Holographic $J/\psi$ production near threshold
and the proton mass problem

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An Assessment of U.S.-Based Electron-Ion Collider Science

Committee on U.S.-Based Electron-Ion Collider Science Assessment

Board on Physics and Astronomy

Division on Engineering and Physical Sciences

A Consensus Study Report of

The National Academies of

SCIENCES • ENGINEERING • MEDICINE

Finding 1: An EIC can uniquely address three profound questions about nucleon-protons—and how they are assembled to form the nuclei of atoms:

• How does the mass of the nucleon arise?
• How does the spin of the nucleon arise?
• What are the emergent properties of dense systems of gluons?
The nucleons

Bound states of the QCD Lagrangian, fundamental building blocks of matter, accounting for 99% of the mass of the visible universe

<table>
<thead>
<tr>
<th></th>
<th>Proton</th>
<th>Neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>938.3MeV</td>
<td>939.6MeV</td>
</tr>
<tr>
<td>Spin</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>Charge</td>
<td>+1</td>
<td>0</td>
</tr>
</tbody>
</table>
What’s mysterious about proton mass? Lattice QCD can explain it.

Proton has spin $\frac{1}{2}$ because it’s a fermion. Why need more explanation?
Spin crisis

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L^q + L^g \]

Quarks’ helicity  Gluons’ helicity  Orbital angular Momentum

Quarks’ helicity accounts for only 25\textendash30\% of the nucleon spin
Mass crisis

$u,d$ quark masses add up to \(~10\text{MeV},\) only 1 % of the proton mass!

Higgs mechanism explains quark masses, but not hadron masses!

In relativity, mass and energy are equivalent. Kinetic energy counts. What about chiral symmetry breaking?
The Proton Mass
At the heart of most visible matter.
Temple University, March 28-29, 2016

$M_p = 2m_u^\text{eff} + m_d^\text{eff}$

**Speakers**
Stan Brodsky (SLAC)
Xiangdong Ji (Maryland)
Dima Kharchev (Stony Brook & BNL)
Keh-Fei Liu (University of Kentucky)
David Richards (JLab)
Craig Roberts (ANL)
Martin Savage (University of Washington)
Stepan Stepanyan (JLab)

**Workshop Topics**

**$H_{\text{QCD}} = H_q + H_m + H_g + H_a$**
- Quark kinetic and potential energy $H_q = \int d^4x \bar{\psi}(iD\cdot\alpha)\psi$
- Quark masses $H_m = \int d^4x \bar{\psi}m\psi$
- Gluon kinetic and potential energy $H_g = \int d^4x \frac{1}{2} (E^2 - B^2)$
- Trace anomaly $H_a = \int d^4x \frac{9\alpha_s}{16\pi} (E^2 - B^2)$

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**ECT***
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY
Institutional Member of the European Expert Committee NUPECC

**The Proton Mass: At the Heart of Most Visible Matter**
Trento, April 3 - 7, 2017

**Main Topics**

Hadron mass decomposition in terms of constituents:
- Uniqueness of the decomposition, Quark mass, and quark and gluon energy contribution, Anomaly contribution, ...

Hadron mass calculations:
- Lattice QCD (total & individual mass component), Approximated analytical methods, Phenomenological model approaches, ...

Experimental access to hadron mass components:
- Exclusive beam modulation resolution at threshold, nuclear phenomenology through relativistic nuclear structure function
Origin of the proton mass

QCD Lagrangian approximately scale (conformal) invariant. Why is the proton mass nonvanishing in the first place?

Conformal symmetry is explicitly broken by the trace anomaly.

