Proton Spin, Form Factors
from Lattice QCD

Martha Constantinou
Temple University

INT workshop on Probing
Nucleons and Nuclei in High Energy Collisions
October 15, 2018
Proton Spin, Form Factors
... and beyond
from Lattice QCD

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INT workshop on Probing
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A. Introduction

B. Nucleon on the Lattice

C. Nucleon Form Factors
   1. Electromagnetic
   2. Axial

D. Proton Spin

E. Access to x-dependence of PDFs

F. Discussion
In collaboration with

- C. Alexandrou $^{1,2}$
- K. Cichy $^3$
- K. Hadjiyiannakou $^2$
- K. Jansen $^4$
- C. Kallidonis $^2$
- G. Koutsou $^2$
- H. Panagopoulos $^1$
- A. Scapellato $^1$
- F. Steffens $^5$
- A. Vaquero $^6$

1. University of Cyprus
2. The Cyprus Institute
3. Adam Mickiewicz University
4. DESY, Zeuthen
5. Bonn University
6. University of Utah
A. Introduction

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F. Discussion
FFs have been studied for decades as a tool to understand nucleon structure

- Nucleon electric & magnetic radii, magnetic moment extracted from Electromagnetic FFs
- Axial FFs relevant to experiments searching neutrino oscillations
- Intrinsic quark spin obtained from $g_A (G_A(Q^2=0))$
- Total quark spin from $<x>$

Information from experiments not without ambiguities

- Discrepancy of $\langle r_p^2 \rangle$ between electron scattering and muonic hydrogen Lamb shifts
- Large uncertainties in cross section of quasielastic neutrino-nucleon scattering: not well-constrained Axial FFs
- Strange E/M FFs are compatible with zero (HAPPEX collaboration, A4 exper., SAMPLE exper.)

[E. J. Downie, EPJ Conf. 113 (2016) 0502]
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Lattice QCD ideal ab initio formulation to study nucleon form factors
The EIC will address crucial questions in hadron structure:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?

... and measure PDFs in high accuracy

*These questions can be addressed in Lattice QCD*
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*These questions can be addressed in Lattice QCD*

EIC will come at a time when lattice QCD:

- will be well into the exa-scale computing era
- will reliably compute sea quark and gluon contributions
- is expected to reliably compute x-dependence quantities

*Lattice QCD already showing promising results*
Parton Distribution Functions

- Probe of hadron structure (1-D)
- Ideal for description of non-perturbative nature
- Probability densities for a given parton to carry a fraction-$x$ of the hadron momentum
- Well-studied both experimentally and theoretically
- Necessary for analysis of DIS data
- Phenomenological input needed for data interpretation

A. Accardi et al., arXiv:1602.03154

M. Constantinou
Parton Distribution Functions

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Calculation from first principle imperative

A. Accardi et al., arXiv:1602.03154
Lattice QCD is ideal *ab initio* formulation

*PDFs parameterized in terms of off-forward matrix elements of light-cone operators.*
Lattice QCD is ideal *ab initio* formulation

PDFs parameterized in terms of off-forward matrix elements of light-cone operators.

Not accessible in Euclidean lattice
Lattice QCD is ideal *ab initio* formulation

PDFs parameterized in terms of off-forward matrix elements of light-cone operators. Not accessible in Euclidean lattice

On lattice: moments of PDFs (reconstructed via OPE)

Reconstruction difficult task:
- $n > 3$: operator mixing
- Statistical noise increases with high moments

Moments of PDFs have physical interpretation and may serve as benchmark
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Nucleon on the Lattice

Connected

Disconnected Quark loop

Disconnected Gluon loop
Nucleon on the Lattice

Separation between source and sink: excited states investigation

Current insertion: ultra-local, covering derivative, non-local
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**Nucleon on the Lattice**

- **Disconnected Quark loop**
- **Disconnected Gluon loop**

**Connected**

- Separation between source and sink: excited states investigation
- Current insertion: ultra-local, covering derivative, non-local

**Particularly interesting for EIC physics**
Statistical errors significantly increase with:
- decrease of pion mass
- increase of momentum transfer between initial-final state
- increase of source-sink separation $T_{sink}$

