
From color-neutral to color-charged final states

Color neutral:

Color charged:

- New long-time correlations in color-charged case:

\[
\left( \frac{d\sigma}{d^4q} \right)_{t\bar{t}} = \sum_{i j a_1 a_2} \int d^2 b \ e^{i q \cdot T \cdot b} \int d z_1 \int d z_2 \ S(Q, b) \ f_{a_1} \otimes [\text{Tr}(H\Delta)C_1 C_2]_{i j a_1 a_2} \otimes f_{a_2}
\]

- Generate azimuthal asymmetries

- Observable for $\Delta p_\perp$ high compared to $\Lambda_{\text{QCD}}$?

F Hautmann: Institute for Nuclear Theory, University of Washington, October 2018
Color correlations in jet and heavy-flavor production

- Initial state / final state soft-gluon correlations → new “color entanglement” effects?

- A recent quantitative estimate of the size of color correlations for the top quark pair spectrum at the LHC:

Catani, Grazzini, Sargsyan
JHEP 1706 (2017) 017
II. The Parton Branching (PB) method

MOTIVATION

- Provide evolution equation connected in a controllable way with DGLAP evolution of collinear parton distributions

- Applicable over broad kinematic range from low to high transverse momenta, for inclusive as well as non-inclusive observables

- Implementable in Monte Carlo event generators
Parton Branching (PB) method: collinear PDFs

QCD evolution and soft-gluon resolution scale

\[
\tilde{f}_a(x, \mu^2) = \Delta_a(\mu^2) \tilde{f}_a(x, \mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_M} dz \, P_{ab}^{(R)}(\alpha_s(\mu'^2), z) \, \tilde{f}_b(x/z, \mu'^2)
\]

where \( \Delta_a(z_M, \mu^2, \mu_0^2) = \exp \left( -\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz \, z \, P_{ba}^{(R)}(\alpha_s(\mu'^2), z) \right) \)

\( z_M \) is the soft-gluon resolution parameter that separates resolvable and nonresolvable branchings.

- Equivalence to DGLAP evolution equation for \( z_M \to 1 \)

\( P^{(R)} \) represents the real-emission probability, and \( \Delta \) is the no-branching probability.
Parton Branching (PB) method: TMD PDFs

\[
\tilde{\mathcal{A}}_a(x, k, \mu^2) = \Delta_a(\mu^2) \tilde{\mathcal{A}}_a(x, k, \mu_0^2) + \sum_b \int \frac{d^2q'}{\pi q'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(q'^2)} \Theta(\mu^2 - q'^2) \Theta(q'^2 - \mu_0^2) \times 
\int_x^{z_M} dz \ P^{(R)}_{ab}(\alpha_s(q'^2), z) \tilde{\mathcal{A}}_b(x/z, k + (1 - z)q', q'^2)
\]

Solve iteratively: \( \tilde{\mathcal{A}}^{(0)}_a(x, k, \mu^2) = \Delta_a(\mu^2) \tilde{\mathcal{A}}_a(x, k, \mu_0^2) \),

\[
\tilde{\mathcal{A}}^{(1)}_a(x, k, \mu^2) = \sum_b \int \frac{d^2q'}{\pi q'^2} \Theta(\mu^2 - q'^2) \Theta(q'^2 - \mu_0^2) \times 
\frac{\Delta_a(\mu^2)}{\Delta_a(q'^2)} \int_x^{z_M} dz \ P^{(R)}_{ab}(\alpha_s(q'^2), z) \tilde{\mathcal{A}}_b(x/z, k + (1 - z)q', \mu_0^2) \Delta_b(q'^2)
\]

Jung, Lelek, Radescu, Zlebcik & H, JHEP 01 (2018) 070

- A new evolution equation!

\[
\mu = |q_c|/(1 - z)
\]
Validation of the method with semi-analytic result from QCDNUM at LO

Agreement to better than 1 % over several orders of magnitude in x and μ
Validation of the method with semi-analytic result from QCDNUM at NLO