QCD energy-momentum tensor

\[
T_{\mu \nu} = -F_{\mu \lambda} F^{\nu \lambda} + \frac{\eta_{\mu \nu}}{4} F^2 + i \bar{q} \gamma^{(\mu} D^{\nu)} q
\]

\[
\langle P | T_{\mu \nu} | P \rangle = 2 P^\mu P^\nu
\]

\[
T_{\mu} = \frac{\beta(g)}{2g} F^2 + m(1 + \gamma_m(g)) \bar{q} q
\]

\[
\langle P | T_{\mu} | P \rangle = 2 M^2
\]
Proton mass decomposition

Traceless and trace parts of EMT in $d = 4 - 2\epsilon$ dimensions.

$$T^{\mu\nu} = \left( T^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} T^{\alpha}_\alpha \right) + \frac{\eta^{\mu\nu}}{d} T^{\alpha}_\alpha + \frac{\beta(g)}{2g} F^{\rho\sigma} F_{\rho\sigma} + m(1 + \gamma_m(g)) \bar{q} q$$

Work in the rest frame. Mass is the eigenvalue of the Hamiltonian $H = \int d^3 x T^{00}$

Quark/gluon kinetic energy, trace anomaly, quark mass

$$M = M_q + M_g + M_a + M_m$$

Ji (1995)

Alternative approach Lorce (2017) + talk next week
$M_q$ and $M_g$ measurable in DIS

$$M_{q,g} = \frac{3}{4} M A_{q,g}$$

$$A_{q,g}(\mu) = \langle x \rangle_{q,g} = \int_0^1 dx x f_{q,g}(x, \mu)$$

$M_q$, $M_g$, $M_m$ calculable on a lattice ➔ talk by Keh-fei (What about $M_a$?)

Yang, et al. (2018) ($\chi$QCD collaboration)
Can we measure the trace anomaly $\langle P|F_{\mu\nu}F_{\mu\nu}|P\rangle$?

The operator $F_{\mu\nu}F_{\mu\nu}$ is twist-four, highly suppressed in high energy scattering. QCD factorization difficult to establish.

Instead, we should look at low-energy scattering.

Purely gluonic operator. Use quarkonium as a probe.

Kharzeev (1996)
Photo-production of $J/\psi, \Upsilon$

\[ \sigma_{tot} = \int_{t_{min}}^{t_{max}} dt \frac{d\sigma}{dt} \]

New experiments proposed at Jlab (Meziani, talk next week). Possibly also at the EIC!
$J/\psi$ photo-production: general consideration

**High energy**

Rich data & phenomenology

- GPD at small-$x$,
- hard/soft pomeron,
- color dipole, saturation

**Low energy**

Old experiments 40 years ago. (Cornell, SLAC)

Large-$x$ physics, $x \to 1$ near the threshold.

Heavy-quark loop $\to$ **local** two-gluon operators

$\to$ gluonic form factors  Frankfurt, Strikman (2002)

Momentum transfer significant $\Delta_{th} = \sqrt{-t_{th}} \approx 1.5$GeV

Higher twist contributions?
Previous approaches

Kharzeev, Satz, Syamtomov, Zinovjev (1998);

Assume vector meson dominance to relate $\gamma p \to J/\psi p$ to forward $J/\psi p \to J/\psi p$

Compute $\text{Im} T^{J/\psi p}(t = 0) \sim \sigma_{tot}^{J/\psi p}$

Reconstruct $\text{Re} T^{J/\psi p}(t = 0)$ via dispersion relation. $\langle P|F^2|P \rangle$ enters as a subtraction constant.

Brodsky, Chudakov, Hoyer, Laget (2001)

Two-gluon, three-gluon hard scattering
No connection to trace anomaly.

Frankfurt, Strikman (2002)

t-dependence from 2-gluon form factor,
not exponential
No connection to trace anomaly.
Holographic approach

Perturbative approach difficult. Need nonperturbative methods. Use AdS/CFT, or more generally, gauge/string duality. QCD amplitude $\approx$ string amplitude in asymptotically $AdS_5$. 

YH, Yang (2018)
The AdS/CFT correspondence

Maldacena, `97

\[ \text{N=4 super Yang-Mills at strong coupling, large-}N_c \]

\[ \text{equivalent} \]

Type IIB superstring theory on \( AdS_5 \times S^5 \)

\[ ds^2 = R^2 \frac{dz^2}{z^2} - dx^\mu dx_\mu + R^2 d\Omega_5^2 \]

Field theory

operators \( T^{\mu\nu}, F^2, \cdots \)

(anomalous) dimension \( \lambda \)

`t Hooft coupling \( \lambda \)

number of colors \( 1/N_c \)

string

string state \( G_{\mu\nu}, \phi, \cdots \)

mass

curvature radius \( R \)

string coupling constant \( g_s \)
Application of AdS/CFT to high/low energy scattering

High energy : Disaster

Scattering amplitudes dominantly real, in stark contrast to QCD
Graviton exchange gives too strong rise of the cross section $\sigma_{tot} \propto S$
Finite-coupling corrections/modified geometry essential for reasonable phenomenology.