Sources of systematic uncertainties:
- cut-off effects (finite lattice spacing)
- finite volume effects
- contamination from other hadron states
- chiral extrapolation for unphysical pion mass
- renormalization and mixing
Set up of Calculation

Twisted Mass including a clover term (ETM Collaboration)

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$N_f$</th>
<th>$L^3 \times T$</th>
<th>$a$ (fm)</th>
<th>$m_\pi$ (MeV)</th>
<th>$L m_\pi$</th>
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<tbody>
<tr>
<td>Nf2.48c</td>
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<td>0.094</td>
<td>135</td>
<td>2.98</td>
</tr>
<tr>
<td>Nf2.64c</td>
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<td>0.094</td>
<td>132</td>
<td>3.97</td>
</tr>
<tr>
<td>Nf211.64c</td>
<td>2+1+1</td>
<td>$64^3 \times 128$</td>
<td>0.081</td>
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<th>Nf211.64c (conn.)</th>
<th>Nf211.64c (disc.)</th>
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<tbody>
<tr>
<td>$T_{\text{sink}}$</td>
<td>$N_{\text{conf}}$</td>
</tr>
<tr>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>12$\alpha$</td>
<td>625</td>
</tr>
<tr>
<td>14$\alpha$</td>
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Set up of Calculation

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<th>(N_{\text{src}})</th>
<th>Flavor</th>
<th>(N_{\text{conf}})</th>
<th>(N_{\text{src}})</th>
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<tbody>
<tr>
<td>—</td>
<td>—</td>
<td>—</td>
<td>16a</td>
<td>625</td>
<td>16</td>
<td>(u)</td>
<td>750</td>
<td>200</td>
</tr>
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<td>12a</td>
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### Nf211.64c (disc.)

Computer architecture (GPUs) and special techniques allow calculations at the physical point. In this work:

- Hierarchical probing (HP)
- One-end trick for Twisted Mass Fermions (OET)
- Spin-color dilution (SCD)
- Deflation (D)

We study:

- Isovector combination \((u-d)\): only connected
- Flavor decompositions: both connected and disconnected
- Strange & charm contributions purely disconnected
OUTLINE

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Nucleon E/M Form Factors

\* $G_E$: slope of lattice data differs from experiments

\* $G_M$: small-$Q^2$ has improved slope ($T_{\text{sink}} = 1.6\text{fm}$)
**Nucleon E/M Form Factors**

- **$G_E$:** slope of lattice data differs from experiments
- **$G_M$:** small-$Q^2$ has improved slope ($T_{\text{sink}} = 1.6\text{fm}$)
- **Excited states investigations:** $T_{\text{sink}} = 1 - 1.6\text{fm}$

![Graphs showing $G_E$ and $G_M$ vs. $Q^2$](image)

- $Q^2 = 0.06\text{GeV}$
- $T_{\text{low}}$(summation)

![Diagram showing $T_{\text{low}}$(summation) and $M_{T_{\text{low}}}$](image)
Clear signal due to algorithmic advances

Tsink chosen based on excited states and quality of fits
Dipole fit: motivated by vector-meson pole contributions to FFs

\[ G_E(Q^2) = \frac{1}{(1 + Q^2/m_E^2)} , \quad G_M(Q^2) = \frac{G_M(0)}{(1 + Q^2/m_M^2)^2} , \quad \langle r_{E,M}^2 \rangle = \frac{12}{m_{E,M}^2} \]

z-expansion: model-independent, expected to model better the low-\(Q^2\)

\[ G_i(Q^2) = \sum_k a_k z(Q^2)^k , \quad z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut}}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut}}} , \quad t_{cut} = 4m_n^2 , \quad \langle r_{E,M}^2 \rangle = -\frac{6a_i^{E,M}}{4t_{cut}a_0^{E,M}} \]
Nucleon charged radii

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- Estimation of radii strongly depends on small Q²
- Large volume: access to Q² close to zero
Nucleon charged radii

- E/M radii well-studied by several lattice groups
- Agreement among several formulations, but quantities sensitive to systematics, thus one must examine fit methods
- Magnetic moment from $G_M(0)$
Nucleon charged radii