Very good agreement at NLO over all x and mu.
NB: the same approach is designed to work at NNLO.
Stability with respect to resolution scale $z_M$
TMDs and soft gluon resolution effects

Well-defined TMDs require appropriate ordering condition
PB method in xFitter

\[ x f_a(x, \mu^2) = x \int dx' \int dx'' A_{0,b}(x') \tilde{A}^b_a(x'', \mu^2) \delta(x'x'' - x) \]
\[ = \int dx' A_{0,b}(x') \cdot \frac{x}{x'} \tilde{A}^b_a \left( \frac{x}{x'}, \mu^2 \right) \]

- fit to HERA data (using xFitter) with \( Q^2 \geq 3.5 \text{ GeV}^2 \) gives \( \chi^2/ndf \sim 1.2 \)

A Bermudez et al, arXiv:1804.11152

A. Lelek et al REF 2016
TMD distributions from fits to precision HERA data

A Bermudez et al, arXiv:1804.11152

- NLO determination of TMDs with uncertainties
Where to find TMDs? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of the REF Workshop and developed since

- A library of parameterizations and fits of TMDs (LHAPDF-style)

http://tmdlib.hepforge.org
http://tdmplotter.desy.de

- Also contains collinear (integrated) pdfs
Next REF Workshop:
Cracow, 19-22 November 2018

https://indico.cern.ch/event/696311
III. New results and applications

ONGOING WORK:

- Drell-Yan $p_T$ spectrum from convolution of two transverse momentum dependent distributions

- Comparison of parton branching results with analytic TMD resummation (Collins-Soper-Sterman method)

- First implementation for jets (using NLO matrix elements for color-charged final states)
Application of PB method to Z-boson transverse momentum spectrum in Drell-Yan production

- Parton branching TMD defined by using angular ordering
- Scale in running coupling also by angular ordering

\[ \alpha_s (\mu^2 (1-z)^2) \]

\( z_M(\mu) = 1 - q_0 / \mu \)

LHC Electroweak WG Meeting, CERN, June 2018
Z-boson transverse momentum spectrum: soft-gluon angular ordering effects


ATLAS data, EPJC 76 (2016) 291
Comparison with CSS (Collins-Soper-Sterman) resummation

\[ \frac{d\sigma}{d^2q dQ^2 dy} = \sum_{q, \bar{q}} \frac{\sigma^{(0)}}{s} H(\alpha_S) \int \frac{d^2b}{(2\pi)^2} e^{i q \cdot b} A_q(x_1, b, Q) A_{\bar{q}}(x_2, b, Q) + \mathcal{O}\left(\frac{|q|}{Q}\right) \]

where

\[ A_i(x, b, Q) = \exp \left\{ \frac{1}{2} \int_{c_0/b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \left[ A_i(\alpha_S(\mu'^2)) \ln \left(\frac{Q^2}{\mu'^2}\right) + B_i(\alpha_S(\mu'^2)) \right] \right\} G_i^{(NP)}(x, b) \]

\[ \times \sum_j \int_{x}^{1} \frac{dz}{z} C_{ij} \left( z, \alpha_S \left( \frac{c_0}{b^2} \right) \right) f_j \left( \frac{x}{z}, \frac{c_0}{b^2} \right) \]

and the coefficients \( H, A, B, C \) have power series expansions in \( \alpha_S \).

\( \diamond \) The parton branching TMD is expressed in terms of real-emission \( P^{(R)} \):

\[ P^{(R)}(x_1, x_2, x_3) = \frac{1}{x_1 x_2 x_3} \frac{1}{P(x_1, x_2, x_3)} \]

\( \triangleright \) via momentum sum rules, use unitarity to relate \( P^{(R)} \) to virtual emission

\( \triangleright \) identify the coefficients in the two formulations, order by order in \( \alpha_S \), at LL, NLL, \ldots
Comparison with CSS (Collins-Soper-Sterman) resummation

More precisely:

- The parton branching TMD contains Sudakov form factor in terms of

\[ P_{ab}^{(R)}(\alpha_s, z) = K_{ab}(\alpha_s) \frac{1}{1 - z} + R_{ab}(\alpha_s, z) \quad \text{where} \]

\[ K_{ab}(\alpha_s) = \delta_{ab} k_a(\alpha_s), \quad k_a(\alpha_s) = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^n k_a^{(n-1)}, \quad R_{ab}(\alpha_s, z) = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^n R_{ab}^{(n-1)}(z) \]

- Via momentum sum rules, use unitarity to re-express this in terms of

\[ P^{(V)} = P - P^{(R)} \quad \text{where} \]

\[ P_{ab}(\alpha_s, z) = D_{ab}(\alpha_s) \delta(1 - z) + K_{ab}(\alpha_s) \frac{1}{(1 - z)_+} + R_{ab}(\alpha_s, z) \]

is full splitting function (at LO, NLO, etc.)

with \( D_{ab}(\alpha_s) = \delta_{ab} d_a(\alpha_s) \), \( d_a(\alpha_s) = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^n d_a^{(n-1)} \)

- Identify \( d_a(\alpha_s) \) and \( k_a(\alpha_s) \) with resummation formula coefficients (LL, NLL, . . .)
Comparison with CSS (Collins-Soper-Sterman) resummation

- $d_a(\alpha_S)$ and $k_a(\alpha_S)$ perturbative coefficients

\begin{align*}
\text{one-loop:} & \quad d_q^{(0)} = \frac{3}{2}C_F, \quad k_q^{(0)} = 2C_F \\
\text{two-loop:} & \quad d_q^{(1)} = C_F^2 \left( \frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3) \right) + C_F C_A \left( \frac{17}{24} + \frac{11\pi^2}{18} - 3\zeta(3) \right) - C_F T_R N_f \left( \frac{1}{6} + \frac{2\pi^2}{9} \right), \\
& \quad k_q^{(1)} = 2C_F \Gamma, \quad \text{where} \quad \Gamma = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - T_R N_f \frac{10}{9}
\end{align*}

- The k and d coefficients of the PB formalism match, order by order, the A and B coefficients of the CSS formalism
Di-jets from PB method: towards NLO-matched parton-shower Monte Carlo generators with TMDs

- Events by NLO POWHEG 2 jets
- Parton branching TMD (with angular ordering)
- TMD parton shower
Di-jets from PB method: towards NLO-matched parton-shower Monte Carlo generators with TMDs

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Conclusions

- PB method to take into account simultaneously soft-gluon emission at \( z \to 1 \) and transverse momentum \( q_T \) recoils in the parton branchings along the QCD cascade

- potentially relevant for calculations both in collinear factorization and in TMD factorization
  \[ \to \text{cf. parton shower calculations and analytic resummation} \]

- terms in powers of \( \ln (1 - zM) \) can be related to large-\( x \) resummation? \[ \to \text{relevant to near-threshold, rare processes to be investigated at high luminosity} \]

- systematic studies of ordering effects and color coherence
  \[ \to \text{helpful to analyze long-time color correlations?} \]
EXTRA SLIDES
PB method at NNLO

- In NNLO VFNS discontinuities both in $\alpha_S$ and PDFs
- These discontinuities ensure continuity of observables, e.g. $F_2$

Discontinuities in the quark and gluon Sudakov factors

R. Zlebcik, talk at REF 2017, November 2017
PB method at NNLO

The Monte Carlo solution vs QCDNUM

LO  NLO  NNLO

The Monte Carlo evolution implemented up to NNLO and cross-checked against the semi-analytical solution of DGLAP

The solution's uncertainties are mainly statistical
(~ number of generated MC evolutions)

R. Zlebcik, talk at REF 2017, November 2017

Workshop REF2017, Universidad Complutense Madrid, 13-16 November 2017
Effects of coupling's scale and angular ordering in integrated parton distributions
Z-boson pT spectrum including TMD uncertainties

- Cf. predictions from fixed-order + resummed calculations
  Bizon et al., arXiv:1805.05916