Low-energy : Some hope

QCD amplitudes dominantly real at low energy.
Steep rise of the cross section near threshold may be explained by graviton exchanges.
Setup

Quarkonium: open string excitation on a D7 brane

Karch, Katz (2002)

\[ S_{D7} = -T_{D7} \int d^8 \xi e^{-\phi} \sqrt{-\det \left[ G_{ab} + 2\pi \alpha' (F^\gamma_{ab} + F^{J/\psi}_{ab}) \right]} \]
Scattering amplitude in AdS

\[ \langle P | \epsilon \cdot J | P' k \rangle \sim \int d^4 x dz \sqrt{-G} \int d^4 x' dz' \sqrt{-G'} \Phi_\gamma \Phi_{J/\psi} G(zx, z'x') \Phi_P \Phi_{P'} \]

dual to \( T^{\mu \nu} \)

dual to \( F^2 \)
Heavy quark limit

In the heavy-quark limit, one can make connection with the form factors

\[ \langle P | \mathbf{e} \cdot J(0) | P' k \rangle \approx -\frac{2\kappa^2}{f_\psi R^3} \int_0^{z_m} dz \frac{\delta S_{D7}(q, k, z)}{\delta g_{\mu\nu}} \frac{z^2 R^2}{4} \langle P | T^{gTT}_{\mu\nu} | P' \rangle \]

\[ + \frac{2\kappa^2}{f_\psi R^3} \frac{3}{8} \int_0^{z_m} dz \frac{\delta S_{D7}(q, k, z)}{\delta \phi} \frac{z^4}{4} \langle P | \frac{1}{4} F^a_{\mu\nu} F^{a}_{\mu\nu} | P' \rangle \]
Bulk-to-bulk propagator

\[ D(xz; x'z') = \langle \phi(xz) \phi(x'z') \rangle = \frac{2\kappa^2 i}{c R^3} \frac{3}{2\pi^2} \frac{1}{(2u)^4} F \left( 4, \frac{5}{2}, 5; -\frac{2}{u} \right), \]

\[ u = \frac{(z - z')^2 - (x - x')^2}{2zz'} \]

\[ D(x, z \to 0, x'z') \approx \frac{2\kappa^2 i}{c R^3} \frac{3}{2\pi^2} \left( \frac{zz'}{z'^2 - (x - x')^2 + i\epsilon} \right)^4 \]

Boundary-to-bulk propagator

\[ \phi(x'z') = \frac{6i}{\pi^2} \left( \frac{z'}{z'^2 - (x - x')^2 + i\epsilon} \right)^4 \]

\[ \langle F^2(x) \cdots \rangle = \int dx' dz' \phi(x'z') \cdots \]
Nucleon gravitational form factors

\[ \langle P' | T_{q,g}^{\mu \nu} | P \rangle = \bar{u}(P') \left[ A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}(\mu \sigma^{\nu})\Delta_\alpha}{2M} + D_{q,g} \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + \tilde{C}_{q,g} M \eta^{\mu\nu} \right] u(P) \]

Assume the dipole form \( A_g(t) = \frac{A_g(0)}{(1 - t/\Lambda^2)^2} \). t-dependence not exponential.

Neglect \( B_g \).

Gluon D-term unknown.