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- Magnetic moment from $G_M(0)$

Fewer studies for (light quark) disconnected contributions
Left: Data for $T_{\text{sink}} > 1.2\text{fm}$ compatible, but slope different
Right: Lattice data compatible with upper range of neutrino-nucleus cross sections (green band) and with MiniBooNE (red band)

Parametrization of lattice data: Dipole fit, $z$-expansion

Deviations on $\langle r_A \rangle$ from different methods
Light quark disconnected contributions are sizable

$G_s$ small but necessary in order to bring $G_A^{\text{total}}$ close to experimental value

$G_{p}^{u+d,\text{DI}}$ cancels (within uncertainties) $G_{p}^{u+d,\text{CI}}$

$G_{p}^{s}$ suppressed compared to light quark contribution
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DIS experiment (1988) shows surprising results for proton spin

"... $g_1(x)$ for the proton has been determined and its integral over $x$ found to be $0.114 \pm 0.012 \pm 0.026$, in disagreement with the Ellis-Jaffe sum rule. ... These values for the integrals of $g_1$ lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon."

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We need a theoretical formulation to address the proton spin puzzle

Lattice QCD
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Lattice QCD

Spin Sum Rule (Ji)

$$\frac{1}{2} = \sum_q J_q^q + J_G^G = \sum_q \left( L_q^q + \frac{1}{2} \Delta \Sigma^q_q \right) + J_G^G$$

$L_q$: Quark orbital angular momentum

$\Delta \Sigma^q_q$: Intrinsic spin

$J_G$: Gluon spin
Spin structure from first principles

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Extraction from Lattice QCD

\[
J^q = \frac{1}{2} \left( A^q_{20} + B^q_{20} \right), \quad L^q = J^q - \Sigma^q, \quad \Sigma^q = g_A^q
\]

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$$J^q = \frac{1}{2} \left( A^q_{20} + B^q_{20} \right), \quad L^q = J^q - \Sigma^q, \quad \Sigma^q = g^q_A$$

**Necessary computations:**

- Axial Charge
- Quark momentum fraction
- Gluon momentum fraction
The proton spin from LQCD

Axial charge

Excited states:

- $g_A$ u-d: non-negligible
- $\langle x \rangle_{u-d}$: sizable
- $\langle x \rangle_g$: negligible
The proton spin from LQCD

Axial charge

Excited states:

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The proton spin from LQCD

Axial charge

Quark momentum fraction

Gluon momentum fraction
The proton spin from LQCD

Axial charge

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  - u-d: non-negligible
  - \( \langle x \rangle_{u-d} \): sizable
  - \( \langle x \rangle_g \): negligible

Flavor decomposition:
- Similar quality results for isoscalar and disconnected contributions

Quark momentum fraction

Gluon momentum fraction

- ETMC, Twisted Mass+Clover, \( N_f=2 \)
- RQCD, Clover, \( N_f=2+1 \)
- ETMC, Twisted Mass, \( N_f=2+1+1 \)
- RBC/UKQCD, Domain Wall, \( N_f=2+1 \)
- ETMC, Twisted Mass, \( N_f=2 \)
- LHPC, Clover, \( N_f=2+1 \)
- ABM
- NNPDF
- MSTW

Excited states:
- \( g_A \): u-d
  - non-negligible
- \( \langle x \rangle_{u-d} \): sizable
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The proton spin from LQCD

Satisfaction of spin and momentum sum rule is not forced

Striped segments: valence quark contributions (connected)
Solid segments: sea quark & gluon contributions (disconnected)
Satisfaction of spin and momentum sum rule is not forced

Striped segments: valence quark contributions (connected)
Solid segments: sea quark & gluon contributions (disconnected)

Better understanding of the spin distribution

Designed by Z.-E. Meziani

The proton spin from LQCD

We study volume and quenching effects

- Need of $T_{\text{sink}} > 1.3\text{fm}$ to find agreement with experiment
- Volume effects within statistical uncertainties
- Currently increasing statistics for $T_{\text{sink}} = 1.5, 1.7\text{fm}$

Results PRELIMINARY:
- More statistics to collect
- Finalize analyses
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Several ideas how to access PDFs on Lattice

- **Fictitious heavy quark** [W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
- **Higher moments** [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
- **Compton amplitude and OPE** [A. Chambers et al. (QCDSF), arXiv:1703.01153]
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All methods have been investigated on the lattice
Based on matrix elements of spatial operators

\[ \tilde{q}(x, \mu^2, P_3) = \int \frac{dz}{4\pi} e^{-ixP_3z} \langle N(P_3) | \bar{\Psi}(z) \gamma^z A(z,0) \Psi(0) | N(P_3) \rangle \mu^2 \]

\( A(z,0) \): Wilson Line of length \( z \)

Hadron boosted with momentum in spatial direction

\( \tilde{q} \): quasi-PDFs
Reconstruction of light-cone PDFs

Contact with light-cone PDFs feasible:

- Difference reduced as $P$ increases

\[ O \left( \frac{\Lambda_{\text{QCD}}^2}{P_3^2}, \frac{m_N^2}{P_3^2} \right) \]

- Matching procedure in large momentum EFT (LaMET) to relate quasi-PDFs to light-cone PDF

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Reconstruction of light-cone PDFs

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- Matching procedure in large momentum EFT (LaMET) to relate quasi-PDFs to light-cone PDF

First exploratory (nucleon) studies feasible:


Lattice studies of quasi-PDFs
Calculations significantly improved...
Lattice studies of quasi-PDFs

Calculations significantly improved...

[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860],
[Y.-Q. Ma et al., arXiv:1412.2688], [C. Alexandrou et al., arXiv:1504.07455], [H.-N. Li et al., arXiv:1602.07575],
[C. Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726],
[C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193],
[C.Alexandrou et al., arXiv:1803.02685], [J-W Chen et al, arXiv:1803.04393], [C.Alexandrou et al., arXiv:1807.00232], ...
Lattice studies of quasi-PDFs

Calculations significantly improved…

[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860],
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[C. Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726],
[C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193],

… and extended to other hadrons

Recent review: C. Monahan @ Lattice 2018
Lattice studies of quasi-PDFs

2014-15

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Lattice studies of quasi-PDFs

- Simulations at physical point
- Renormalization
- Matching

2014-15

2018

$u - d$

$x$

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Nf=2 twisted mass fermions & clover term

Ensemble parameters:

<table>
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<tr>
<th>$\beta=2.10$, $c_{SW}=1.57751$, $a=0.0938(3)(2)$ fm</th>
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<tr>
<td>$48^3 \times 96$  $a\mu = 0.0009$, $m_N = 0.932(4)$ GeV</td>
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<tr>
<td>$L = 4.5$ fm      $m_\pi = 0.1304(4)$ GeV, $m_\pi L = 2.98(1)$</td>
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Parameters of Calculation

[C. Alexandrou et al., (PRL), arXiv:1803.02685], [C. Alexandrou et al., arXiv:1807.00232]

* Nf=2 twisted mass fermions & clover term
* Ensemble parameters:

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<tr>
<td>$L$</td>
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<tr>
<td>$a\mu$</td>
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* Nucleon momentum & statistics:

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<th>$P = \frac{6\pi}{L}$ (0.83 GeV)</th>
<th>$P = \frac{8\pi}{L}$ (1.11 GeV)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Ins.</td>
<td>$N_{\text{conf}}$</td>
<td>$N_{\text{meas}}$</td>
</tr>
<tr>
<td>------</td>
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<td>------------------</td>
</tr>
<tr>
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<td>9600</td>
</tr>
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</tr>
<tr>
<td>$\gamma_5 \gamma_3$</td>
<td>65</td>
<td>6240</td>
</tr>
</tbody>
</table>
Parameters of Calculation

[C. Alexandrou et al., (PRL), arXiv:1803.02685], [C. Alexandrou et al., arXiv:1807.00232]

∗ Nf=2 twisted mass fermions & clover term

∗ Ensemble parameters:

| $\beta$=2.10, | $c_{SW}$=1.57751, | $a$=0.0938(3)(2) fm |
| $48^3 \times 96$ | $\mu = 0.0009$ | $m_N = 0.932(4)$ GeV |
| $L = 4.5$ fm | $m_\pi = 0.1304(4)$ GeV | $m_\pi L = 2.98(1)$ |