Use a model based on the asymptotic formula and quark counting rule.

\[ D_g(t) = \frac{16}{3n_f} D_q(t) = \frac{16}{3n_f} \frac{-0.1}{(1 - t/\Lambda)^3} \]

\( \tilde{C}_g \) related to trace anomaly \( \langle P | (T_{q,g})^\mu_\mu | P \rangle = 2M (A_{q,g} + 4\tilde{C}_{q,g}) \) Nontrivial!
Quark and gluon contributions to trace anomaly

Energy momentum tensor consists of quark and gluon parts.

\[ T^\mu_\mu = (T_q)^\mu_\mu + (T_g)^\mu_\mu = \frac{\beta}{2g} F^2 + m(1 + \gamma_m) \bar{\psi}\psi \]

Can we compute \((T_q)^\mu_\mu\) and \((T_g)^\mu_\mu\) separately?

In dimensional regularization,

\[ (T_q)^\mu_\mu = m \bar{\psi}\psi \]

\[ (T_g)^\mu_\mu = \frac{\beta}{2g} F^2 + m\gamma_m \bar{\psi}\psi \]

for the bare operators

What about the renormalized operators \((T_{q,g}^R)^\mu_\mu\)?
Two-loop result in the $\overline{\text{MS}}$ scheme

\[
\eta_{\mu\nu} T_{gR}^{\mu\nu} = \frac{1}{2M^2} \langle P | \left\{ \frac{\alpha_s}{4\pi} \left( \frac{14}{3} C_F (m \bar{\psi} \psi)_R - \frac{11}{6} C_A (F^2)_R \right) \\
+ \left( \frac{\alpha_s}{4\pi} \right)^2 \left[ \left( C_F \left( \frac{812C_A}{27} - \frac{22n_f}{27} \right) + \frac{85C_F^2}{27} \right) (m \bar{\psi} \psi)_R + \left( \frac{28C_An_f}{27} - \frac{17C_A^2}{3} + \frac{5C_Fn_f}{54} \right) (F^2)_R \right] \right\} | P \rangle
\]

\[
\eta_{\mu\nu} T_{qR}^{\mu\nu} = \frac{1}{2M^2} \langle P | \left\{ (m \bar{\psi} \psi)_R + \frac{\alpha_s}{4\pi} \left( \frac{4}{3} C_F (m \bar{\psi} \psi)_R + \frac{1}{3} n_f (F^2)_R \right) \\
+ \left( \frac{\alpha_s}{4\pi} \right)^2 \left[ (m \bar{\psi} \psi)_R \left( C_F \left( \frac{61C_A}{27} - \frac{68n_f}{27} \right) - \frac{4C_F^2}{27} \right) + (F^2)_R \left( \frac{17C_An_f}{27} + \frac{49C_Fn_f}{54} \right) \right] \right\} | P \rangle
\]

\[
A_q^R (\mu \to \infty) = \frac{3n_f}{4C_F + n_f}
\]

\[
\bar{C}_q^R (\mu \to \infty) = -\frac{1}{4} \left( \frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0} \right) \approx -0.15
\]
Numerical results

\[ M_m = \frac{1}{4} \left\langle P | m(1 + \gamma_m) \bar{\psi} \psi | P \right\rangle - \frac{b}{4} M \]

\[ M_a = \frac{1}{4} \left\langle P | \frac{\beta}{2g} F^2 | P \right\rangle - \frac{b}{4} M \]

\[ W = \sqrt{s_{\gamma p}} = 4.3 \text{ GeV} \]

\[ d\sigma_r \equiv \left( \frac{d\sigma}{dt} \right)_{b=0} \]

\[ d\sigma_r \equiv \left( \frac{d\sigma}{dt} \right)_{b=1} \]
Towards EIC

Jlab: Fixed-target, low energy
EIC: collider, high energy

Better control on $y = E^\gamma / E^e$.

Simulate the final states by boosting Jlab final state particles.  

Talk by A. Deshpande  
@Proton mass workshop (2017)

Is $\Upsilon$ more useful?  

$W_{\gamma p}^{th} = 10.4 \text{GeV}$

$t_{min} \approx -8.1 \text{GeV}^2$
Conclusion

• Origin of the nucleon mass → important goal of EIC.
• Look at heavy-quarkonium production near threshold. Cross section sensitive to $\langle P|F^2|P'\rangle$.
• Precise $t$-dependence of gravitational form factors very welcome ← models, lattice
• Use more realistic AdS/QCD models.
• First principle/model independent approach?