∗ Nucleon momentum & statistics:

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∗ Excited states investigation:

$T_{sink} = 8a, 9a, 10a, 12a \quad (T_{sink} = 0.75, 0.84, 0.94, 1.13$ fm)
Challenges of calculation

Noise-to-signal ratio increases with:
- Hadron momentum boost
- Simulations at the physical point
- Source-sink separation
Challenges of calculation

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Noise problem must be tamed to investigate uncertainties
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Noise problem must be tamed to investigate uncertainties

Momentum smearing
[G. Bali et al., PRD93, 094515 (2016)]

Momentum smearing helps reach higher momenta
Challenges of calculation

Noise-to-signal ratio increases with:
- Hadron momentum boost
- Simulations at the physical point
- Source-sink separation

Noise problem must be tamed to investigate uncertainties

Momentum smearing helps reach higher momenta
But limitations in max momentum due to comput. cost

M. Constantinou
Bare matrix elements

Unpolarized:
- Initial studies used $\gamma^\mu$ in same direction with Wilson line
- Mixing with higher twist revealed perturbatively
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Abandoned
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Unpolarized:
★ Initial studies used $\gamma^\mu$ in same direction with Wilson line
★ Mixing with higher twist revealed perturbatively
★ No mixing for $\gamma^0$ (perpendicular to Wilson line)

Abandoned

★ Similar general features for polarized and transversity
★ Highest priority: deliver reliable results

M. Constantinou
Excited states contamination

Analyses techniques:

◆ Single-state fit,
◆ Two-state fit, \( P = 1.4 \text{ GeV} \)
◆ Summation method

Conclusions:

◆ \( t_{\text{sink}} = 8a \) heavily contaminated by excited states
◆ \( t_{\text{sink}} = 9a - 10a \) not consistent within uncertainties

M. Constantinou
Excited states contamination

Analyses techniques:
* Single-state fit,          Two-state fit,           Summation method

Conclusions:
* Tsink=8a heavily contaminated by excited states
* Tsink=9a-10a not consistent within uncertainties
* Crucial to have same error for reliable 2-state fit
* Excited states worsen as momentum $P$ increases
* For momenta in this work, Tsink=1fm is safe
Non-predictible behavior (depends in $z$ value)
Real and imaginary part affected differently

Conclusions:

• Excited states uncontrolled for $T_{sink} < 1$ fm
• Multi-sink analysis demands same accuracy for all data
Towards light-cone PDFs

Evolution of lattice data (P=1.4GeV):
Towards light-cone PDFs

Evolution of lattice data ($P=1.4$ GeV):

- Fourier Transform of renormalized matrix elements

Unpolarized

\[ \tilde{q} \]

\[ u - d \]

Polarized

\[ \Delta \tilde{q} \]

\[ \Delta u - \Delta d \]
Towards light-cone PDFs

Evolution of lattice data ($P=1.4$GeV):
- Fourier Transform of renormalized matrix elements
- Matching of quasi-PDFs (LaMET)

[C. Alexandrou et al., (PRL), arXiv:1803.02685]
Towards light-cone PDFs

Evolution of lattice data ($P=1.4\,\text{GeV}$):

* Fourier Transform of renormarmalized matrix elements
* Matching of quasi-PDFs (LaMET)
  [C. Alexandrou et al., (PRL), arXiv:1803.02685]
* Target Mass Corrections ($m_N/P$: finite)
Towards light-cone PDFs

Evolution of lattice data (P=1.4GeV):

- Fourier Transform of renormalized matrix elements
- Matching of quasi-PDFs (LaMET) [C. Alexandrou et al., (PRL), arXiv:1803.02685]

Lattice PDFs similar behavior as phenomenological fits
Towards light-cone PDFs

Nucleon boost dependence:

Unpolarized

Polarized

Increasing momentum approaches the phenomenological fits a saturation of PDFs for $p=8\pi/L$ and $p=10\pi/L$

$0<x<0.5$: Lattice polarized PDF overlap with phenomenology

Negative $x$ region: anti-quark contribution

$x\sim 1$: affected by finite nucleon momentum
Towards light-cone PDFs

Nucleon boost dependence:

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Increasing momentum approaches the phenomenological fits, leading to a saturation of PDFs for $p=8\pi/L$ and $p=10\pi/L$.

$0<x<0.5$: Lattice polarized PDF overlap with phenomenology.

Negative $x$ region: anti-quark contribution.

$x \sim 1$: affected by finite nucleon momentum.
Mild dependence on nucleon momentum

Integral of PDF ($g_T=1.09(11)$) compatible with results from moments  

[C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]
Mild dependence on nucleon momentum

Integral of PDF ($g_T = 1.09(11)$) compatible with results from moments [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]

Lattice data from quasi-PDFs more accurate than SIDIS

SIDIS improved with $g_T^{\text{Lat}}$ constraints, but *ab initio* quasi-PDFs statistically more accurate
A. Introduction

B. Nucleon on the Lattice

C. Nucleon Form Factors
   1. Electromagnetic
   2. Axial

D. Proton Spin

E. Access to x-dependence of PDFs

F. Discussion
**Summary:**

- **Simulations at the physical point for nucleon structure**
- **Investigations of systematic uncertainties:**
  - $g_A$ agreement with experiment for $T_{\text{sink}} \sim 1.5 \text{fm}$
  - Slope of Axial & E/M form factors sensitive to $T_{\text{sink}}$
- **Disconnected contributions to FFs computed for $u$, $d$, $s$:**
  - Necessary to bring $g_{u+d}$ in agreement with experiment
  - Large contributions to $G_{u+d}$ that partly cancels connected part
  - Strange contributions to FFs non-negligible
- **Significant progress on other direction (quasi-PDFs)**
Future Investigations:

**FFs & proton spin**
- Increase statistics and addition of separations larger than 1.5fm
- Two additional 2+1+1 ensembles (physical point, same physical volume)

**quasi-PDFs**
- Increase of momentum seems a natural next step
  - **BUT** is a major challenge if reliable results is the goal
- Other directions should be pursued, e.g. 2-loop matching
THANK YOU
Pion mass dependence

Simulations at physical $m_\pi$ crucial for above conclusions

Unpolarized
Pion mass dependence

Simulations at physical $m_\pi$ crucial for above conclusions

$\begin{align*}
6\pi/32, & \quad 8\pi/32, & \quad 10\pi/32, & \quad CJ15, & \quad ABMP16, & \quad NNPDF3.1 \\
6\pi/32, B55, & \quad 10\pi/48, \text{ phys.point}, & \quad \text{CJ15}, & \quad \text{ABMP16}, & \quad \text{NNPDF3.1}
\end{align*}$

$u - d$:

$m_\pi = 375$ MeV:
Lattice data saturate away from phenomenology.
Pion mass dependence

Simulations at physical $m_\pi$ crucial for above conclusions

Unpolarized

$m_\pi = 375$ MeV:
Lattice data saturate away from phenomenology

$m_\pi = 132, 375$ MeV
Significant pion mass dependence
Alternative Fourier

* Standard Fourier (SF) :

\[ \tilde{q}(x) = 2P_3 \int_{z_{\text{max}}}^{\pm z_{\text{max}}} \frac{dz}{4\pi} e^{ixP_3} h(z) \]

can be written using integration by parts (DF):

\[ \tilde{q}(x) = h(z) \frac{e^{ixP_3}}{2\pi ix} \bigg|_{z_{\text{max}}}^{\pm z_{\text{max}}} - \int_{z_{\text{max}}}^{\pm z_{\text{max}}} \frac{dz}{2\pi} e^{ixP_3} h'(z) \]

[H.W. Lin et al., arXiv:1708.05301]

- Truncation at \( z_{\text{max}} \) (SF) vs neglecting surface term (DF)
  (latter non-negligible numerically)
- Oscillations reduced for DF, but small-x not well-behaved
- SF, DF different systematics, but DF uses interpolated data instead of raw ME ⇒ enhanced cut-off effects
Renormalized ME have no dependence on the stout smearing